

Multigroup Anisotropic Neutron Transport

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Outline

References

Problem Formulation

References

- ▶ C. E. SIEWERT, A spherical harmonics method for multi-group or non-grey radiation transport, J. Quant. Spectrosc. Radiat. Transfer, 49 (1993), pp. 95–106.
- ▶ C. T. KELLEY, Multilevel source iteration accelerators for the linear transport equation in slab geometry, Trans. Th. Stat. Phys., 24 (1995), pp. 679–708.

Multigroup Anisotropic Problem: I

$$\mu \frac{\partial I}{\partial x}(x, \mu) + \Sigma I(x, \mu) = \frac{1}{2} \int_{-1}^1 C(\mu, \mu') I(x, \mu') d\mu' + q(x, \mu).$$

Here, $0 < x < L = 30$ and $\mu \in [-1, 0) \cup (0, 1]$.

Σ is a given 6×6 diagonal matrix, the source term $q(x, \mu)$ and 6×6 scattering cross section matrix C have expansions

$$q(x, \mu) = \sum_{i=0}^3 P_i(\mu) q_i(x), \quad C(\mu, \mu') = \sum_{i=0}^3 P_i(\mu) P_i(\mu') T_i$$

The unknown I is a R^6 valued function.

Legendre Polynomials

P_i is the i th degree Legendre polynomial.

$$\int_{-1}^1 P_j(\mu) P_i(\mu) d\mu = \delta_{ij} \frac{2}{2i+1}$$

$$P_0(\mu) = 1; P_1(\mu) = \mu;$$

$$P_2(\mu) = \frac{1}{2}(3\mu^2 - 1); P_3(\mu) = \frac{1}{2}(5\mu^3 - 3\mu)$$

Multigroup Anisotropic Problem: II

In this example $L = 30$. Boundary Conditions:

$$F_l(\mu) = (1, 0, 0, 0, 0, 0)^T, F_r(\mu) = (0, 0, 0, 0, 0, 0)^T.$$

Data: Σ and T_i are in `transport_data.m`.

$$\frac{1}{2} \int_{-1}^1 C(\mu, \mu') I(x, \mu') d\mu' + q(x, \mu).$$

in Legendre Polynomials and we'll find them.

Fluxes: I

Define, for $0 \leq k \leq 3$,

$$f_k(x) = \int_{-1}^1 P_k(\mu) I(x, \mu) d\mu$$

This is an R^6 -valued function of x .

Use the expansion for C

$$C(\mu, \mu') = \sum_{i=0}^3 P_i(\mu) P_i(\mu') T_i,$$

and get ...

Fluxes: II

$$\begin{aligned}\frac{1}{2} \int_{-1}^1 C(\mu, \mu') I(x, \mu') d\mu' &= \sum_{i=0}^3 \int_{-1}^1 T_i P_i(\mu) \int_{-1}^1 P_i(\mu') I(x, \mu') d\mu' \\ &= \sum_{k=0}^3 T_k T_k f_k(x)\end{aligned}$$

This transforms the Transport equation into

Discretization: I

$$\mu \frac{\partial I}{\partial x}(x, \mu) + \Sigma I(x, \mu) = S(x, \mu) \equiv \sum_{k=0}^3 T_k P_k(\mu) f_k(x) + q(x, \mu).$$

So, if I know the 24 fluxes $\{f_k\}$ I can compute I as I did in the scalar case.

We will do this with the same discretization as before. Let

$$\phi_i^k \approx f_k(x_i) \text{ and } \psi_i^j \approx I(x_i, \mu_j)$$

then ...

Discretization: II

$$\mu_j \frac{\psi_{i+1}^j - \psi_i^j}{h} + \Sigma \frac{\psi_{i+1}^j + \psi_i^j}{2} = \frac{S_{i+1}^j + S_i^j}{2},$$

where

$$S_i^j = \sum_{k=0}^3 P_k(\mu_j) T_k \phi_i^k + q(x_i, \mu_j).$$

$q \equiv 0$ in this example.

Forward Sweep

So, if $\mu_j > 0$ you can do a forward sweep

$$\psi_{i+1}^j = \left(\mu_j I + \frac{h}{2} \Sigma \right)^{-1} \left(h \frac{S_{i+1}^j + S_i^j}{2} + \left(\mu_j I - \frac{h}{2} \Sigma \right) \psi_i^j \right)$$

for $i = 1, \dots, N - 1$.

Backward Sweep

The backward sweep for $\mu_j < 0$ is

$$\psi_i^j = \left(-\mu_j I + \frac{h}{2} \Sigma \right)^{-1} \left(h \frac{S_{i+1}^j + S_i^j}{2} + \left(-\mu_j I - \frac{h}{2} \Sigma \right) \psi_{i+1}^j \right)$$

for $i = N - 1, \dots, 1$.

Source Iteration Map

So, given ϕ you can compute ψ . You have solved the transport equation when

$$\phi^k = \mathcal{S}(\phi)^k \equiv \sum_{j=1}^{N_A} \psi_i^j P_k(\mu_j) w_j$$

This is the source iteration map for this problem.

This is tricky. Each ψ_i^k and ψ_j^j is a vector in R^6 . So there are a total of $6 \times 4 \times N$ unknowns in this problem.

The Project

Your job is to

- ▶ Solve the problem with source iteration and match the fluxes on the next page.
- ▶ Solve the problem with GMRES/BiCGSTAB/TFQMR. This will require that you figure out a way to map the $24 \times N$ unknowns into a single vector and back.
- ▶ Solve the problem with the multilevel method.

The Answers: Table 3 from Siewert

 P_N method for multi-group or non-gray radiation transportTable 3. The group fluxes $\Psi_0(\tau)$.

Group	$\tau/\tau_0 = 0.0$	$\tau/\tau_0 = 0.25$	$\tau/\tau_0 = 0.5$	$\tau/\tau_0 = 0.75$	$\tau/\tau_0 = 1.0$
1	1.09	1.6205(-4)	4.8524(-8)	1.4567(-11)	4(-15)
2	2.30(-1)	3.7447(-2)	1.9639(-3)	1.0277(-4)	1.79(-6)
3	2.92(-1)	1.8547(-1)	9.7989(-3)	5.1278(-4)	4.37(-6)
4	3.06(-2)	2.6281(-2)	1.3884(-3)	7.2654(-5)	4.39(-7)
5	6.00(-4)	6.1078(-4)	3.2260(-5)	1.6882(-6)	7.91(-9)
6	7.31(-6)	7.2593(-6)	3.8325(-7)	2.0056(-8)	7.94(-11)

Rules and Reality

- ▶ Projects due by Friday, July 8, 17:00 European time.
- ▶ You are pretty much on your own after Friday. So you should formulate a plan of attack before the end of the week.
 - ▶ Allan will be on a two week vacation. He will not be in room 12.
 - ▶ Tim will be working on other things before leaving for Canada and China. He will not be in room 10.
- ▶ I can answer short questions via email, but not the kind of questions many of you have been asking me this week.