Multigroup Anisotropic Neutron Transport

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Outline

References

- C. E. Siewert, A spherical harmonics method for multi-group or non-grey radiation transport, J. Quant. Spectrosc. Radiat. Transfer, 49 (1993), pp. 95–106.
- C. T. Kelley, Multilevel source iteration accelerators for the linear transport equation in slab geometry, Trans. Th. Stat. Phys., 24 (1995), pp. 679–708.

Multigroup Anisotropic Problem: I

$$\mu \frac{\partial I}{\partial x}(x, \mu) + \Sigma I(x, \mu) = \frac{1}{2} \int_{-1}^{1} C(\mu, \mu') I(x, \mu') \, d\mu' + q(x, \mu).$$

Here, $0 < x < L = 30$ and $\mu \in [-1, 0) \cup (0, 1]$.
Σ is a given 6×6 diagonal matrix, the source term $q(x, \mu)$ and 6×6 scattrering cross section matrix C have epansions

$$q(x, \mu) = \sum_{i=0}^{3} P_i(\mu) q_i(x), C(\mu, \mu') = \sum_{i=0}^{3} P_i(\mu) P_i(\mu') T_i$$

The unknown I is a R^6 valued function.

Legendre Polynomials

P_i is the ith degree Legendre polynomial.

$$\int_{-1}^{1} P_j(\mu)P_i(\mu)\,d\mu = \delta_{ij}\frac{2}{2i+1}$$

$$P_0(\mu) = 1; P_1(\mu) = \mu;$$

$$P_2(\mu) = \frac{1}{2}(3\mu^2 - 1); P_3(\mu) = \frac{1}{2}(5\mu^3 - 3\nu)$$

Multigroup Anisotropic Problem: II

In this example $L = 30$. Boundary Conditions:

$$F_l(\mu) = (1,0,0,0,0,0)^T, F_r(\mu) = (0,0,0,0,0,0)^T.$$

Data: Σ and T_i are in `transport_data.m`.

$$\frac{1}{2}\int_{-1}^{1} C(\mu,\mu') I(x,\mu')\, d\mu' + q(x,\mu).$$

in Legendre Polynomials and we'll find them.

Fluxes: I

Define, for $0 \leq k \leq 3$,

$$f_k(x) = \int_{-1}^{1} P_k(\mu) I(x, \mu) \, d\mu$$

This is an R^6-valued function of x.
Use the expansion for C

$$C(\mu, \mu') = \sum_{i=0}^{3} P_i(\mu) P_i(\mu') T_i,$$

and get ...

Fluxes: II

$$
\begin{aligned}
\frac{1}{2}\int_{-1}^{1} C(\mu,\mu')I(x,\mu')\,d\mu' &= \sum_{i=0}^{3} \int_{-1}^{1} T_i P_i(\mu) \int_{-1}^{1} P_i(\mu')I(x,\mu')\,d\mu' \\
&= \sum_{k=0}^{3} T_k P_k(\mu) f_k(x)
\end{aligned}
$$

This transforms the Transport equation into

Discretization: I

$$\mu \frac{\partial I}{\partial x}(x,\mu) + \Sigma I(x,\mu) = S(x,u) \equiv \sum_{k=0}^{3} T_k P_k(\mu) f_k(x) + q(x,\mu).$$

So, if I know the 24 fluxes $\{f_k\}$ I can compute I as I did in the scalar case.
We will do this with the same discretization as before. Let

$$\phi_i^k \approx f_k(x_i) \text{ and } \psi_i^j \approx I(x_i, \mu_j)$$

then ...

Discretization: II

$$\mu_j \frac{\psi^j_{i+1} - \psi^j_i}{h} + \Sigma \frac{\psi^j_{i+1} + \psi^j_i}{2} = \frac{S^j_{i+1} + S^j_i}{2},$$

where

$$S^j_i = \sum_{k=0}^{3} P_k(\mu_j) T_k \phi^k_i + q(x_i, \mu_j).$$

$q \equiv 0$ in this example.

Forward Sweep

So, if $\mu_j > 0$ you can do a forward sweep

$$\psi^j_{i+1} = \left(\mu_j I + \frac{h}{2}\Sigma\right)^{-1} \left(h\frac{S_{i+1} + S_i}{2} + \left(\mu_j I - \frac{h}{2}\Sigma\right)\psi^j_i\right)$$

for $i = 1, \ldots, N - 1$.

Backward Sweep

The backward sweep for $\mu_j < 0$ is

$$\psi_i^j = \left(-\mu_j I + \frac{h}{2}\Sigma\right)^{-1} \left(h\frac{S_{i+1} + S_i}{2} + \left(-\mu_j I - \frac{h}{2}\Sigma\right)\psi_{i+1}^j\right)$$

for $i = N-1, \dots, 1$.

Source Iteration Map

So, given ϕ you can compute ψ. You have solved the transport equation when

$$\phi^k = \mathcal{S}(\phi)^k \equiv \sum_{j=1}^{N_A} \psi_i^j P_k(\mu_j) w_j$$

This is the source iteration map for this problem.
This is tricky. Each ψ_i^k and ψ_i^j is is a vector in R^6. So there are a total of $6 \times 4 \times N$ unknowns in this problem.

The Project

Your job is to

- Solve the problem with source iteration and match the fluxes on the next page.
- Solve the problem with GMRES/BiCGSTAB/TFQMR. This will require that you figure out a way to map the $24 \times N$ unkowns into a single vector and back.
- Solve the problem with the multilevel method.

The Answers: Table 3 from Siewert

P_N method for multi-group or non-gray radiation transport

Table 3. The group fluxes $\Psi_0(\tau)$.

Group	$\tau/\tau_0 = 0.0$	$\tau/\tau_0 = 0.25$	$\tau/\tau_0 = 0.5$	$\tau/\tau_0 = 0.75$	$\tau/\tau_0 = 1.0$
1	1.09	1.6205(–4)	4.8524(–8)	1.4567(–11)	4(–15)
2	2.30(–1)	3.7447(–2)	1.9639(–3)	1.0277(–4)	1.79(–6)
3	2.92(–1)	1.8547(–1)	9.7989(–3)	5.1278(–4)	4.37(–6)
4	3.06(–2)	2.6281(–2)	1.3884(–3)	7.2654(–5)	4.39(–7)
5	6.00(–4)	6.1078(–4)	3.2260(–5)	1.6882(–6)	7.91(–9)
6	7.31(–6)	7.2593(–6)	3.8325(–7)	2.0056(–8)	7.94(–11)

Rules and Reality

- Projects due by Friday, July 8, 17:00 European time.
- You are pretty much on your own after Friday. So you should formulate a plan of attack before the end of the week.
  - Allan will be on a two week vacation. He will not be in room 12.
  - Tim will be working on other things before leaving for Canada and China. He will not be in room 10.
- I can answer short questions via email, but not the kind of questions many of you have been asking me this week.