Course 02402/02323 Introducerende Statistik

Forelæsning 1: Intro, R og beskrivende statistik

Per Bruun Brockhoff

DTU Compute, Statistik og Dataanalyse
Bygning 324, Rum 220
Danmarks Tekniske Universitet
2800 Lyngby – Danmark
e-mail: perbb@dtu.dk

DTU Compute
Department of Applied Mathematics and Computer Science

Oversigt

DTU Compute
Department of Applied Mathematics and Computer Science

Praktisk Information

- Undervisning: Tirsdage 13-17/fredage 8-12
- Generel daglig agenda:
  - FØR undervisningsmodulet: læs det annoncerede i eNoten!
  - 2x45 minutters forelæsning (ugens pensum)
  - 2 timers øvelser (Enote Excersises (eNe) og små quiz-spørgsmål)
  - EFTER undervisningsmodulet: Test dig selv med online eksamens-quiz
- Skriftlig eksamen: Lørdag 30/05.
- OBLIGATORISKE projekter: 2 stk - skal godkendes for at kunne gå til eksamen.

DTU Compute
Department of Applied Mathematics and Computer Science

Praktisk Information

- Hjemmeside: introstat.compute.dtu.dk
  - Læsemateriale: eNote
  - Forelæsningsplan
  - Øvelser & besvarelser
  - Slides
  - Podcasts af nye og gl. forelæsninger(På dansk OG engelsk)
  - Quizzer
- Campusnet: www.campusnet.dtu.dk
  - Meddelelser, visse (få)dokumenter
  - Links til interessante historier
  - Projekter - beskrivelse OG aflevering

DTU Compute
Department of Applied Mathematics and Computer Science

Introduction to Statistics - a primer

New England Journal of medicine:

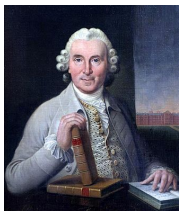
EDITORIAL: Looking Back on the Millennium in Medicine, *N Engl J Med*, 342:42-49, January 6, 2000.
`http://www.nejm.org/doi/full/10.1056/NEJM200001063420108`

Millennium list

- Elucidation of Human Anatomy and Physiology
- Discovery of Cells and Their Substructures
- Elucidation of the Chemistry of Life
- **Application of Statistics to Medicine**
- Development of Anesthesia
- Discovery of the Relation of Microbes to Disease
- Elucidation of Inheritance and Genetics
- Knowledge of the Immune System
- Development of Body Imaging
- Discovery of Antimicrobial Agents
- Development of Molecular Pharmacotherapy

James Lind

"One of the earliest clinical trials took place in 1747, when James Lind treated 12 scorbutic ship passengers with cider, an elixir of vitriol, vinegar, sea water, oranges and lemons, or an electuary recommended by the ship's surgeon. The success of the citrus-containing treatment eventually led the British Admiralty to mandate the provision of lime juice to all sailors, thereby eliminating scurvy from the navy." (See also `http://en.wikipedia.org/wiki/James_Lind`).



John Snow

"The origin of modern epidemiology is often traced to 1854, when John Snow demonstrated the transmission of cholera from contaminated water by analyzing disease rates among citizens served by the Broad Street Pump in London's Golden Square. He arrested the further spread of the disease by removing the pump handle from the polluted well." (See also `http://en.wikipedia.org/wiki/John_Snow_(physician)`).

Google - *Big Data*

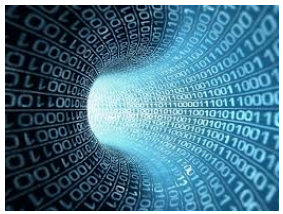
A quote from New York Times, 5. August 2009, from the article titled "For Today's Graduate, Just One Word: Statistics" is:

"I keep saying that the sexy job in the next 10 years will be statisticians," said Hal Varian, chief economist at Google. "And I'm not kidding.""

(Og Politiken, 12/2 2014 - se links i CampusNet)

IBM - *Big Data*

"The key is to let computers do what they are good at, which is trawling these massive data sets for something that is mathematically odd," said Daniel Gruhl, an I.B.M. researcher whose recent work includes mining medical data to improve treatment. "And that makes it easier for humans to do what they are good at - explain those anomalies."



Intro Case historier: IBM Big data, Novo Nordisk small data, Skive fjord

- Gæsteforedrag af:
  - Tirsdag: Senior Scientist Hanne Refsgaard, Novo Nordisk A/S
- Bliver IKKE streamet OG podcasted.
- IBM og Skive Fjord podcasts ligger på hjemmesiden allerede

Introduktion til Statistik

- Hvordan behandle (eller analysere) data?
- Hvad er tilfældig variation?
- Statistik er et værktøj til at træffe beslutninger:
  - Hvor mange computere har vi solgt det sidste år?
  - Hvad er forventet pris af en aktie?
  - Er maskine A mere effektiv end maskine B ?
- Statistik er et metodefag, der kan anvendes inden for de fleste fagområder, og er derfor et meget vigtigt værktøj

Statistik og Ingeniører

- Statistik er et vigtigt værktøj i problemløsning
- Analyse af data
- Kvalitetforbedring
- Forsøgsplanægning
- Forudsigelse af fremtidige værdier
- .. og meget mere!

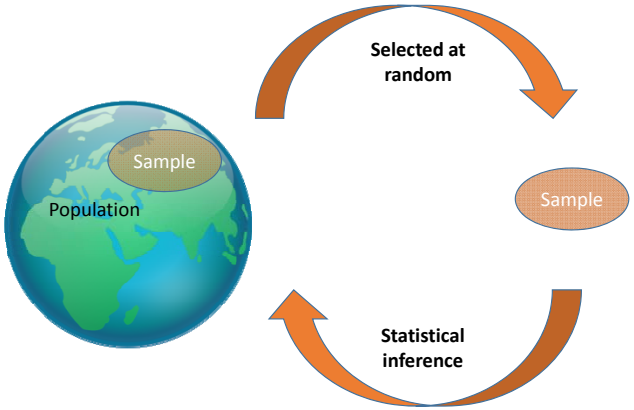
Statistik

- Moderne statistik har baggrund i sandsynlighedsregning og beskrivende statistik

Statistik

- Statistik handler ofte om at analysere en *stikprøve*, der er taget fra en *population*
- Baseret på stikprøven, prøver vi at generalisere (eller udtale os) om populationen
- Det er derfor vigtigt, at stikprøven er *repræsentativ* for populationen

Statistik



Kapitel 2: Nøgletal

- Vi anvender en række *nøgletal* for at opsummere og beskrive data (og stokastiske variable)
  - Gennemsnit $\bar{x}$, Median, Fraktiler
  - Varians s^2, Standardafvigelse s
  - Covarians og korrelation

Gennemsnit

- Gennemsnittet er et nøgletal, der angiver tyngdepunkt eller centrering af data
- Gennemsnit:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

- Vi siger, at $\bar{x}$ er et *estimat* af middelværdien

Median

Medianen er et også nøgletal, der angiver tyngdepunkt eller centrering af data. I nogle tilfælde, f.eks. hvis man har ekstreme værdier, er medianen at foretrække frem for middelværdien

- Median:
  Den midterste observation (i den sorterede rækkefølge)

Eksempel: Højder af unge mænd:

- Stikprøve (Sample):

```
x <- c(185, 184, 194, 180, 182)
```

  n=5
- **Gennemsnit**:

$$\bar{x} = \frac{1}{5}(185 + 184 + 194 + 180 + 182) = 185$$

- **Median**, først ordn data: 180 182 184 185 194
  Og så vælg det midterste (idet n er ulige)(3'te) tal: 184
- Hvad nu hvis en person på 235cm tilføjes til data:
  Median = 184.5
  Mean = 193.33

Varians og standardafvigelse

Variansen (eller standardafvigelsen) siger noget om hvor meget data spreder:

- Varians

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

- Standardafvigelse (spredning) (=standard deviation)

$$s = \sqrt{s^2} = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

Eksempel: Højder af unge mænd:

- **Data** n=5:
  185 184 194 180 182
- **Varians**, $s^2 =$

$$\frac{1}{4}((185-185)^2 + (184-185)^2 + (194-185)^2 + (180-185)^2 + (182-185)^2)$$

$$= 29$$

- **Standardafvigelse**, $s = \sqrt{s^2} =$

$$s = \sqrt{29} = 5.385$$

Variationskoefficient

Standardafvigelse og variansen er nøgletal for den absolutte variation. Hvis man gerne vil sammenligne variationen mellem forskellige datasæt, er det en god idé at anvende et relativt nøgletal, nemlig variationskoefficienten:

$$V = \frac{s}{\bar{x}} \cdot 100$$

Fraktiler(=percentiles=quantiles)

Medianen beregnes som det punkt, der deler data ind i to halvdele. Man kan naturligvis finde andre punkter, der deler data ind i andre dele, og det man kalder fraktiler.

Ofte beregner man fraktilerne

- $0, 25, 50, 75, 100$ % fraktiler
- Bemærk: 50% fraktilen svarer til medianen

Fraktiler(=percentiles=quantiles), Definition 1.6

Den p'te fraktil, også kaldet *quantile*, kan defineres ud fra følgende procedure:

1. Ordn de n observationer fra mindst til størst: $x_{(1)}, \ldots, x_{(n)}$.
2. Beregn pn.
3. Hvis pn er et helt tal: Midl den pn'te og $(pn+1)$'te ordnede observationer

$$\text{Den } p\text{'te fraktil } = \left(x_{(np)} + x_{(np+1)}\right)/2 \tag{1}$$

4. Hvis pn er et ikke-helt tal: tag den "næste" i den ordnede liste:

$$\text{Den } p\text{'te fraktil } = x_{(\lceil np \rceil)} \tag{2}$$

hvor $\lceil np \rceil$ er *ceiling*("loftet") af np, dvs. det mindste hele tal større end np.

DTU Compute
Department of Applied Mathematics and Computer Science

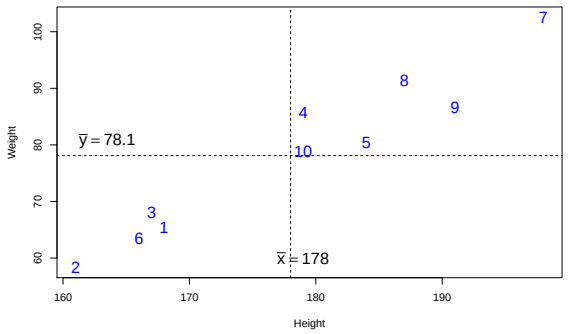
Eksempel: Højder af unge mænd:

- **Data,** n=5:
  185 184 194 180 182
- **Nedre kvartil**, Q_1, først ordn data: 180 182 184 185 194
  Og så vælg det rigtige baseret på $np = 1.25$:
  $Q_1 = 182$
- **Øvre kvartil**, Q_3, først ordn data: 180 182 184 185 194
  Og så vælg det rigtige baseret på $np = 3.75$:
  $Q_3 = 185$

DTU Compute
Department of Applied Mathematics and Computer Science

Kovarians og Korrelation - mål for sammenhæng

Heights (x_i)	168	161	167	179	184	166	198	187	191	179
Weights (y_i)	65.5	58.3	68.1	85.7	80.5	63.4	102.6	91.4	86.7	78.9



DTU Compute
Department of Applied Mathematics and Computer Science

Kovarians og Korrelation - Def. 1.17 og 1.18

The sample covariance is given by

$$s_{xy} = \frac{1}{n-1}\sum_{i=1}^{N}(x_i - \bar{x})(y_i - \bar{y}) \tag{3}$$

The sample correlation coefficient is given by

$$r = \frac{1}{n-1}\sum_{i=1}^{N}\left(\frac{x_i - \bar{x}}{s_x}\right)\left(\frac{y_i - \bar{y}}{s_y}\right) = \frac{s_{xy}}{s_x \cdot s_y} \tag{4}$$

where s_x and s_y is the sample standard deviation for x and y respectively.

DTU Compute
Department of Applied Mathematics and Computer Science

Kovarians og Korrelation - mål for sammenhæng

Student	1	2	3	4	5	6	7	8	9	10
Heights (x_i)	168	161	167	179	184	166	198	187	191	179
Weights (y_i)	65.5	58.3	68.1	85.7	80.5	63.4	102.6	91.4	86.7	78.9
$(x_i - \bar{x})$	-10	-17	-11	1	6	-12	20	9	13	1
$(y_i - \bar{y})$	-12.6	-19.8	-10	7.6	2.4	-14.7	24.5	13.3	8.6	0.8
$(x_i - \bar{x})(y_i - \bar{y})$	126.1	336.8	110.1	7.6	14.3	176.5	489.8	119.6	111.7	0.8

$$\begin{aligned} s_{xy} &= \frac{1}{9}(126.1 + 336.8 + 110.1 + 7.6 + 14.3 + 176.5 + 489.8 \\ &\quad +119.6 + 111.7 + 0.8) \\ &= \frac{1}{9} \cdot 1493.3 \\ &= 165.9 \end{aligned}$$

$$s_x = 12.21, \quad \text{and} \quad s_y = 14.07$$

$$r = \frac{165.9}{12.21 \cdot 14.07} = 0.97$$

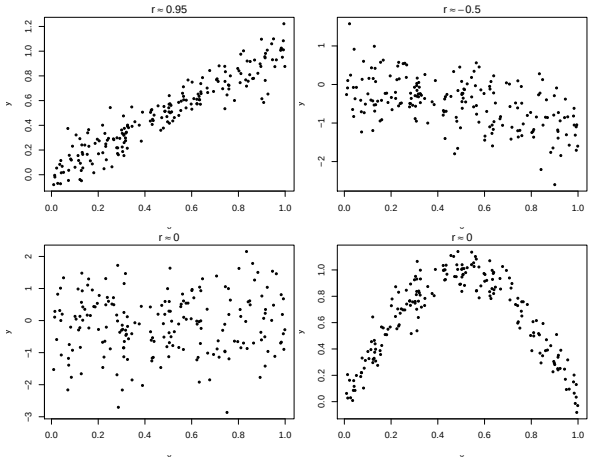
DTU Compute
Department of Applied Mathematics and Computer Science

Korrelation - egenskaber

- r is always between -1 and 1: $-1 \leq r \leq 1$
- r measures the degree of linear relation between x and y
- $r = \pm 1$ if and only if all points in the scatterplot are exactly on a line
- $r > 0$ if and only if the general trend in the scatterplot is positive
- $r < 0$ if and only if the general trend in the scatterplot is negative

DTU Compute
Department of Applied Mathematics and Computer Science

Korrelation



DTU Compute
Department of Applied Mathematics and Computer Science

Plots/figurer

- Kvantitative data:
  - Scatter plot (xy plot)
  - Histogram
  - Kumulativ fordeling
  - Boxplots
- Antalsdata:
  - Bar charts (pareto diagram)
  - Pie charts

DTU Compute
Department of Applied Mathematics and Computer Science

Software: R

- Installer R og Rstudio på egen computer.
- Introduceres i eNoten
- Er integreret i alt vi gør
- Globalt og hurtigt voksende open source beregningsmiljø
- ADVAAAARRRSEEEEL: R kan IKKE erstatte vores hjerner!!!! (Læs sektion 1.5.4!)

Software: R

```
> ## Adding numbers in the console
> 2+3
```

[1] 5

```
> y <- 3
```

```
> x <- c(1, 4, 6, 2)
> x
```

[1] 1 4 6 2

```
> x <- 1:10
> x
```

[1] 1 2 3 4 5 6 7 8 9 10

Software: R

```
## Sample Mean and Median (data from eNote)
x <- c(168,161,167,179,184,166,198,187,191,179)
mean(x)
```

[1] 178

```
median(x)
```

[1] 179

```
## Sample variance and standard deviation
var(x)
```

[1] 149.11

```
sd(x)
```

[1] 12.211

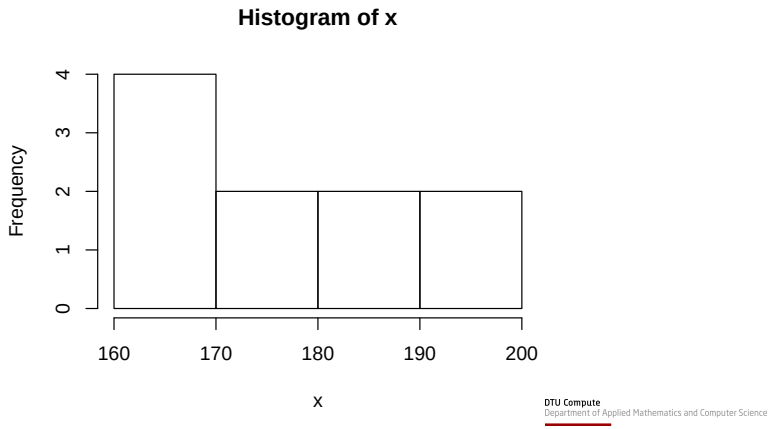
Software: R

```
## Sample quartiles
quantile(x,type=2)

##   0%  25%  50%  75% 100%
##  161  167  179  187  198
```

```
## Sample quantiles 0%, 10%,..,90%, 100%:
quantile(x,probs=seq(0, 1, by=0.10),type=2)

##    0%   10%   20%   30%   40%   50%   60%   70%   80%   90%  100%
## 161.0 163.5 166.5 168.0 173.5 179.0 184.0 187.0 189.0 194.5 198.0
```

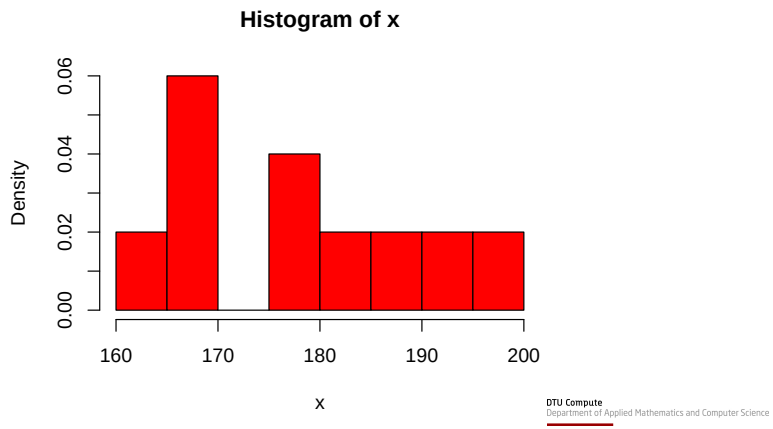
Software: R

```
## A histogram of the heights:
 hist(x)
```



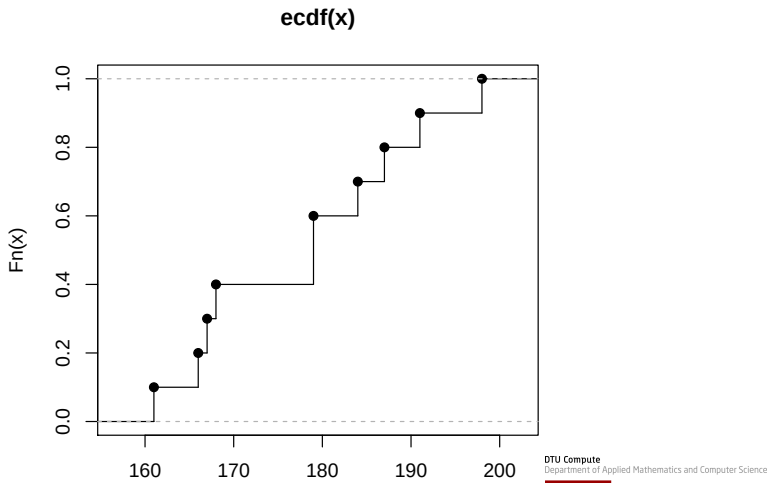
Software: R

```
## A density histogram of the heights:
hist(x,freq=FALSE,col="red",nclass=8)
```



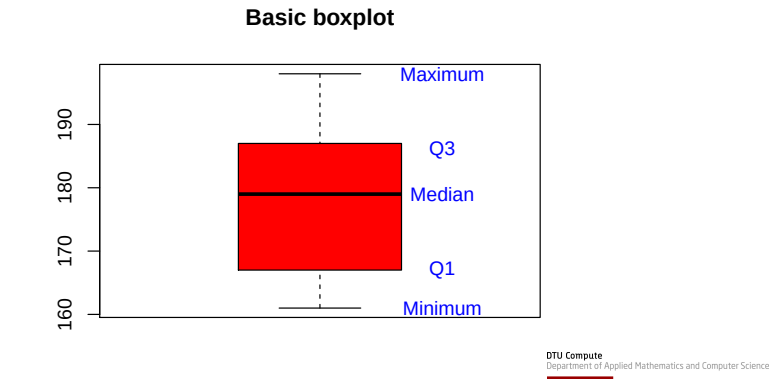
Software: R

```
plot(ecdf(x),verticals=TRUE)
```



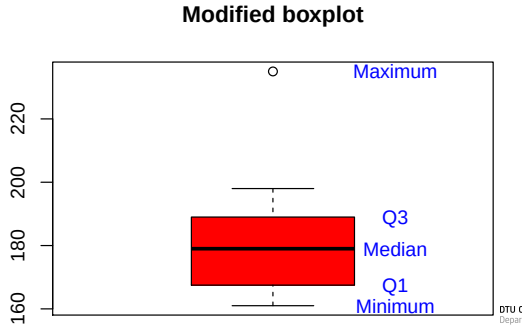
Software: R

```
## A basic boxplot of the heights: (range=0 makes it "basic")
boxplot(x,range=0,col="red",main="Basic boxplot")
text(1.3,quantile(x),c("Minimum","Q1","Median","Q3","Maximum"),
     col="blue")
```



Software: R

```
## A modified boxplot of the heights with an
## extreme observation, 235cm added:
## The modified version is the default
boxplot(c(x,235),col="red",main="Modified boxplot")
text(1.3,quantile(c(x,235)),c("Minimum","Q1","Median","Q3"
                              ,"Maximum"),col="blue")
```



DTU Compute
Department of Applied Mathematics and Computer Science

Næste uge:

- Sandsynlighed, diskrete fordelinger - kapitel 2 i eNoten

DTU Compute
Department of Applied Mathematics and Computer Science

Oversigt

DTU Compute
Department of Applied Mathematics and Computer Science