Improved Approximate String Matching and Regular Expression Matching on Ziv-Lempel Compressed Texts

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Approximate String Matching

- The *edit distance* between two strings is the minimum number of insertions, deletions, and substitutions needed to convert one string to the other. E.g., edit-distance("cocoa", "cola") = 2.

- Let P and Q be strings and let k (integer > 0) be an *error threshold*.

- The approximate string matching problem is to find all ending positions of substrings in Q whose edit distance to P is at most k.

Results

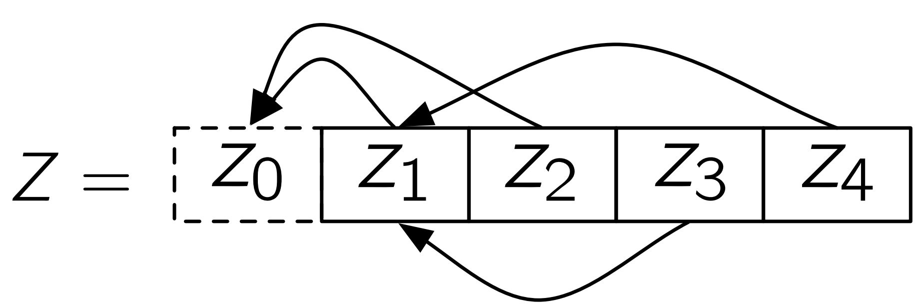
Time	Space	Reference
$O(um)$	$O(m)$	[Sellers1980]
$O(uk)$	$O(m)$	[LV1989]
$O\left(\frac{uk^4}{m} + u\right)$	$O(m)$	[CH2002]

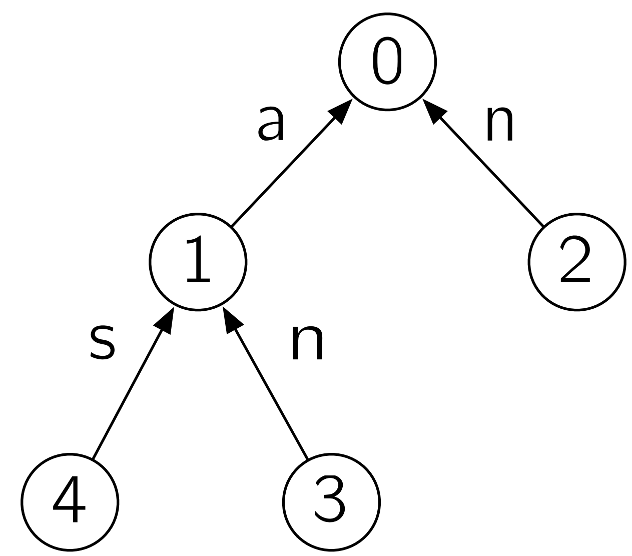
$|P| = m$ and $|Q| = u$

Ziv-Lempel 1978 compression

$Q = \text{ananas}$

$Z = (0,\text{a})(0,\text{n})(1,\text{n})(1,\text{s})$





Approximate String Matching on ZL78 compressed texts

- Let P be a string and Z be a ZL78 compressed representation of a string Q.
- Given P and Z, the *compressed approximate string matching problem* is to solve the approximate string matching for P and Q without decompressing Z.
- Goal: Do it more efficiently than decompressing Z and using the best (uncompressed) approximate string matching algorithm.

Applications

- Textual data bases (e.g. DNA sequence collections) issues:
  - Save space = keep data in compressed form.
  - Search efficiently.
- Solution: Compressed string matching algorithms.

Results

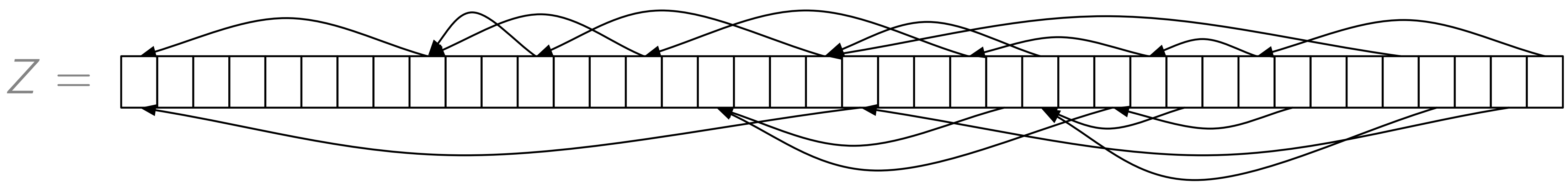
- Let $|P| = m$ and $|Z| = n$.
- Kärkkäinen, Navarro, and Ukkonen [KNU2003]:
    - $O(nmk + \mathrm{occ})$ time and $O(nmk)$ space.
- Our result (Theorem 1): For any parameter $\tau \geq 1$:
    - $O(n(\tau + m + t(m, 2m + 2k, k)) + \mathrm{occ})$ expected time and
    - $O(n/\tau + m + s(m, 2m + 2k, k)) + \mathrm{occ})$ space.

Example Results

<table>
<tr><th>Time</th><th>Space</th><th colspan="2">Reference</th></tr>
<tr><td>$O(nmk + \mathrm{occ})$</td><td>$O(nmk)$</td><td colspan="2">[KNU2003]</td></tr>
<tr><td>$O(nmk + \mathrm{occ})$</td><td>$O\left(\frac{n}{mk} + m + \mathrm{occ}\right)$</td><td>LV +
$\tau = mk$</td><td rowspan="2">This paper</td></tr>
<tr><td>$O(nk^4 + nm + \mathrm{occ})$</td><td>$O\left(\frac{n}{k^4 + m} + m + \mathrm{occ}\right)$</td><td>CH +
$\tau = k^4 + m$</td></tr>
</table>

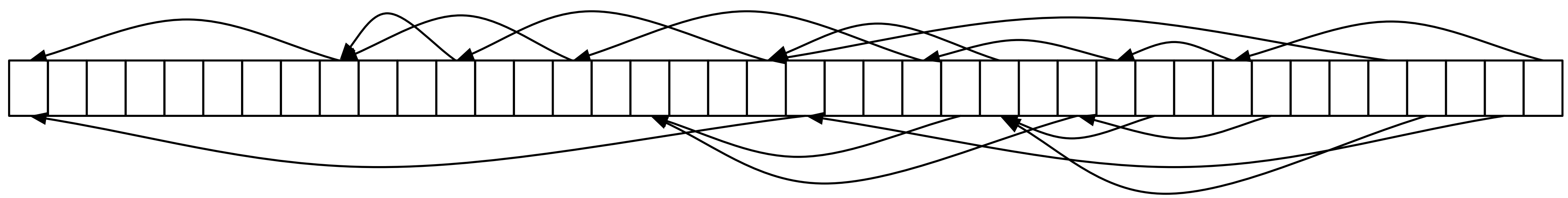
$|P| = m$ and $|Z| = n$

Selecting Compression Elements



- For parameter $\tau \geq 1$, select a subset C of the compression elements of Z such that:
  - $|C| = O(n/\tau)$.
  - From any compression element z_i, the distance (minimum number of references) to any compression element in C is at most 2τ.

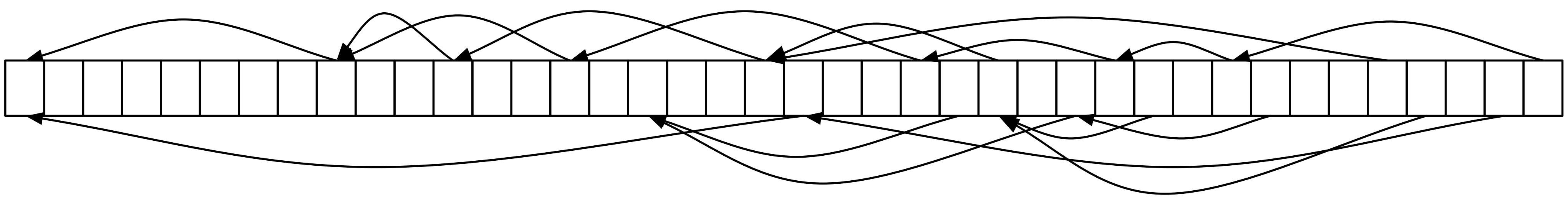
Selecting Compression Elements

$Z =$ 

- Maintain C using dynamic perfect hashing while scanning Z from left-to-right.
- Initially, set $C = \{z_0\}$.
- To process element z_{i+1} follow references until we encounter $y \in C$:
  - If the distance l from z_{i+1} to y is less than 2τ we are done.
  - Otherwise ($l = 2\tau$), insert element the element at distance τ into C.

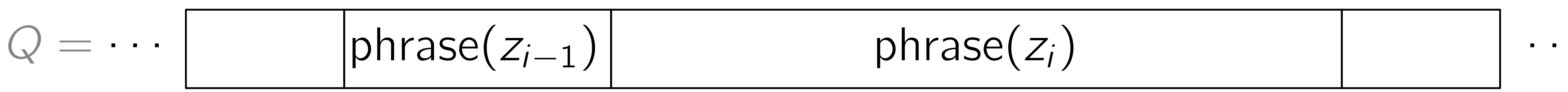
Selecting Compression Elements

$Z =$



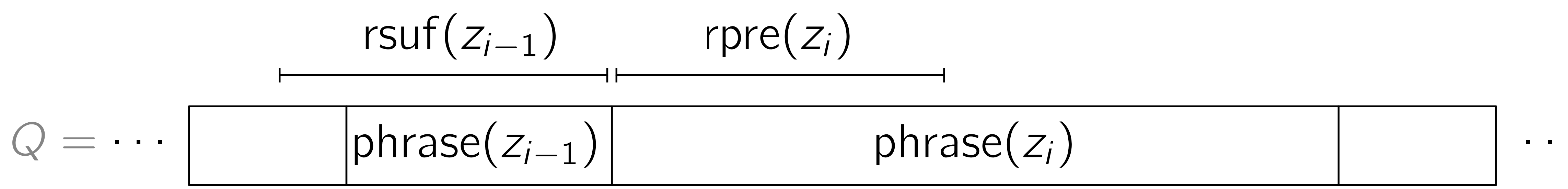
- Lemma: For any parameter $\tau \geq 1$, C is constructed in
  - $O(n\tau)$ expected time and
  - $O(n/\tau)$ space.

Computing Matches



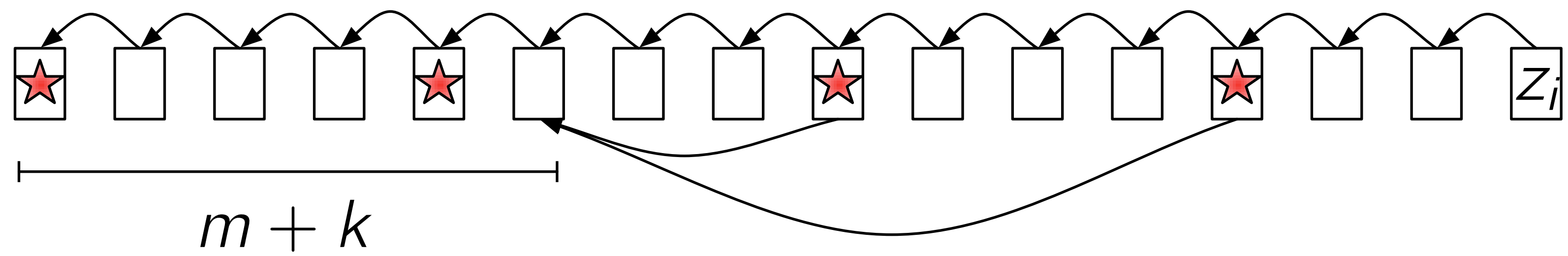
- Strategy:
  - Process Z from left-to-right.
  - At z_i we compute all matches ending in the substring encoded by z_i.

Computing Overlapping Matches



- Let $[u_i, u_i + l_i - 1]$ be the positions in Q of $\text{phrase}(z_i)$.
- Goal: Find all *overlapping matches* for z_i, i.e., the matches starting before u_i and ending in $[u_i, u_i + l_i - 1]$.
- Decompress substrings $\text{rpre}(z_i)$ and $\text{rsuf}(z_{i-1})$ of length $m + k$ around u_i.
- Run favorite (uncompressed) approximate string matching algorithm to find matches of P in $\text{rsuf}(z_{i-1}) \cdot \text{rpre}(z_i)$. Add offset to these to get the overlapping matches for z_i.

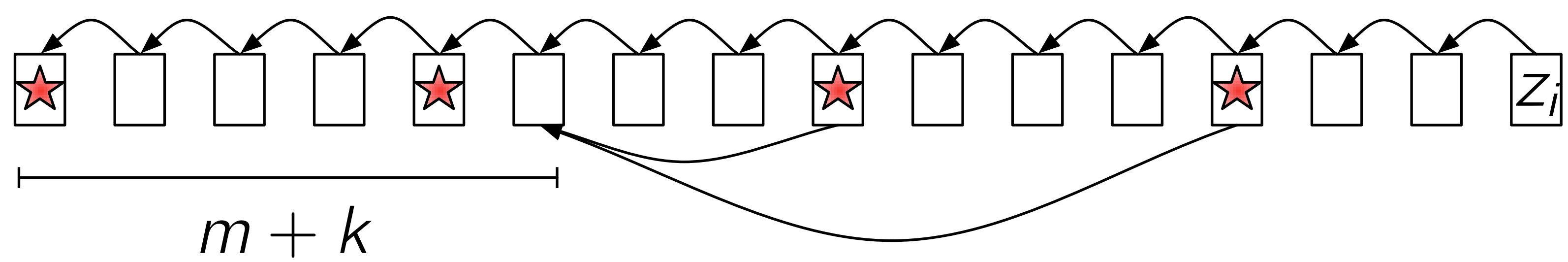
Computing the Relevant Prefix and Suffix



- For parameter $\tau \geq 1$, select a subset C of the compression elements of Z according to Lemma 1.

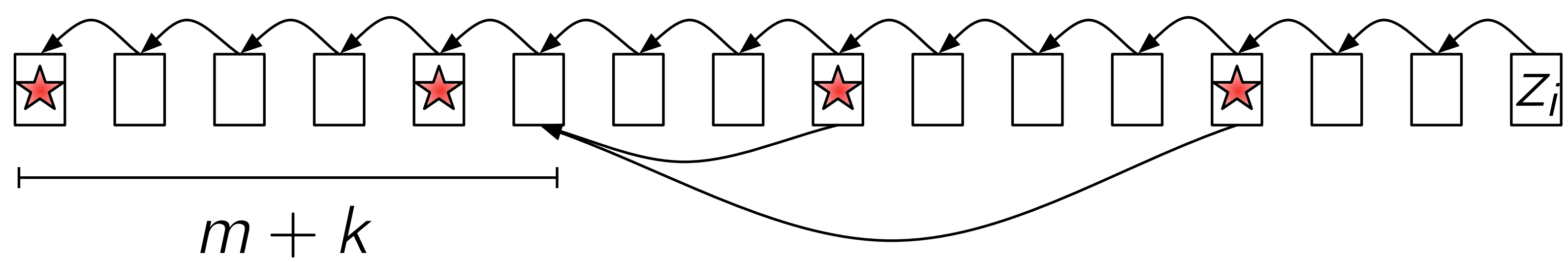
- For each element in C at distance more than $m + k$ from z_0 add “shortcut” to element at distance $m + k$.

Computing the Relevant Prefix



- Follow references to nearest element in C.
- Follow shortcut if present.
- Compute the relevant prefix by decompressing length $m + k$ substring.

Computing the Relevant Suffix



- Follow references to decompress substring of length $m + k$.

- If the phrase is shorter than $m + k$, recursively apply to z_{i-1} until we have $m + k$ characters.

Analysis

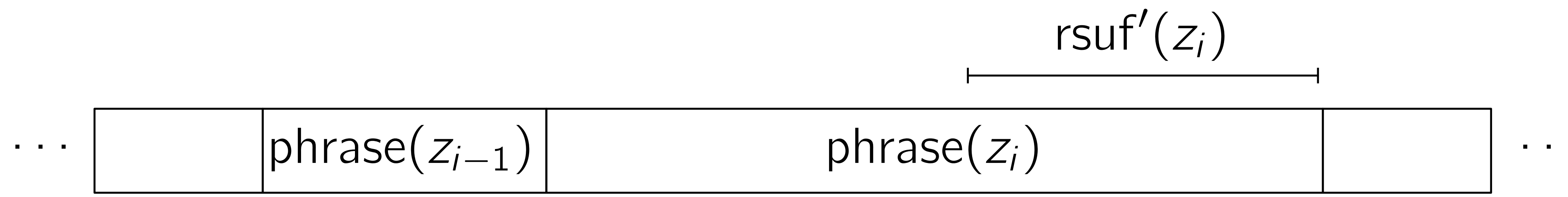
- Time = preprocess + n(find nearest element + decompress + match) =

$$O(n\tau + n(\tau + m + t(m, 2m + 2k, k))$$

- Space = preprocess + decompress + match =

$$O(n/\tau + m + s(m, 2m + 2k, k))$$

Computing Internal Matches



- Goal: Find all *internal matches* for z_i, i.e., all matches starting and ending within $[u_i, u_i + l_i - 1]$.

- Compute and store all the internal match sets indexed by compression elements using dynamic perfect hashing.

- Decompress substring $\text{rsuf}'(z_i)$ of length $\min(l_i, m + k)$ ending at $u_i + l_i - 1$.

- Internal matches for z_i =
  (internal matches for $\text{reference}(z_i)$) $\bigcup$ (matches of P in $\text{rsuf}'(z_i)$)

Analysis

- Time = n (decompress + match + internal matches) =

$$O(n(m + t(m, m + k, k)) + \mathrm{occ})$$

- Space = decompress + match + total number of internal matches =

$$O(m + s(m, m + k, k) + \mathrm{occ})$$

Putting the Pieces Together

- Merging overlapping and internal matches we get *all* matches for z_i ending within $[u_i, u_i + l_i - 1]$.
- Implies Theorem 1: For any parameter $\tau \geq 1$:
  - $O(n(\tau + m + t(m, 2m + 2k, k)) + \mathrm{occ})$ expected time and
  - $O(n/\tau + m + s(m, 2m + 2k, k)) + \mathrm{occ})$ space.
- Does not hold for ZLW compressed texts, unless $\Omega(n)$ space is used.
- For $\Omega(n)$ space the bounds hold in the worst-case and work for both ZL78 and ZLW.

Regular Expression Matching

- A *regular expression* is a generalized pattern composed from simple characters using union, concatenation, and Kleene star.

- Given a regular expression R and a string Q the *regular expression matching problem* is to find all ending positions of substrings in Q that matches a string in the language generated by R.

Regular Expression Matching

- Let $|R| = m$ and $|Q| = u$.
- Classic solution [Thompson1968]: $O(um)$ time and $O(m)$ space.
- Several improvements based on the Four-Russian technique or word-level parallelism [Myers1992, NR2004, BFC2005, Bille2006].

Compressed Regular Expression Matching

- Let $|R| = m$ and $|Z| = n$.
- Navarro [Navarro2003] simplified and without word-level parallel techniques:
  - $O(nm^2 + \mathrm{occ} \cdot m \log m)$ time and $O(nm^2)$ space.
- Our result (Theorem 2): For any parameter $\tau \geq 1$:
  - $O(nm(m + \tau) + \mathrm{occ} \cdot m \log m)$ time and
  - $O(nm^2/\tau + nm)$ space.
- E.g. $\tau = m$ gives $O(nm^2 + \mathrm{occ} \cdot m \log m)$ time and $O(nm)$ space.

Remarks

- Compressed strings are large and therefore $\Omega(n)$ space space may not be feasible for large texts.
- Our result for compressed approximate string matching is one of the very few algorithms for compressed matching that uses $o(n)$ space.
- More sublinear space compressed string matching algorithms are needed!