Technical University of Denmark *Page 1 of 28 sider.*

Written examination: 28. May 2017

Course name and number: **Introduction to Statistics (02323 and 02402)**

Aids and facilities allowed: All

The questions were answered by

_____ _____ _____

(student number) (signature) (table number)

There are 30 questions of the "multiple choice" type included in this exam divided on 11 exercises. To answer the questions you need to fill in the prepared 30-question multiple choice form (on 6 seperate pages) in CampusNet. **There is one and only one correct answer to each question**.

5 points are given for a correct answer and -1 point is given for a wrong answer. ONLY the following 5 answer options are valid: 1, 2, 3, 4 or 5. If a question is left blank or another answer is given, then it does not count (i.e. "0 points"). Also, if more answers are given to a single question, which in fact is technically possible in the online system, it will not count (i.e. "0 points"). The number of points corresponding to specific marks or needed to pass the examination is ultimately determined during censoring.

The final answer of the exercises should be given by filling in and submitting via the exam module in CampusNet. The table sheet here is ONLY to be used as an "emergency" alternative.

Exercise	I.1	I.2	II.1	III.1	III.2	III.3	III.4	III.5	III.6	IV.1
Question	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Answer										

Exercise	IV.2	IV.3	V.1	V.2	V.3	VI.1	VI.2	VII.1	VII.2	VII.3
Question	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
Answer										

Exercise	VII.4	VII.5	VIII.1	VIII.2	IX.1	IX.2	X.1	XI.1	XI.2	XI.3
Question	(21)	(22)	(23)	(24)	(25)	(26)	(27)	(28)	(29)	(30)
Answer										

In case of "emergency": Remember to provide your **study number**. The questionnaire contains 28 pages. Please check that your questionnaire contains them all.

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Multiple choice questions: *Note that not all the suggested answers are necessarily meaningful. In fact, some of them are very wrong but under all circumstances there is one and only one correct answer to each question.*

Exercise I

The island of Moen is the first official Dark Sky area in the Nordic area. A man went there to watch stars and shooting stars. It was a clear night and he had read that he should expect to see 2 shoothing stars per minute.

It can be assumed that the intensity of shooting stars is constant and that they arrive independently of each other.

Question I.1 (1)

Which distribution gives the best description of the waiting time in minutes between two shooting stars?

1 ☐ An exponential distribution with $\lambda = 2$

2 ☐ A Poisson distribution with $\lambda = 2$

3 ☐ A Normal distribution with $\mu = 1$ and $\sigma = \sqrt{2}$

4 ☐ A uniform distribution with $\alpha = 0$ and $\beta = 2$

5 ☐ A Poisson distribution with $\lambda = 1/2$

Question I.2 (2)

What is the probability that there is no shooting stars in a given 4 minutes interval?

1 ☐ 0.8824969

2 ☐ 0.1353353

3 ☐ 0.9996645

4 ☐ 3.3546263×10^{-4}

5 ☐ There must have been at least one shooting star in a given 4 minutes interval.

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Exercise II

When inspecting cars the combustion is tested by measuring the exhaust gasses. Assume that a given car has a true exhaust of particles of 0.12 g/km where the EURO2 norm limit is 0.08 g/km. The measurement has a standard deviation of 0.02 g/km. It is assumed that the measurements are normally distributed around the true exhaust. A car is tested by a single measurement.

Question II.1 (3)

What is the probability that the car is tested as having too high exhaust of particles?

1 ☐ 0.6113513

2 ☐ 0.0227501

3 ☐ 0.3886487

4 ☐ 0.9772499

5 ☐ 0.6113513

Continues on page 4

Exercise III

In a purification experiment, the so-called yield was measured after dosing a certain amount of enzyme. The response variable was the yield percentage in relation to the theoretical highest obtainable level (X). Data from 10 different test samples from the experiment were:

x_i
74.7
74.2
74.1
69.6
75.4
76.3
76.7
75.6
72.0
74.3
$\bar{x} = 74.29$
$s = 2.115$

Question III.1 (4)

What is the 80% percentile for these data using the definition from the book?

1 ☐ 74.10

2 ☐ 74.50

3 ☐ 75.95

4 ☐ 74.29

5 ☐ 75.60

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Question III.2 (5)

Assuming that $X \sim N(\mu, \sigma^2)$ and applying the usual estimated parameters ($\mu = \bar{x}$ and $\sigma = s$), what is the only statement that can be correct:

1 ☐ More than 99% of the population is within $[72.18, 76.40]$ (In R: `mean(x) + c(-1, 1) * sd(x)`)

2 ☐ More than 99% of the population is within $[70.06, 78.52]$ (In R: `mean(x) + c(-1, 1) * 2 *sd(x)`)

3 ☐ Around 95% of the population is within $[72.18, 76.40]$ (In R: `mean(x) + c(-1, 1) * sd(x)`)

4 ☐ More than 99% of the population is within $[67.95, 80.63]$ (In R: `mean(x) + c(-1, 1) * 3 *sd(x)`)

5 ☐ Less than 95% of the population is within $[67.95, 80.63]$ (In R: `mean(x) + c(-1, 1) * 3 *sd(x)`)

Question III.3 (6)

What is the 95% confidence interval for the mean?

1 ☐ $2.262 \pm 74.29\frac{2.115}{\sqrt{10}} = [-47.42, 51.95]$

2 ☐ $74.29 \pm 1.812\frac{2.115}{\sqrt{9}} = [73.01, 75.57]$

3 ☐ $74.29 \pm 1.96\frac{2.115}{\sqrt{9}} = [72.91, 75.67]$

4 ☐ $74.29 \pm 2.262\frac{2.115}{\sqrt{10}} = [72.78, 75.80]$

5 ☐ $74.29 \pm 1.96\frac{2.115^2}{\sqrt{10}} = [71.52, 77.06]$

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Question III.4 (7)

If you make a 95% confidence interval for the standard deviation, which quantiles should then be used?

1 ☐ `qnorm(0.025)` and `qnorm(0.975)`

2 ☐ `qchisq(0.025, 9)` and `qchisq(0.975, 9)`

3 ☐ `qf(0.025, 9, 9)` and `qf(0.975, 9, 9)`

4 ☐ `qt(0.025, 9)` and `qt(0.975, 9)`

5 ☐ `qunif(0.025)` and `qunif(0.975)`

Question III.5 (8)

The p-value for the hypothesis test of $H_0 : \mu = 70$ is:

1 ☐ `2*(1-pt(4.29/(2.115/sqrt(10)), 9))`

2 ☐ `2*(1-pnorm(70/(2.115/sqrt(10))))`

3 ☐ `(1-pt(-4.29/(2.115/sqrt(9)), 10))`

4 ☐ `1-pnorm(-4.29/(2.115/sqrt(10)))`

5 ☐ `1-qt(2.115/4.29, 9)`

Question III.6 (9)

In a new experiment which is in the planning phase, a 95% confidence interval for the mean with an expected width of around 1 is wanted. Assume that the standard deviation is 2.115. How large a sample does it approximately require to achieve this desired precision?

1 ☐ 5

2 ☐ 1230

3 ☐ 4

4 ☐ 100

5 ☐ 69

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Exercise IV

In a purification experiment, two different doses of an enzyme have been investigated called d_1 and d_2, with the purpose to investigate a possible effect on the yield. The response variable was the yield percentage in relation to the theoretical highest obtainable level. Data from 19 different test samples from the experiment were:

Dose d_1	Dose d_2
74.7	79.6
74.2	77.5
74.1	82.5
69.6	76.7
75.4	78.2
76.3	76.7
76.7	76.6
75.6	78.1
72.0	79.2
74.3	
$\bar{x}_1 = 74.29$	$\bar{x}_2 = 78.34$
$s_1 = 2.115$	$s_2 = 1.898$

The following was run in R:

```
x1 <- c(74.7, 74.2, 74.1, 69.6, 75.4, 76.3, 76.7, 75.6, 72.0, 74.3)
x2 <- c(79.6, 77.5, 82.5, 76.7, 78.2, 76.7, 76.6, 78.1, 79.2)
t.test(x2, x1)

##
##  Welch Two Sample t-test
##
## data:  x2 and x1
## t = 4.4041, df = 17, p-value = 0.0003878
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  2.112129 5.996760
## sample estimates:
## mean of x mean of y
##  78.34444  74.29000
```

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Question IV.1 (10)

What is the 99% confidence interval for the difference in means between dose d_2 and dose d_1?

1 ☐ $[2.45, 5.66]$

2 ☐ $[1.39, 6.72]$

3 ☐ $[1.97, 6.14]$

4 ☐ $[2.11, 6.00]$

5 ☐ $[74.29, 78.34]$

Question IV.2 (11)

The conclusion of the usual t-test (based on $\alpha = 0.05$) for this situation is (both conclusion and argument must be correct):

1 ☐ The two variances are significantly different as the p-value is small

2 ☐ The two means are significantly different as the p-value is large

3 ☐ The two means are approximately equal as the p-value is large

4 ☐ The two means are approximately equal as the p-value is small

5 ☐ The two means are significantly different as the p-value is small

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Question IV.3 (12)

A new study with 2 doses is planned. It is assumed that the standard deviation within each group is 2, and that a t-test on level $\alpha = 0.05$ should be carried out. The following things are run in R:

```
power.t.test(n=30, delta=2, sd=2, sig.level=0.05)

##
##      Two-sample t test power calculation
##
##               n = 30
##           delta = 2
##              sd = 2
##       sig.level = 0.05
##           power = 0.9677083
##     alternative = two.sided
##
## NOTE: n is number in *each* group

power.t.test(power=0.80, delta=1, sd=2, sig.level=0.05)

##
##      Two-sample t test power calculation
##
##               n = 63.76576
##           delta = 1
##              sd = 2
##       sig.level = 0.05
##           power = 0.8
##     alternative = two.sided
##
## NOTE: n is number in *each* group
```

Only one of the following statements is true. Which?

1 ☐ The risk that a study with $n = 30$ in each group does not find a significant difference between the means, if the real difference is 1, is around 97%

2 ☐ The chance that a study with $n = 30$ in each group finds a significant difference between the means, if the real difference is 1, is around 97%

3 ☐ The chance that a study with $n = 64$ in each group finds a significant difference between the means, if the real difference is 1, is around 80%

4 ☐ The risk that a study with $n = 64$ in each group does not find a significant difference between the means, if the real difference is 1, is around 80%

5 ☐ The risk that a study with $n = 30$ in each group does not find a significant difference between the means, if the real difference is 2, is around 97%

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Exercise V

In Danish power plants materials are being burned to generate electricity and in this combustion CO2 is emitted. It's a gas which enhances the Greenhouse effect and therefore contributes to warming up the atmosphere. This has many negative consequences, and it is of interest to reduce these emissions. This is done by introducing more wind and solar energy production into the system.

Each day, CO2 emissions are calculated (in grams of CO2 equivalent gas) per kWh electricity produced in Denmark based on data from ENTSO-E about the production.

Column 1 of the table below shows the date. In Column 2, average values of CO2 emissions are given for the 15 days with the highest wind energy production in the period from December 1, 2016 to April 1, 2017. Column 3 in the table shows electricity generation with coal, this column is not used in the first two questions.

t	co2intensity (gCO2eq/kWh)	coal (MW)
2016-12-02	230	1016
2016-12-25	205	817
2016-12-26	203	746
2017-01-01	212	948
2017-01-05	292	1448
2017-01-12	260	1398
2017-02-08	317	1409
2017-02-12	321	1578
2017-02-21	235	1102
2017-02-22	268	1325
2017-02-23	233	1187
2017-03-01	253	1195
2017-03-02	260	1093
2017-03-16	212	976
2017-03-22	250	1095

The data is read into R in a data.table `X` and the following is run:

```
## Put observations in x
x <- X$co2intensity
## Number of simulated samples
k <- 100000
n <- length(x)

## Simulation
simsamples <- replicate(k, sample(x, replace = TRUE))
## Calculate the mean of each simulated sample
simmeans <- apply(simsamples, 2, mean)
```

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```r
## Quantiles of the differences gives the CI
quantile(simmeans, c(0.005, 0.995))

##     0.5%    99.5%
## 227.2667 275.2000

quantile(simmeans, c(0.01, 0.99))

##       1%      99%
## 229.3333 272.8000

quantile(simmeans, c(0.025, 0.975))

##     2.5%    97.5%
## 232.3333 269.2667

quantile(simmeans, c(0.05, 0.95))

##       5%      95%
## 235.0000 265.9333

quantile(simmeans, c(0.1, 0.9))

##      10%      90%
## 238.0667 262.4000
```

Question V.1 (13)

A 99% bootstrapped confidence interval for the mean of the CO2-intensity is wanted, without assumptions about the distribution. What is the correct interval?

1 ☐ $[227, 275]$

2 ☐ $[229, 273]$

3 ☐ $[232, 269]$

4 ☐ $[235, 266]$

5 ☐ $[238, 262]$

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Question V.2 (14)

Which one of the following statements is not correct? Each statement is about a null hypothesis for the mean level of CO2 intensity μ_{CO2} at high wind energy production, and the conclusion is drawn based on the results of the R code? (Note again: there are 4 true and 1 false statements - you must find the false statement!)

1 ☐ The null hypothesis $H_0 : \mu_{CO2} = 200$ would have been rejected on a 10% significance level

2 ☐ The null hypothesis $H_0 : \mu_{CO2} = 220$ would have been rejected on a 5% significance level

3 ☐ The null hypothesis $H_0 : \mu_{CO2} = 230$ would have been rejected on a 5% significance level

4 ☐ The null hypothesis $H_0 : \mu_{CO2} = 270$ would have been rejected on a 5% significance level

5 ☐ The null hypothesis $H_0 : \mu_{CO2} = 270$ would have been rejected on a 1% significance level

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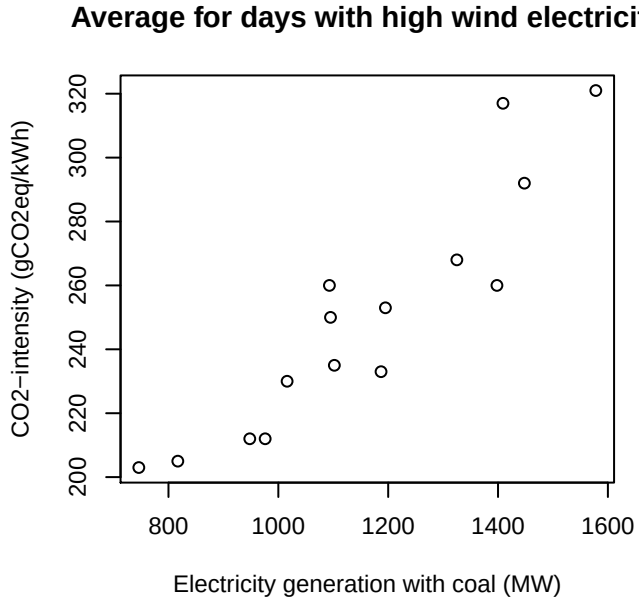
Question V.3 (15)

In this question, the relationship between the electricity generation with coal (Column 3) and the CO2-intensity at high wind electricity generation (Column 2) is investigated.

To visualize the relation the following scatter plot with the observations is created:



Based on a consideration of the plot, which of the following statements is the most correct conclusion?

1 ☐ The correlation is approximately -1.2

2 ☐ The correlation is approximately 0.1

3 ☐ The correlation is approximately 0

4 ☐ The correlation is approximately 0.9

5 ☐ The correlation is approximately 1.2

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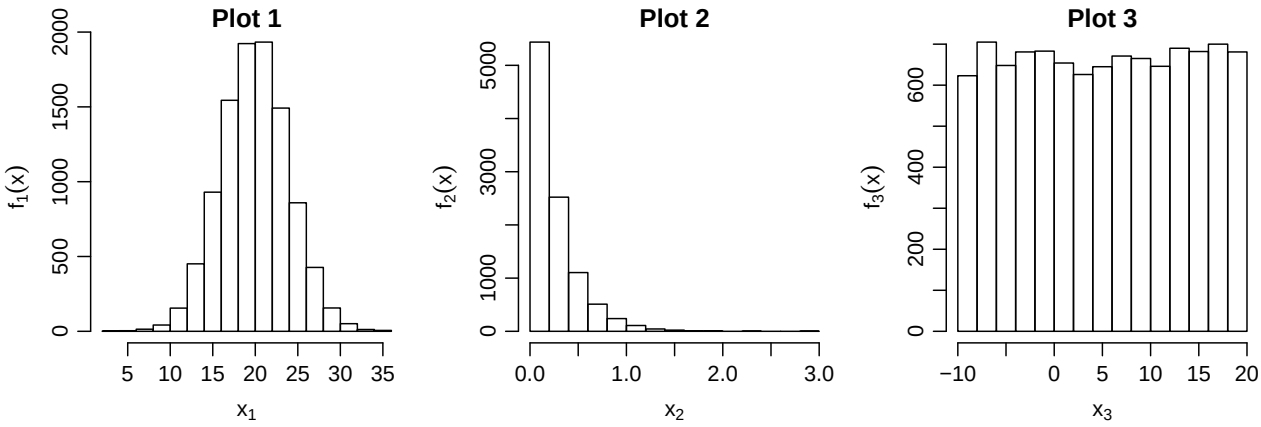
Exercise VI

This exercise is about random variables and simulation.

Question VI.1 (16)

The following three histograms are of values from simulations of $n = 10000$ observations from three distributions:



Which of the following distributions are simulated (both the ordering and parameter values must be correct)?

1 ☐ Plot 1: $X_1 \sim N(10, 4^2)$, Plot 2: $X_2 \sim \text{Exp}(4)$ and Plot 3: $X_3 \sim U(5, 20)$

2 ☐ Plot 1: $X_1 \sim U(5, 20)$, Plot 2: $X_2 \sim \text{Exp}(1)$ and Plot 3: $X_3 \sim N(20, 4^2)$

3 ☐ Plot 1: $X_1 \sim U(-10, 20)$, Plot 2: $X_2 \sim \text{Exp}(1)$ and Plot 3: $X_3 \sim N(20, 4^2)$

4 ☐ Plot 1: $X_1 \sim N(20, 4^2)$, Plot 2: $X_2 \sim \text{Exp}(4)$ and Plot 3: $X_3 \sim U(-10, 20)$

5 ☐ Plot 1: $X_2 \sim \text{Exp}(4)$, Plot 2: $X_1 \sim N(10, 4^2)$ and Plot 3: $X_3 \sim U(5, 20)$

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Question VI.2 (17)

Let the random variables $X_i \sim N(2, 4^2)$ for $i = 1, \ldots, 20$ be i.i.d. and define the following random variables as function of these

$$\bar{X} = \frac{1}{20} \sum_{i=1}^{20} X_i,$$

$$S = \sqrt{\frac{1}{19} \sum_{i=1}^{20} (X_i - \bar{X})^2},$$

$$Y = \frac{\bar{X} - 2}{S}.$$

Which distribution does the random variable Y follow?

1 ☐ Y follows the normal distribution $N(0, 1^2)$

2 ☐ Y follows the normal distribution $N(0, 4^2)$

3 ☐ Y follows the χ^2-distribution with 20 degrees of freedom

4 ☐ Y follows t-distribution with 20 degrees of freedom

5 ☐ Y follows t-distribution with 19 degrees of freedom

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Exercise VII

A recreational runner wants to measure the effect of his training. For this purpose, he has measured values of average pulse (beats per minute), weeks in the training program and speed (km/h), for a particular stretch he runs frequently.

The recreational runner has decided to measure the effect by examining whether the average speed increases over time (weeks).

Data reading in R is:

```
week <- c(1, 1, 1, 3, 3, 4, 5, 5, 5, 5, 6, 6, 7, 7, 8, 9, 9, 10, 12, 12, 13, 13,
          15, 15, 15, 16, 16)

pulse <- c(137.6, 140.1, 143.0, 148.6, 135.6, 139.0, 155.8, 135.0, 149.0, 133.0,
           135.3, 139.8, 137.2, 137.9, 136.8, 134.6, 152.3, 131.9, 137.2, 160.3,
           130.9, 130.9, 131.8, 131.4, 135.6, 138.6, 136.3)

speed <- c(10.01, 10.02, 10.39, 11.86,  9.65, 10.40, 12.60,  9.80, 11.52,  9.59,
           10.26, 10.42, 10.05, 10.48, 10.03, 10.29, 12.22, 10.27, 10.80, 13.79,
           10.40,  9.49, 10.09, 10.34, 11.18, 11.33, 11.34)
```

Initially, parameters in the following model are estimated

$$\text{speed}_i = \beta_0 + \beta_1 \cdot \text{week}_i + \epsilon_i, \quad \varepsilon_i \sim N(0, \sigma^2).$$

Further it is assumed that the ε_i's are independent.

The result of the estimation in R is:

```
 summary(lm(speed~week))

##
## Call:
## lm(formula = speed ~ week)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -1.3909 -0.6058 -0.3407  0.2741  2.9492
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 10.36053    0.38514  26.901   <2e-16 ***
## week         0.04003    0.04046   0.989    0.332
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.008 on 25 degrees of freedom
## Multiple R-squared:  0.03766,Adjusted R-squared:  -0.0008306
## F-statistic: 0.9784 on 1 and 25 DF,  p-value: 0.3321
```

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Question VII.1 (18)

At the significance level $\alpha = 0.05$ what is the conclusion in relation to increased speed (both argument and conclusion must be correct)?

1 ☐ There is no significant effect of the training, since $0.0377 < 0.05$

2 ☐ There is a significant effect of the training, since $26.9 > t_{0.975}$ (where the degrees of freedom for the t-distribution is 25)

3 ☐ There is no significant effect of the training, since $0.332 > 0.05$

4 ☐ There is a significant effect of the training, since $0.04 < 0.05$

5 ☐ There is a significant effect of the training, since $0.989 > 0.95$

Question VII.2 (19)

With the model above what is the 95% confidence interval for β_0?

1 ☐ $10.36 \pm 1.008 \cdot 1.96$

2 ☐ $10.36 \pm \frac{0.385}{\sqrt{25}} \cdot 2.06$

3 ☐ $0.040 \pm 0.0405 \cdot 1.96$

4 ☐ $0.040 \pm \frac{0.989}{25} \cdot 1.96$

5 ☐ $10.36 \pm 0.385 \cdot 2.06$

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The recreational runner now decides to investigate a model for the relationship between pulse and velocity (ignoring weeks).

The result of the estimation in R is:

```
summary(lm(speed ~ pulse))

##
## Call:
## lm(formula = speed ~ pulse)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.78371 -0.40486 -0.00015  0.35978  0.96661
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -5.06134    1.82173  -2.778   0.0102 *
## pulse        0.11324    0.01308   8.659 5.39e-09 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.5139 on 25 degrees of freedom
## Multiple R-squared:  0.7499,Adjusted R-squared:  0.7399
## F-statistic: 74.98 on 1 and 25 DF,  p-value: 5.388e-09
```

Based on the above model, the recreational runner wants an uncertainty interval for a new run. He uses as the assumption that he can hold an average pulse of 160 beats per minute.

As an aid to the task he has calculated the following number:

```
length(week)

## [1] 27

c(mean(week), var(week))

## [1]  8.222222 23.871795

c(mean(pulse), var(pulse))

## [1] 139.09259  59.38225

c(mean(speed), var(speed))

## [1] 10.689630  1.015396
```

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Question VII.3 (20)

What is the 95% prediction interval for the speed?

1 ☐ $-5.06 + 0.113 \cdot 160 \pm 2.06 \cdot 0.51 \cdot \sqrt{\frac{1}{27} + \frac{(139.1-160)^2}{27 \cdot 59.38}}$

2 ☐ $-5.06 + 0.113 \cdot 160 \pm 2.06 \cdot 0.51 \cdot \sqrt{1 + \frac{1}{27} + \frac{(139.1-160)^2}{26 \cdot 1.02}}$

3 ☐ $-5.06 + 0.113 \cdot 160 \pm 2.06 \cdot 0.51 \cdot \sqrt{1 + \frac{1}{27} + \frac{(139.1-160)^2}{26 \cdot 59.38}}$

4 ☐ $-5.06 + 0.113 \cdot 160 \pm 2.06 \cdot 0.51^2 \cdot \sqrt{\frac{1}{27} + \frac{(139.1-160)^2}{59.38}}$

5 ☐ $-5.06 + 0.113 \cdot 160 \pm 2.06 \cdot 0.51 \cdot \sqrt{\frac{1}{27} + \frac{(139.1-160)^2}{59.38^2}}$

The recreational runner now decides to estimate a multiple regression model that contains both weeks and pulse:

$$\text{speed}_i = \beta_0 + \beta_1 \cdot \text{week}_i + \beta_2 \cdot \text{pulse}_i + \epsilon_i; \quad \epsilon_i \sim N(0, \sigma^2)$$

The result from R is:

```
summary(lm(speed ~ week + pulse))

##
## Call:
## lm(formula = speed ~ week + pulse)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.59154 -0.13508 -0.00055  0.15562  0.43438
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -8.033531   0.945936  -8.493 1.08e-08 ***
## week         0.094014   0.010414   9.028 3.48e-09 ***
## pulse        0.129052   0.006603  19.545 3.02e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.2501 on 24 degrees of freedom
## Multiple R-squared:  0.9431,Adjusted R-squared:  0.9384
## F-statistic:   199 on 2 and 24 DF,  p-value: 1.148e-15
```

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Question VII.4 (21)

At level $\alpha = 0.05$ which of the following statements is correct (both conclusion and argument must be correct)?

1 ☐ Neither the effect of weeks nor the effect of pulse is significant as $0.094 > 0.05$ and $0.129 > 0.05$

2 ☐ Both the effect of weeks and the effect of pulse is significant as $0.01 < 0.05$ and $0.0066 < 0.05$

3 ☐ Since $0.094 < 0.129$ the effect of weeks is significant, while the effect of pulse is not significant

4 ☐ Both the effect of weeks and the effect of pulse is significant since $3.5 \cdot 10^{-9} < 0.05$ and $3.0 \cdot 10^{-16} < 0.05$

5 ☐ Neither the effect of weeks nor the effect of pulse is significant since $3.5 \cdot 10^{-9} < 0.05$ and $3.0 \cdot 10^{-16} < 0.05$

Question VII.5 (22)

Which statement about the interpretation of the model is correct?

1 ☐ When weeks increase by 0.094 the pulse increase by 0.129

2 ☐ For a given pulse the expected speed increase with 0.094km/h per week.

3 ☐ The model is meaningless since $-8.03 < 0$ and the speed must be positive

4 ☐ Since the degree of explanation is about 0.25, the model have explained about 3/4 of the variation

5 ☐ Since all parameters are significant, it can be seen that all model assumptions are fulfilled

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Exercise VIII

A person has twice evaluated the sharpness (`Sharpness`) for each of 12 different setups (`Treat`) of images on computer screens, ie. 24 observations of sharpness in total split on 12 setups. The scale is a continuous scale from 0 to 15, in practice, done by marking the value on a line.

The result of the usual analysis of variance of these data gave the following R output, however, some of the values are replaced by the letters A, ..., F:

```
Analysis of Variance Table

Response: Sharpness
          Df Sum Sq Mean Sq F value Pr(>F)
Treat      A   93.7     C       E      F
Residuals  B   51.9     D
```

Question VIII.1 (23)

What are the values of A and B?

1 ☐ A=11 and B=12

2 ☐ A=1 and B=22

3 ☐ A=93.7/51.9 and B= 51.9/22

4 ☐ A=12 and B=24

5 ☐ A=11 and B=23

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Question VIII.2 (24)

Similarly, another person evaluated the sharpness (`Sharpness`) a number of times for each of different setups (Treat) of images on computer screens. The result of the usual analysis of variance of these data gave the following R output, however, some of the values are replaced by the letters G,. . . ,J:

```
Analysis of Variance Table

Response: Sharpness
          Df Sum Sq Mean Sq F value Pr(>F)
Treat      7  111.5     G       I      J
Residuals 32   88.4     H
```

What is the value of J?

1 ☐ 1.261

2 ☐ 5.766

3 ☐ 0.0002

4 ☐ 0.2188

5 ☐ 0.3002

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Exercise IX

Eight experts have each assessed the sharpness (`Sharpness`) for each of 12 different setups (`Treat`) of images on computer screens, ie. 96 observations of sharpness in total split on 12 setups. The scale is a continuous scale from 0 to 15, in practice, done by marking the value on a line:

	Setup 1	Setup 2	Setup 3	Setup 4	Setup 5	Setup 6	Setup 7	Setup 8	Setup 9	Setup 10	Setup 11	Setup 12
Person 1	9.30	4.70	6.60	8.80	5.90	7.20	7.60	5.50	8.10	8.20	6.40	7.40
Person 2	10.20	7.00	8.80	10.70	9.80	7.00	9.20	9.60	8.00	11.80	8.90	10.20
Person 3	11.50	9.50	8.00	12.90	10.00	8.20	11.50	6.40	8.60	11.20	7.70	11.00
Person 4	11.90	6.60	8.20	12.70	5.40	9.00	4.90	8.10	10.10	12.90	8.20	8.70
Person 5	10.70	4.20	5.40	11.40	8.30	7.10	6.80	3.80	9.60	8.60	3.80	10.80
Person 6	10.90	9.10	7.10	11.40	8.60	5.90	8.50	10.50	6.40	11.70	9.50	7.10
Person 7	8.50	5.00	6.30	10.80	6.80	4.60	4.70	8.80	6.70	10.00	5.50	7.50
Person 8	12.60	8.90	10.70	13.50	11.40	8.90	9.50	8.60	7.40	13.50	8.70	9.80

The result of a usual twoway analysis of variance of these data gave the following R-output:

```
## Analysis of Variance Table
##
## Response: Sharpness
##           Df Sum Sq Mean Sq F value    Pr(>F)
## Person     7 122.42 17.4881  8.4596 1.212e-07 ***
## Setup     11 224.28 20.3894  9.8630 6.864e-11 ***
## Residuals 77 159.18  2.0673
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Question IX.1 (25)

What are the conclusions of the usual hypothesis tests for such an analysis? (Both conclusions and arguments must be correct)

1 ☐ There is a difference between the mean sharpness for persons, but not for setups

2 ☐ There is a difference between the mean sharpness for setups, but not for persons

3 ☐ There is a difference between the variances for persons, but not for setups

4 ☐ There is a difference between the mean sharpness for both setups and persons

5 ☐ There is a difference between the variances for setups, but not for persons

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Question IX.2 (26)

Which probability distribution has been used to find the p-value provided for `Setup` in the output?

1 ☐ The z-distribution (= standard normal distribution)

2 ☐ The t-distribution with 159 degrees of freedom

3 ☐ The χ^2-distribution with 159 degrees of freedom

4 ☐ The F-distribution with degrees of freedom 7 and 11

5 ☐ The F-distribution with degrees of freedom 11 and 77

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Exercise X

In a questionnaire survey under an Introduction to Statistics lecture the participants were asked about different topics. This assignment will cover the analysis of the answers to one of the questions. There were 32 respondents in total.

The question asked was: "Are you worried that we don't do enough to stop climate change?". To this the students answered the following:

Answer	Count
Yes	27
No	5

Question X.1 (27)

Using the "Plus 2" correction when calculating the usual 95% confidence interval for the proportion of students who are worried about the climate (answering yes), one gets:

1 ☐ $0.844 \pm 2.04 \cdot \frac{0.844}{36} = [0.796, 0.892]$

2 ☐ $0.806 \pm 1.69 \cdot \frac{0.806}{36} = [0.768, 0.844]$

3 ☐ $0.806 \pm 1.96\sqrt{\frac{0.806 \cdot 0.194}{36}} = [0.677, 0.935]$

4 ☐ $0.844 \pm 1.69\sqrt{\frac{0.844 \cdot 0.156}{32}} = [0.736, 0.952]$

5 ☐ $0.844 \pm 1.96\sqrt{\frac{0.844 \cdot 0.156}{36}} = [0.725, 0.963]$

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Exercise XI

In a questionnaire survey 114 respondents were asked about their traffic preferences. Two questions were asked, which had the following three identical answer options: "Car", "Bike" and "Train or bus".

The two questions were: "If you have 10 km to DTU from your home, what kind of transportation would you prefer during the summer (they take about equal time)?", and: "if you have 10 km to DTU from your home, which kind of transportation would you prefer during the winter (they take about equal time)?".

The following distribution of answers were observed:

		Winter		
		Car	Bike	Train or bus
Summer	Car	27	2	4
	Bike	20	22	11
	Train or bus	13	3	12

The following is run in R:

```
## The data table
tbl <- matrix(c(27, 20, 13, 2, 22, 3, 4, 11, 12), nrow = 3)
rownames(tbl) <- c("Car", "Bike", "Trainorbus")
colnames(tbl) <- c("Car", "Bike", "Trainorbus")
tbl

##            Car Bike Trainorbus
## Car         27    2          4
## Bike        20   22         11
## Trainorbus  13    3         12

## Row sums (distribution for summer)
margin.table(tbl, 1)

##        Car       Bike Trainorbus
##         33         53         28

## Column sums (distribution for winter)
margin.table(tbl, 2)

##        Car       Bike Trainorbus
##         60         27         27

## Chi2-test
chisq.test(tbl, correct = FALSE)

##
##  Pearson's Chi-squared test
##
## data:  tbl
## X-squared = 27.608, df = 4, p-value = 1.498e-05
```

Continues on page 27

Question XI.1 (28)

What is the expected count of preferences for: bike in the summer and car in the winter, under the null hypothesis: there is independence between traffic preferences in summer and winter in the surveyed population?

1 ☐ $e_{11} = 114 \cdot \frac{33}{114} \cdot \frac{60}{114} = 17.37$

2 ☐ $e_{23} = 114 \cdot \frac{53}{114} \cdot \frac{27}{114} = 12.55$

3 ☐ $e_{12} = 114 \cdot \frac{33}{114} \cdot \frac{27}{114} = 7.816$

4 ☐ $e_{33} = 114 \cdot \frac{28}{114} \cdot \frac{27}{114} = 6.877$

5 ☐ $e_{21} = 114 \cdot \frac{53}{114} \cdot \frac{60}{114} = 27.89$

Question XI.2 (29)

What is the conclusion at significance level 1% of the test for independence of traffic preferences in summer and winter (both conclusion and argument must be correct)?

1 ☐ No significant dependence between traffic preferences is found since the p-value > 0.01

2 ☐ A significant dependence between traffic preferences is found since the p-value < 0.01

3 ☐ No significant dependence between traffic preferences is found since the p-value < 0.01

4 ☐ A significant dependence between traffic preferences is found since the p-value > 0.01

5 ☐ The question cannot be answered with the given information

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Question XI.3 (30)

There are 60 out of 114 who prefer to drive car in the winter. Would the following null hypothesis

$$H_0 : p_{\text{car,winter}} = 50\%,$$

be rejected at the 5% significance level with the usual test (both conclusion and the p-value must be correct)?

1 ☐ Yes, since the p-value is $0.024 < 0.05$

2 ☐ No, since the p-value is $0.57 > 0.05$

3 ☐ No, since the p-value is $0.40 > 0.05$

4 ☐ Yes, since the p-value is $0.089 > 0.05$

5 ☐ No, since the p-value is $0.21 > 0.05$

THE EXAM IS FINISHED. ENJOY THE SUMMER!