Approximate Message Passing

Exercise 1

Implement a Matlab script for generating measurement from random Gaussian design matrix $\mathbf{A} \in \mathbb{R}^{M \times N}$, k-sparse vector $\mathbf{x} \in \mathbb{R}^M$ and additive Gaussian noise $\mathbf{w} \in \mathbb{R}^M$. Choose the parameters, $N > M > k$. Implement the AMP algorithm as in slide 20 of the lecture.
Try to answer the following questions by simulations:

- Implement the state/evolution. Does it require the ground-truth? Does the prediction (State evolution) follow the simulation?
- What happens if the design matrix $\mathbf{A}$ is not zero-mean?
- What happens if you run the algorithm without the Onsager-term? Does it follow the state evolution?
- Use a piece-wise image as input from (?) and implement/use the proximal TV operator in Matlab for the non linear input function $\eta(\cdot)$. Can you find and analytic solution for the variance $< \eta' >$? Otherwise try using Monte Carlo method in slide 43. Does the algorithm converge? Are there any free parameters in the simulation? (The code for the proximal operator code and the input image will be provided).

Exercise 2

Implement a Matlab script for generating measurement from random Gaussian design matrix $\mathbf{A} \in \mathbb{R}^{M \times N}$, k-sparse vector $\mathbf{x} \in \mathbb{R}^M$ and additive Gaussian noise $\mathbf{w} \in \mathbb{R}^M$. Choose the parameters, $N > M > k$. Implement the SVD-VAMP algorithm as in slide 45 of the lecture.
Try to answer the following questions:

- Simulate the model with k-sparse input signal and Gaussian noise as in Exercise 1. Is the MSE vs iterations plot of SVD-AMP different from AMP?
- What happens if $\mathbf{A}$ is generated as random row sub/selection of a Fourier matrix?
- Generate a sparse/view sinogram with AIRTool by randomly sub-selecting the full view sinogram and use the SVD decomposition of the forward operator. Run the SVD-VAMP algorithm.