A Locally Adaptive Normal Distribution — Errata —

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This document provides a list of issues for the paper "A Locally Adaptive Normal Distribution" [1]. Please contact Georgios Arvanitidis in case you find further mistakes.

In a Riemannian manifold the covariance matrix is related to the precision matrix as

$$\boldsymbol{\Sigma} = \int_{\mathcal{M}} \text{Log}_{\boldsymbol{\mu}}(\mathbf{x})\text{Log}_{\boldsymbol{\mu}}(\mathbf{x})^{\intercal} p_{\mathcal{M}}(\mathbf{x} \mid \boldsymbol{\mu},\, \boldsymbol{\Gamma})\mathrm{d}\mathcal{M}(\mathbf{x}), \tag{1}$$

where $\boldsymbol{\Gamma}$ is the precision matrix, under the normal distribution

$$p_{\mathcal{M}}(\mathbf{x} \mid \boldsymbol{\mu},\, \boldsymbol{\Gamma}) = \frac{1}{\mathcal{C}(\boldsymbol{\mu}, \boldsymbol{\Gamma})} \exp\left(-\frac{1}{2}\langle \text{Log}_{\boldsymbol{\mu}}(\mathbf{x}), \boldsymbol{\Gamma} \cdot \text{Log}_{\boldsymbol{\mu}}(\mathbf{x})\rangle\right). \tag{2}$$

In the case of a flat manifold the $\boldsymbol{\Gamma} = \boldsymbol{\Sigma}^{-1}$. So in the main paper, the term "covariance matrix" related to the distribution $p_{\mathcal{M}}(\mathbf{x} \mid \boldsymbol{\mu},\, \boldsymbol{\Gamma})$ should be instead considered as "precision matrix".

The ODE which defines the geodesic curve is wrong, and the correct one is [2]

$$\ddot{\gamma}(t) = -\frac{1}{2}\mathbf{M}^{-1}(\gamma(t))\left[2(\mathbb{I}_d \otimes \dot{\gamma}(t)^{\intercal})\frac{\partial \text{vec}[\mathbf{M}(\gamma(t))]}{\partial \gamma(t)}\dot{\gamma}(t) - \frac{\partial \text{vec}[\mathbf{M}(\gamma(t))]}{\partial \gamma(t)}^{\intercal}(\dot{\gamma}(t) \otimes \dot{\gamma}(t))\right]. \tag{3}$$

However, in the diagonal Riemannian metric case the difference in the resulting logarithmic map is negligible, especially, when the two points are close. An empirical example can be seen in Fig. 1.

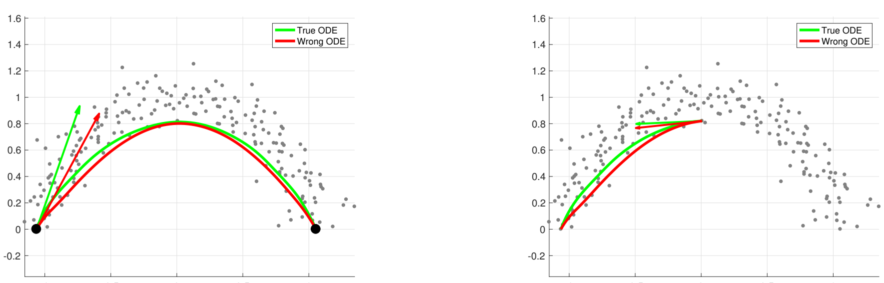

Figure 1: Comparison of the true vs the wrong ODE system.

References

[1] Georgios Arvanitidis, Lars Kai Hansen and Søren Hauberg, "A Locally Adaptive Normal Distribution", Advances in Neural Information Processing Systems (NIPS), 2016.

[2] Georgios Arvanitidis, Lars Kai Hansen and Søren Hauberg, "Latent Space Oddity: on the Curvature of Deep Generative Models", International Conference on Learning Representations (ICLR), 2018.