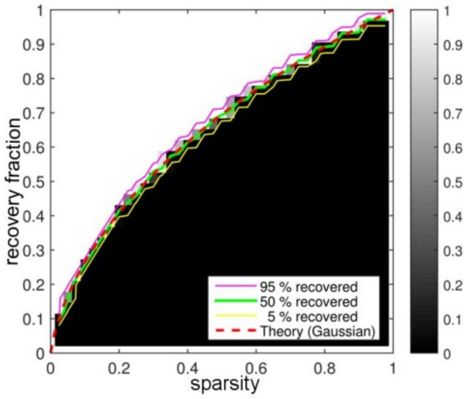
Summary of the Major Achievements

March 22, 2015

The project is divided into 5 scientific tasks A: “Catalogue” of Prior Information, B: Incorporation of Prior Information, C: Sparse Approximation, D: Large-Scale Algorithms, and E: Software, Test, and Validation. Our work covers all 5 tasks, and many research activities relate to more than one of them. A common theme is that we consider *underdetermined problems* with few projections or measurements; with enough data any classical reconstruction method can give good reconstructions, while in the limited-data case it is essential to use prior information to compensate for the lack of data.

The Role of Sparsity in CT (tasks B and C; joint work with Univ. of Chicago, Technical Univ. of Braunschweig)

Reliable imaging from few projections (to reduce X-ray dose) is an important example of how a prior (here, sparsity) can significantly improve CT reconstruction. We published a trilogy of papers which, using extensive carefully designed computer simulations, determine the critical amount of data needed for accurate reconstruction of sparse images. We established a so-called phase-transition phenomenon, i.e., how the critical amount of data depends on the image sparsity. An underlying theory known as Compressed Sensing (CS) exists only for special randomized sampling setups involving Gaussian matrices, however our results demonstrate similar behavior in CT. Our preliminary results show that phase transitions can accurately predict the sufficient amount of data in large-scale CT.

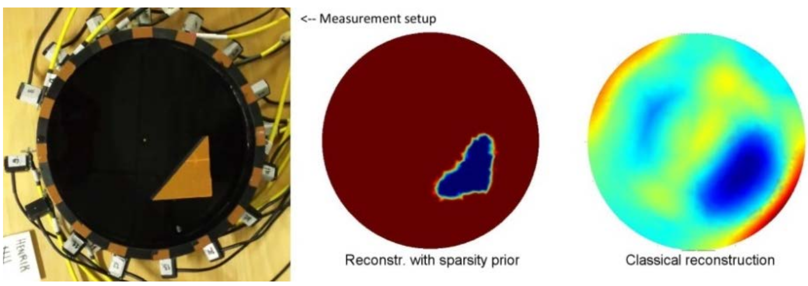


The figure on the right shows that in parallel-beam CT, the fraction of accurately reconstructed images at a given sparsity abruptly changes from 0 to 1 once a critical number of measurements is reached. Surprisingly, this agrees almost perfectly with the theoretical phase transition for Gaussian matrices. Ongoing work seeks to extend the CS theory to explain such observations, and involve further investigation of how to use the phase transition to accurately predict the sufficient amount of data in real-world CT scanners.

Sparsity Priors in EIT (tasks B, C, and D; joint work with Helsinki Univ. and Univ. of Eastern Finland)

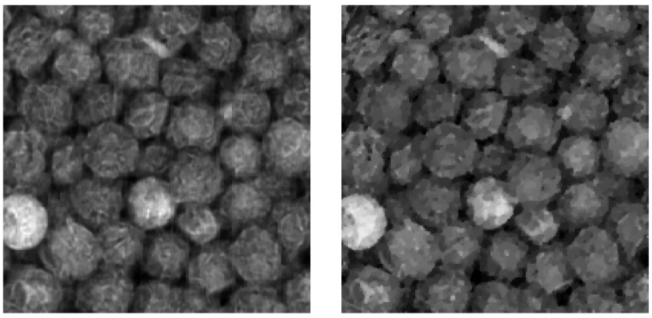
Precise localization and correct intensity in computational EIT is a challenging problem. In two submitted papers we demonstrate (by simulations) that prior information about sparsity of the solution greatly improves the reconstruction, and that our 1-norm prior works better than a total variation prior. The improvements are especially pronounced for situations where only partial boundary data are available.

We use a Tikhonov formulation where the sparsity is enforced via a spatially varying regularization parameter. This also allows the use of spatial information that can be found directly from the data via factorization and monotonicity methods. Our studies involve both 2D and 3D problems in a FEM formulation, and for the latter situation we developed large-scale simulation code for the continuum model and the complete electrode model. Our algorithms were tested on real data showing that they are superior to the classical methods.



Training Images as Priors (tasks A and B; joint work with Tufts Univ. and Univ. of Helsinki)

Some priors take the form of cross-section images of the object, and this information must be used in a fast, reliable, and computationally efficient manner. We developed an algorithmic framework for this: From a set of training images we use techniques from machine learning to form a dictionary that captures the desired features, and we then compute a reconstruction with a sparse representation in this dictionary. Simulations show that for textural images our approach is superior to other methods used for limited-data situations.



Reconstr. using training images   Reconstr. using total variation

Our main contribution is a careful study of how to stably compute a dictionary through a regularized non-negative matrix or tensor factorization, and how this dictionary affects the reconstruction quality. We also demonstrate the advantage of using a tensor formulation of the problem, which is more natural for working with dictionaries of images than a standard matrix formulation, and which leads to much sparser representations because tensors better allow for identifying spatial coherence in the training images. The figure shows that our method outperforms total variation that fails to capture the texture.

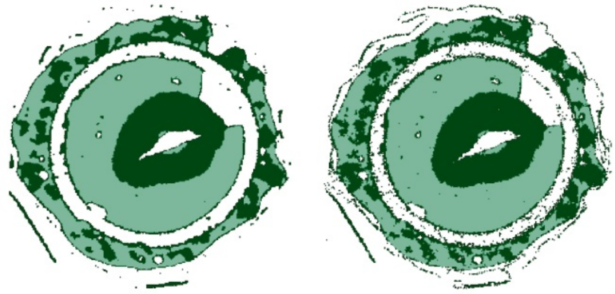
Large-Scale Reconstruction Methods (tasks D and E; joint work with DTU Computing Center, CWI in Amsterdam, Linköping Univ., and Iran Univ. of Science and Technology)

This work has two aspects – as support for our other HD-Tomo activities, and as research in its own right. We wish to highlight the following achievements:

- We developed and implemented a new, efficient, and robust algorithm for incorporating subspace priors in a Krylov subspace iterative method, improving on an existing and flawed approach.
- We established a link between incremental gradient (first-order) optimization methods and classical row-action methods. This enables us to incorporate priors such as TV and to handle other problems – for example, with a new step length strategy the ART algorithm can solve problems with Poisson noise.
- We studied the convergence and implementation of row-action methods. In particular we obtained rigorous insight into the semi-convergence of ART, and – through implementations of several block algorithms on multi/many-core computers – we decided which algorithm to include in our software.

Simultaneous Reconstruction and Segmentation (task A and B)

TV + classical segmentation   Our combined method



We develop formulations and algorithms for simultaneous reconstruction and segmentation, where our prior takes the form of a hidden Markov measure field for the segmentation classes, and we show the stabilizing effect of this combined approach. Experiments with real data (see the figure) show that our approach is better able to identify thin structures in the image than the classical approach, where reconstruction and segmentation are two separate processes.

Novel and/or unconventional methodologies

At the midway point of the project, it is clear that the most important themes for our research are associated with the following analytical and computational state-of-the-art techniques:

- Various aspects of **sparsity** as a means to impose priors about "simplicity" on the reconstruction, either through an analysis operator such as TV or through synthesis via sparse representation/approximation.
- Variational formulations and **convex optimization** to formulate and solve the computational large-scale optimization problems through first-order methods.
- Elements of **machine learning** to extract and represent priors that do not conform to a strictly analytical or variational formulation.
- Techniques for specific **non-Gaussian noise** models where priors from reference data provide well-defined denoising models and stable algorithms

Inter and cross disciplinary developments

HD-Tomo has initiated two other research projects in tomography: a project with DTU Physics on 6D reconstructions in materials science, and a Danish research grant for impedance tomography with hybrid data. Also we started collaboration with the ASTRA software developers on block algebraic iterative methods.

Knowledge and technology transfer

Knowledge transfer takes the classical forms of courses, workshops, and conference attendance.

- Workshop: *Sparse Tomo Days*, March 26-28.
- Five seminars by Prof. Bill Lionheart: *Sufficient Data for Stable Reconstruction*, *Ray Transforms*, *EIT for Beginners*, *Rich Tomography*, and *Inverse Problems in Security Screening*.
- Online short-course *Algebraic Iterative Reconstruction Methods.*
- PhD course *Discrete Inverse Problems*.

The PI, Per Christian Hansen, was invited speaker at:

- The COST workshop *Advanced X-Ray Tomography: Experiment, Modeling, and Algorithms*, Feb. 10-14, 2014.
- *Householder Symposium on Numerical Linear Algebra*, Spa, Belgium, June 8-13, 2014.
- *Oberwolfach Workshop on Mathematics and Algorithms in Tomography*, August 10-16, 2014.

Others

A formatted version of the report with hyperlinks and several figures is attached as a pdf file.

Keywords (alphabetical order)

Absorption CT
Algebraic iterative methods
Compressed sensing
EIT
First-order methods
ℓ_1 optimization
Large-scale algorithms
Limited data
Machine learning
Materials science
Non-Gaussian noise
Porous media
Segmentation
Sparsity
Spatial priors
Tensor factorization
Total variation
Training data

Publishable Brief Summary of the Main Achievements of the Project

Tomography is the science of "seeing inside objects" – we send signals through an object, and from measurements of the response we compute a 3D representation of the object's interior. In computed tomography (CT), we use the computer to synthesize an object's 3D structure from the measurements by solving millions of equations. A decisive factor behind the human vision system to recognize objects is the ability to use *prior information* – an organized accumulation of experience with other objects. The key to high-quality CT reconstruction is to get the computer to do the same, by means of advanced mathematics.

In the HD-Tomo project we develop the enabling mathematical technology and next-generation algorithms for *high-definition tomography* – sharper images with more reliable details – by using and further developing the most recent and sophisticated advances in mathematics. Our goal is to make it possible to incorporate many different types available prior information in a flexible way.

Highlight 1: Sparsity and Low-Dose CT

In medical CT and materials science we want to minimize the X-ray dose and shorten the measurement time, resulting in problems with limited and noisy data. To compensate for this we may assume that the desired tomographic image is "simple" – in mathematical terms, *sparse* – and use methods from compressed sensing to compute reliable reconstructions. We demonstrate that the amount of data sufficient for accurately reconstructing sparse CT images using these methods depends in a simple way on the image sparsity: To each sparsity corresponds a certain critical number of measurements at which the fraction of fully reconstructed images abruptly changes from 0 to 1. This so-called phase-transition behavior for CT is new insight that will further stimulate the analysis and use of compressed sensing in CT.

Highlight 2: Improved Localization in Electrical Impedance Tomography (EIT)

EIT can be used for industrial process monitoring, and due to constraints on the measurement geometry we may have only partial data available. To overcome this challenge we incorporate the fact that the objects of interest stand out from the background, and this allows us to compute 3D reconstructions with superior localization and contrast compared to other EIT methods. Our software has been verified on data from tank experiments done at University of Eastern Finland. Ongoing research seeks to quantify (through an eigenfunction analysis) the obtainable resolution and the optimal measurement configurations.

Highlight 3: Training Images and Machine Learning

Certain priors take the form of cross-section images of the object, providing a set of so-called training images which the reconstruction should resemble. We developed a mathematical and computational framework – based on machine learning techniques involving matrices and tensors – to use such training images in a fast, reliable, and computationally efficient manner. Simulations show that for textural images our approach is superior to other methods used for limited-data situations.

Highlight 4: Algorithms and Software

Incorporation of prior information in 3D tomographic reconstructions as described above requires the development of new large-scale algorithms and software. We focus on computational methods, such as row-action methods, that lend themselves to utilization of many-core and GPU computers. In addition to the software development we derive theoretical results for the fast convergence of these methods.

Major Problems/Difficulties

Scientific/Technical problems

There are not scientific or technical problems.

Support provided by the Host Institution

The support provided by the host institution (DTU) is excellent.

Others

- Even with international postings, it has been surprisingly difficult to find suited PhD students with a high enough level of mathematics for this project.
- The Danish immigration authorities have a very long processing time, resulting in a delay of 3–4 months between a position is accepted and the person is able to move to Denmark and start working.

The Research Group

The Assistant Professors, Post Docs, and PhD students, were all hired after international postings, in order to create the best team. All team members are affiliated with the Section for Scientific Computing at the Dept. of Applied Mathematics and Computer Science (DTU Compute) at the Technical University of Denmark.

Permanent faculty

- Assistant Professor Martin S. Andersen
- Assistant Professor Yiqiu Dong
- Professor Per Christian Hansen - Principal Investigator
- Associate Professor Kim Knudsen

Post Docs

- Lauri Harhanen, Post Doc project: Formulation and Application of Priors in Spectral CT (start May 1, 2015).
- Jakob Sauer Jørgensen, Post Doc project: Computations with Sparse Representations (started August 1, 2013)

PhD students (all PhD projects last 3 years)

- Hari Om Aggrawal, PhD project: Priors for Temporal Tomographic Image Reconstruction (starts April 1, 2015).
- Henrik Garde, PhD project: Prior Information in Inverse Boundary Problems (started March 1, 2013).
- Rasmus Dalgas Rasmussen, PhD project: Segmentation-Driven Tomographic Reconstruction (started September 1, 2014).
- Mikhail (Mike) Romanov, PhD project: Statistical Priors in Variational Reconstruction Methods (started November 1, 2012)
- Federica Sciacchitano, DTU Compute. PhD project: Image Reconstruction under Non-Gaussian Noise (started September 1, 2013).
- Marie Foged Schmidt, PhD project: Prior-Driven Diffusion Regularization for Inverse Problems (started December 2014).
- Sara Soltani, PhD project: Training Sets in Large-Scale Reconstruction Methods (started September 1, 2012)

Publication List

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- D. Chen, M. E. Kilmer, and P. C. Hansen, *"Plug-and-play" edge-preserving regularization*, Electronic Transactions on Numerical Analysis, 41 (2014), pp. 465-477.
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- J. S. Jørgensen, C. Kruschel, and D. Lorenz, *Testable uniqueness conditions for empirical assessment of undersampling levels in total variation-regularized x-ray CT*, Inverse Problems in Science and Engineering, published online (2014); DOI: 10.1080/17415977.2014.986724
- J. S. Jørgensen and E. Y. Sidky, *How little data is enough? Phase-diagram analysis of sparsity-regularized X-ray CT*, Phil. Trans. Royal Soc. A, **to appear** May 4, 2015 (special issue “X-ray tomographic reconstruction for materials science”). Available from: arxiv.org/abs/1412.6833
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