Technical University of Denmark *Page 1 of 27 pages.*

Written examination: 20. August 2017

Course name and number: **Introduction to Statistics (02323 og 02402)**

Aids and facilities allowed: All

The questions were answered by

(student number) (signature) (table number)

There are 30 questions of the "multiple choice" type included in this exam divided on 9 exercises. To answer the questions you need to fill in the prepared 30-question multiple choice form (on 6 seperate pages) in CampusNet.

5 points are given for a correct answer and -1 point is given for a wrong answer. ONLY the following 5 answer options are valid: 1, 2, 3, 4 or 5. If a question is left blank or another answer is given, then it does not count (i.e. "0 points"). Hence, if more than one answer option is given to a single question, which in fact is technically possible in the online system, it will not count (i.e. "0 points"). The number of points corresponding to specific marks or needed to pass the examination is ultimately determined during censoring.

The final answers should be given in the exam module in CampusNet. The table sheet here is ONLY to be used as an "emergency" alternative (remember to provide your study number if you hand in the sheet).

Exercise	I.1	I.2	I.3	II.1	II.2	II.3	III.1	III.2	IV.1	IV.2
Question	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Answer										

Exercise	IV.3	IV.4	IV.5	V.1	V.2	V.3	V.4	V.5	VI.1	VI.2
Question	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
Answer										

Exercise	VI.3	VI.4	VI.5	VII.1	VIII.1	VIII.2	VIII.3	IX.1	IX.2	IX.3
Question	(21)	(22)	(23)	(24)	(25)	(26)	(27)	(28)	(29)	(30)
Answer										

The questionnaire contains 27 pages.

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Multiple choice questions: *Note that not all the suggested answers are necessarily meaningful. In fact, some of them are very wrong but under all circumstances there is one and only one correct answer to each question.*

Exercise I

A swimming team goes on an weekly training camp with a focus on training the swimming stroke front crawl. A test is carried out where the time, for each swimmer swimming the same distance in front crawl, is measured. The test is carried out before and after the camp.

The measured times are stored (in the same order for the swimmers) in the following vectors in R: `before` holds the times before and `after` holds the times after the training camp.

The following hypothesis must be tested

$$\mu_{\text{after}} - \mu_{\text{before}} = 0$$
$$\mu_{\text{after}} - \mu_{\text{before}} \neq 0$$

where μ_{before} and μ_{after} denotes the mean times for the entire team before and after the camp.

Question I.1 (1)

Which of the following `R`-calls correctly calculates the p-value for a t-test of the hypothesis?

1 ☐ `t.test(after, before, mu=0)`

2 ☐ `t.test(after, before, mu=-10)`

3 ☐ `t.test(after, before, mu=10)`

4 ☐ `t.test(after, mu=10)`

5 ☐ `t.test(after-before, mu=0)`

Question I.2 (2)

The p-value of the test was calculated to 0.00287. Can the null hypothesis be rejected at significance level $\alpha = 5\%$ (both conclusion and argument must be correct)?

1 ☐ Yes, since the p-value is below the significance level the null hypothesis is rejected

2 ☐ No, since the p-value is below the significance level the null hypothesis is accepted

3 ☐ Yes, since the p-value is over the significance level the null hypothesis is rejected

4 ☐ No, since the p-value is over the significance level the null hypothesis is accepted

5 ☐ More information is needed in order to decide against the null hypothesis

Question I.3 (3)

Each day at the training camp, there is a random drawing about who should do the dishes. There must be 4 each day for doing the dishes and there are in total 35 participants. For each participant there is equally high probability of being drawn each day. Calculate the probability that a participant will not do the dishes at all during training camp, which includes 7 evenings with dish washing.

1 ☐ 1 - $\binom{7}{0} \cdot 0.144^0 \cdot (1 - 0.144)^{7-0} = 0.57$

2 ☐ $\binom{5}{2} \cdot 0.144^2 \cdot (1 - 0.144)^{5-2} = 0.09$

3 ☐ $\binom{7}{7} \cdot 0.798^7 \cdot (1 - 0.798)^{7-7} = 0.21$

4 ☐ $\binom{7}{7} \cdot 0.886^7 \cdot (1 - 0.886)^{7-7} = 0.43$

5 ☐ $\binom{5}{2} \cdot 0.886^2 \cdot (1 - 0.886)^{5-2} = 0.01$

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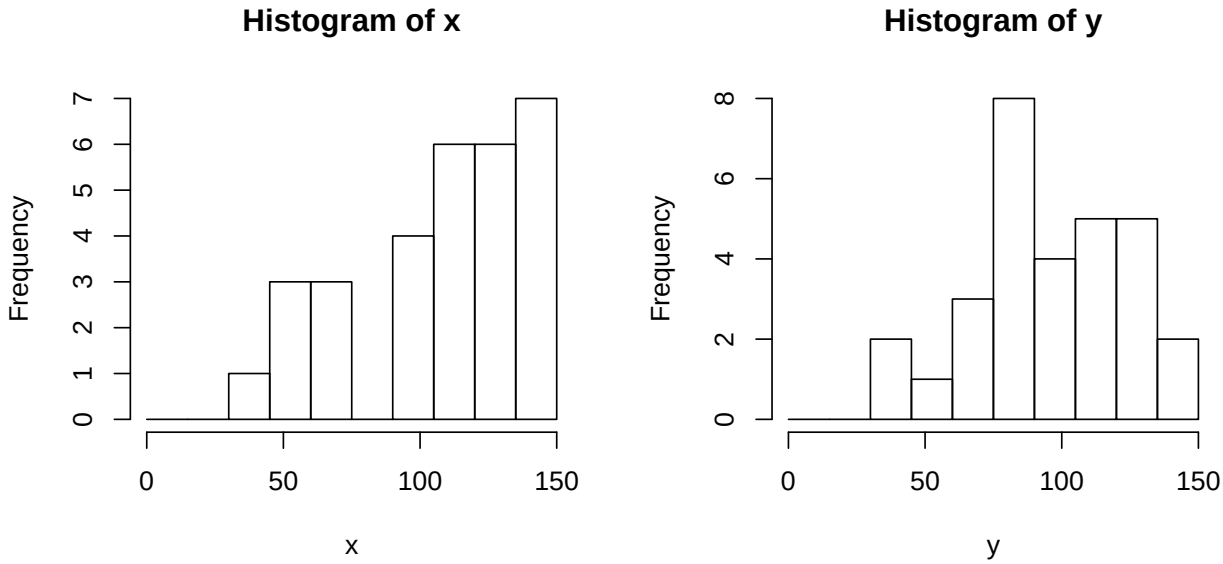
Exercise II

In connection with the exam in Introduction to Statistics, it is desired to examine whether foreign students are doing well. The score of the exam is calculated as a number between -30 and 150, as there are 30 questions and a wrong answer gives -1 point and a correct answer gives 5 points. There can only be given one answer to each question.

Two random samples of the score has been taken: one for foreign students (`x`) and one for Danish students (`y`). Each sample has 30 observations.

Question II.1 (4)

To assess the most appropriate analysis, a histogram is plotted of each sample:



What is the most appropriate statement based on the given information?

1 ☐ Nothing indicates that the samples don't come from symmetrical distributed populations

2 ☐ The samples cannot be assumed to come from symmetrical distributed populations. This is supported by the histograms, in particular the distribution of `x` appears to be right-skewed

3 ☐ The samples cannot be assumed to come from symmetrical distributed population. This is supported by the histograms, in particular the distribution of `x` appear to be left-skewed

4 ☐ The populations from which the samples are taken can both be assumed to be exponentially distributed

5 ☐ The populations from which the samples are taken can both be assumed to be normally distributed

Question II.2 (5)

It is decided that the best analysis is included in the following R code:

```
## Number of simulations
k <- 10000
## Simulate each sample k times
simxsamples <- replicate(k, sample(x, replace=TRUE))
simysamples <- replicate(k, sample(y, replace=TRUE))
## Calculate the sample mean differences
simmeandifs <- apply(simxsamples,2,mean) - apply(simysamples,2,mean)
## Quantiles of the differences gives the CI
quantile(simmeandifs, c(0.005,0.995))

##  0.5% 99.5%
## -9.23 31.63

quantile(simmeandifs, c(0.025,0.975))

##   2.5%  97.5%
## -4.125 26.106

## CI for the median differences
simmediandifs <- apply(simxsamples,2,median) - apply(simysamples,2,median)
quantile(simmediandifs, c(0.005,0.995))

##   0.5%  99.5%
## -10.42  43.05

quantile(simmediandifs, c(0.025,0.975))

##   2.5%  97.5%
## -3.975 39.525
```

Which of the following statements is correct?

1 ☐ Non-parametric bootstrap confidence intervals have been calculated for differences between two populations

2 ☐ Parametric bootstrap confidence intervals have been calculated for differences between two populations

3 ☐ Confidence intervals for differences between two populations have been calculated under the assumption of normal distributions

4 ☐ Confidence intervals for differences between two populations have been calculated under the assumption of exponential distributions

5 ☐ Confidence intervals for differences between two populations have been calculated under the assumption of Poisson distributions

Question II.3 (6)

The following hypothesis should be tested at significance level $\alpha = 5\%$

$$H_0 : q_{0.5,\mathrm{x}} = q_{0.5,\mathrm{y}}$$
$$H_1 : q_{0.5,\mathrm{x}} \neq q_{0.5,\mathrm{y}}$$

where $q_{0.5,\mathrm{x}}$ denotes the 50% quantile for foreign students and $q_{0.5,\mathrm{y}}$ denotes the 50% quantile for Danish students.

Which of the following statements is correct (not all of the statements are necessarily meaningful)?

1 ☐ H_0 is rejected and it can be concluded that Danish students perform significantly better than foreign students at the indicated level of significance

2 ☐ H_0 is rejected and it can be concluded that foreign students perform significantly better than Danish students at the indicated level of significance

3 ☐ H_0 is not rejected and it cannot be concluded that Danish students perform significantly different than foreign students at the indicated level of significance

4 ☐ H_0 is not rejected and it can be concluded that Danish students perform significantly different than foreign students at the indicated level of significance

5 ☐ None of the above statements are correct

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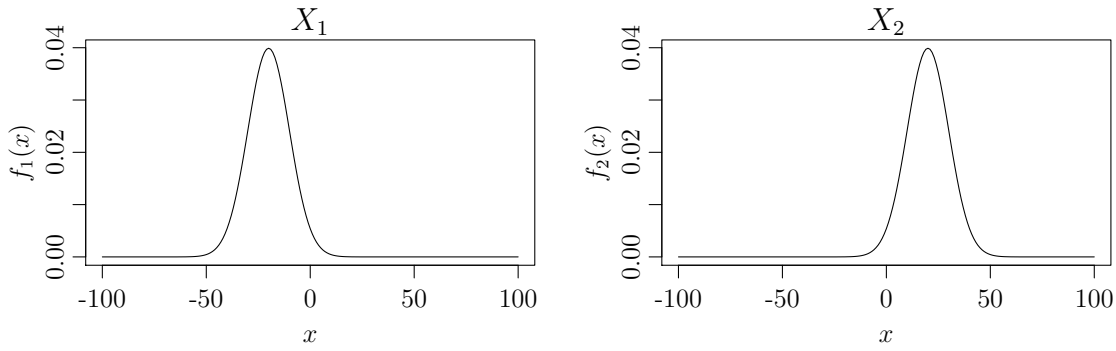
Exercise III

Let two independent random variables be given by

$$X_1 \sim N(-20, 10^2) \quad \text{and} \quad X_2 \sim N(20, 10^2).$$

Their probability density functions (pdfs) are then:

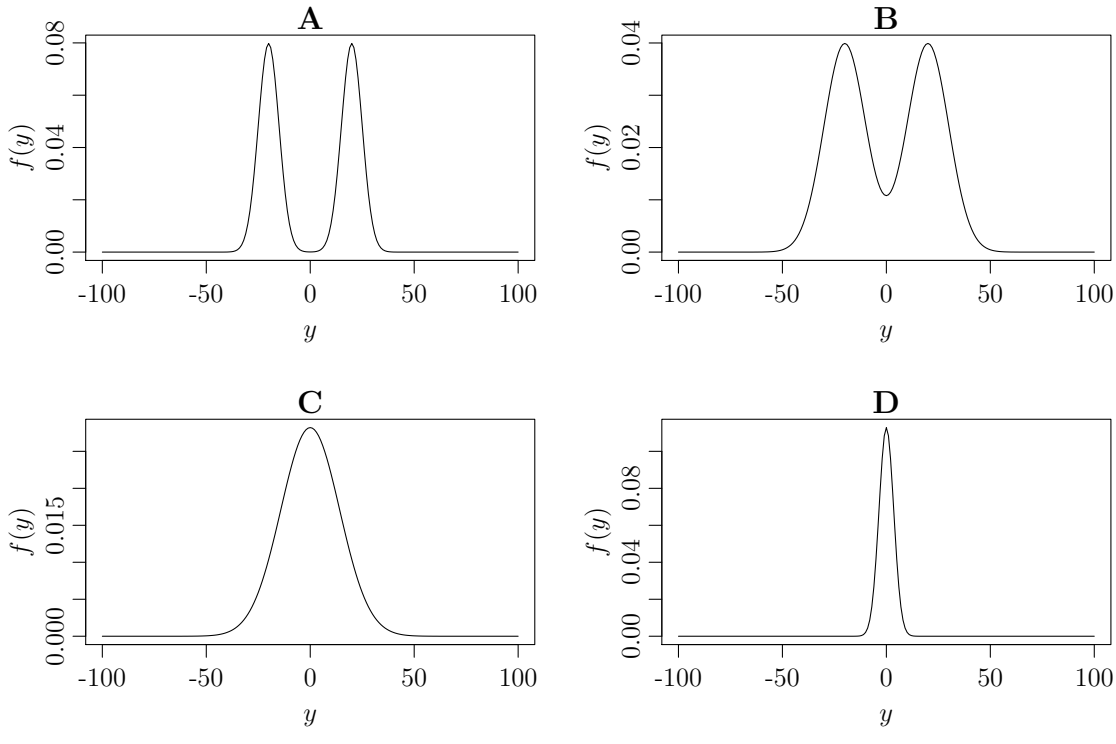


Question III.1 (7)

Now a new random variable is defined by

$$Y = X_1 + X_2.$$

Which of the following plots is then the pdf for Y?



1 ☐ Plot A

2 ☐ Plot B

3 ☐ Plot C

4 ☐ Plot D

5 ☐ None of the shown plots can be close to the pdf of Y

Question III.2 (8)

Assuming X_1 and X_2 each represent a population and the test for difference in mean value with the commonly used non-paired t-test should be carried out. What is the smallest sample size $n = n_1 = n_2$ that must be taken from each population, at significance level $\alpha = 5\%$, in order to achieve a power of the test of at least 99%?

1 ☐ $n = 4$ observations in each sample

2 ☐ $n = 12$ observations in each sample

3 ☐ $n = 38$ observations in each sample

4 ☐ $n = 69$ observations in each sample

5 ☐ $n = 248$ observations in each sample

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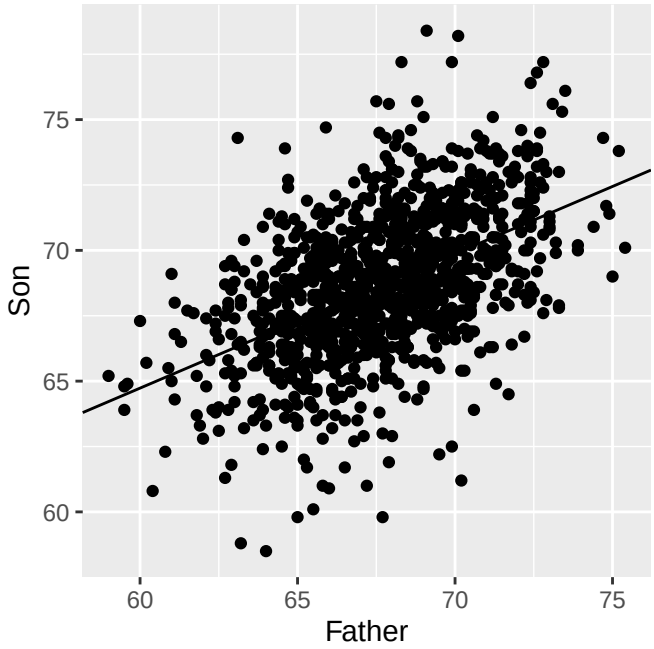
Exercise IV

The figure below shows the relation between the height of about 1000 fathers and their sons measured in inches:



The shown regression line describes the fit of the following model

$$Y_i = \beta_0 + \beta_1 \cdot x_i + \varepsilon_i, \quad \varepsilon_i \sim N(0, \sigma^2) \text{ and i.i.d.},$$

where Y_i is the height of the i'th son and x_i is the height of the i'th father.

Question IV.1 (9)

Which of the following statements is a correct description of the regression line?

1 ☐ The line describes an estimate of the mean height of the sons as a function of their fathers mean height

2 ☐ The line describes an estimate of the linear correlation between the average height of father and son

3 ☐ The line describes an estimate of the fathers mean height as a function of the height of their sons

4 ☐ The line describes an estimate of the sons mean height as a function of the height of their fathers

5 ☐ The line describes the height of a son as a function of the height of the father

Question IV.2 (10)

It is chosen to analyze the data with the following `R` code, where `fs` is a data frame with the columns `Son` and `Father` holding the observed heights:

```
summary(fit <- lm(Son ~ Father, data=fs))
```

Which gives the following result where two numbers are replaced by letters:

```
Call:
lm(formula = Son ~ Father, data = fs)

Residuals:
    Min      1Q  Median      3Q     Max
-8.8910 -1.5361 -0.0092  1.6359  8.9894

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 33.89280          A   18.49   <2e-16 ***
Father       0.51401          B   19.00   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 2.438 on 1076 degrees of freedom
Multiple R-squared:  0.2512, Adjusted R-squared:  0.2505
F-statistic: 360.9 on 1 and 1076 DF,  p-value: < 2.2e-16
```

How large a proportion of the variation in the height of sons is not explained by the height of the fathers?

1 ☐ Approximately 25%

2 ☐ Approximately 50%

3 ☐ Approximately 75%

4 ☐ Approximately 86.5%

5 ☐ Approximately 66%

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Question IV.3 (11)

What is the estimate of the standard deviation of the coefficient for `Father`?

1 ☐ $\hat{\sigma}_{\beta_1} = 0.514/2.438 = 0.211$

2 ☐ $\hat{\sigma}_{\beta_1} = 2.438/1076 = 0.00227$

3 ☐ $\hat{\sigma}_{\beta_1} = 0.514 \cdot 19.00 = 9.77$

4 ☐ $\hat{\sigma}_{\beta_1} = 33.89/18.49 = 1.83$

5 ☐ $\hat{\sigma}_{\beta_1} = 0.514/19.0 = 0.027$

Question IV.4 (12)

Given the following calculations in `R`, what is a 95% confidence interval for the mean height of sons of fathers who are 75 inches tall?

```
mean(fs$Father); var(fs$Father)

## [1] 67.68683
## [1] 7.539566

mean(fs$Son); var(fs$Son)

## [1] 68.68423
## [1] 7.930949
```

1 ☐ $33.893 + 0.514 \cdot 75 \pm 1.96 \cdot 2.438 \cdot \sqrt{\frac{1}{1078} + \frac{(75-67.687)^2}{7.540 \cdot (1078-1)}}$

2 ☐ $33.893 + 0.514 \cdot 75 \pm 1.96 \cdot 2.438 \cdot \sqrt{\frac{1}{1078} + \frac{(75-67.687)^2}{7.540}}$

3 ☐ $33.893 + 0.514 \cdot 75 \pm 1.96 \cdot 2.438^2 \cdot \sqrt{\frac{1}{1078} + \frac{(75-67.687)^2}{7.540 \cdot (1078-1)}}$

4 ☐ $33.893 + 0.514 \cdot 75 \pm 1.96 \cdot 2.438 \cdot \sqrt{\frac{1}{1077} + \frac{(75-67.687)^2}{7.540 \cdot (1077-1)}}$

5 ☐ $33.893 + 0.514 \cdot 75 \pm 1.65 \cdot 2.438^2 \cdot \sqrt{\frac{1}{1077} + \frac{(75-67.687)^2}{7.540 \cdot (1077-1)}}$

Question IV.5 (13)

Now information about each family's monthly income is obtained and the following model is setup

$$Y_i = \beta_0 + \beta_1 \cdot x_{1,i} + \beta_2 \cdot x_{2,i} + \varepsilon_i, \quad \varepsilon_i \sim N(0, \sigma^2) \text{ and i.i.d.},$$

where Y_i is the height of the i'th son, $x_{1,i}$ is height of the i'th father, and $x_{2,i}$ is the income for the i'th family.

Under the following two assumptions:

- Rich families eat better and a better diet has a significant positive effect, which gives the sons of the family a higher growth
- There is independence between the father's height and the family's income

what is the consequence of adding the income into the model (not all answers are necessarily meaningful)?

1 ☐ Inclusion of income in the model will contribute to reducing the residual variance ($\hat{\sigma}^2$) and the uncertainty of the regression coefficient for the father's height (β_1) will be reduced

2 ☐ Inclusion of income in the model will contribute to reducing the residual variance ($\hat{\sigma}^2$), but this will not affect the uncertainty of the regression coefficient for the father's height (β_1)

3 ☐ As the fathers height is independent of the fathers income, the inclusion of income in the model will not affect the estimate of β_1 or the uncertainty of it

4 ☐ Inclusion of income in the model will use one more degree of freedom, such that a confidence interval for β_1 may be expected to be wider than if incomes were not included in the model

5 ☐ One must expect a high degree of multicollinearity between the estimates of β_1 and β_2, so the model must be reduced to a simple linear regression model

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Exercise V

In humans there are a variety of different genetic determined blood type systems. The most well-known are probably the AB0- and Rhesus-systems. Another blood type system is the so-called MN blood type system, which is determined by a single gene Glycophorin A (GPA). In the GPA-gene there are two alleles M and N, such that a human may have the genotype (blood type) MM, MN, or NN.

The distribution of blood types in the MN blood type system is now seeked estimated from a sample of volunteer students of two different Philippine universities. One university, University of the Philippines-Diliman, here shortened UPD, is the country's largest university where students come from all over the country. The second university, Isabela State University, here abbreviated ISU, is a small university where the students primarily come from the local area. The following table lists the distribution of genotypes among the students in the samples from the two universities:

Bloodtype	UDP	ISU
MM	19	43
MN	15	7
NN	17	9

Question V.1 (14)

State the χ^2 test statistic and the conclusion of the test in which the MN blood type distribution in the two universities are compared (both test size and conclusion must be correct).

1 ☐ The test statistic is $\chi^2 = 14.15$, its distribution has 2 degrees of freedom and the test shows that there is some evidence for a difference in the MN blood type distribution at the two universities

2 ☐ The test statistic is $\chi^2 = 14.15$, its distribution has 2 degrees of freedom and the test shows that there is very strong evidence for a difference in the MN blood type distribution at the two universities

3 ☐ The test statistic is $\chi^2 = 3.76$, its distribution has 2 degrees of freedom and the test shows that there is not found any evidence for a difference in the MN blood type distribution at the two universities

4 ☐ The test statistic is $\chi^2 = 4.57$, its distribution has 1 degree of freedom and the test shows that there is evidence for a difference in the MN blood type distribution at the two universities

5 ☐ The test statistic is $\chi^2 = 3.76$, its distribution has 1 degree of freedom and the test shows that there is weak evidence for a difference in the MN blood type distribution at the two universities

Question V.2 (15)

A biological population is said to be in Hardy-Weinberg (HW) equilibrium if the proportion of genotypes can be written as

$$p_{\text{MM}} = p^2,$$
$$p_{\text{MN}} = 2pq,$$
$$p_{\text{NN}} = q^2.$$

Where p and q are the allele frequencies for M and N, respectively. They are calculated by

$$p = \frac{2 \cdot X_{\text{MM}} + X_{\text{MN}}}{2n},$$
$$q = \frac{2 \cdot X_{\text{NN}} + X_{\text{MN}}}{2n},$$

where $X_{\text{bloodtype}}$ is the observed number of the blood type and n is the sample size. Thus for UDP

$$p_{\text{MN}} = 2 \cdot \frac{2 \cdot X_{\text{MM}} + X_{\text{MN}}}{2n} \cdot \frac{2 \cdot X_{\text{NN}} + X_{\text{MN}}}{2n} = 0.4992,$$

is set as the proportion of MN blood type under HW-equilibrium.

A simple test to decide whether the population on UDP is not in HW-equilibrium can therefore be of the hypothesis

$$H_0 : p_{\text{MN,UDP}} = 0.4992$$
$$H_1 : p_{\text{MN,UDP}} \neq 0.4992$$

i.e. if the observed proportion of MN blood type on UDP $p_{\text{MN,UDP}}$ is equal to the proportion under HW-equilibrium.

We want to test whether it can be rejected that the genotypes on UDP are in HW-equilibrium. What is the usually applied test statistic for this test?

1 ☐ The test statistic is $\chi^2 = 2(1.99 + 4.30 + 2.32) = 17.2$

2 ☐ The test statistic is $z_{\text{obs}} = \frac{15-25.46}{\sqrt{25.46 \cdot (1 - \frac{25.46}{51})}} = -2.93$

3 ☐ The test statistic is $z_{\text{obs}} = \frac{15-51}{\sqrt{51 \cdot (1 - \frac{15}{51})}} = -6.00$

4 ☐ The test statistic is $\chi^2 = (1.99^2 + 4.30^2 + 2.32^2)/2 = 13.9$

5 ☐ The test statistic is $\chi^2 = 1.99 + 4.30 + 2.32 = 8.6$

Question V.3 (16)

For another type of test for HW-equilibrium the test statistic is found to $\chi^2 = 24.52$ and under the null hypothesis it will follow a χ^2-distribution with 1 degree of freedom. What is the p-value and conclusion of the test using a significance level of 0.001?

1 ☐ p-value is `pchisq(24.52, df=1)` ≈ 1 and the hypothesis of HW-equilibrium cannot be rejected

2 ☐ p-value is `1 - pchisq(24.52, df=1)` < 0.001 and the hypothesis of HW-equilibrium cannot be rejected

3 ☐ p-value is `1 - pchisq(24.52, df=1)`< 0.001 and the hypothesis of HW-equilibrium is rejected

4 ☐ p-value is `1 - pnorm(sqrt(24.52))` < 0.001 and the hypothesis of HW-equilibrium cannot be rejected

5 ☐ p-value is `1 - pnorm(sqrt(24.52))` < 0.001 and the hypothesis of HW-equilibrium is rejected

Question V.4 (17)

For theoretical reasons it has been suggested that the frequencies of genotypes MM and NN in the underlying population are the same and it is now of interest to investigate this on the basis of the observations from UDP. Assuming that the proportions for MM and NN are independent a 90% confidence interval for the difference in the proportion of MM and NN ($p_{\text{MM,UDP}} - p_{\text{NN,UDP}}$) is given by:

1 ☐ $2/51 \pm 1.64\sqrt{\frac{19\cdot 32}{51^3} + \frac{17\cdot 34}{51^3}}$

2 ☐ $2/51 \pm 1.96\sqrt{\frac{19\cdot 32}{51^3} + \frac{17\cdot 34}{51^3}}$

3 ☐ $2/51 \pm 1.64\sqrt{\frac{19\cdot 32}{51^2} + \frac{17\cdot 34}{51^2}}$

4 ☐ $2/51 \pm 1.96\sqrt{\frac{19\cdot 32}{51^2} + \frac{17\cdot 34}{51^2}}$

5 ☐ $2/51 \pm 1.68\sqrt{\frac{19\cdot 32}{51^3} + \frac{17\cdot 34}{51^3}}$

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Question V.5 (18)

What is the usual test statistic for the test that the proportions of MM and NN are equal at UDP, i.e. $H_0 : p_{\text{MM,UDP}} = p_{\text{NN,UDP}}$?

1 ☐ $z_{\text{obs}} = \frac{2}{51\sqrt{\frac{19\cdot 32}{51^2}+\frac{17\cdot 34}{51^2}}}$

2 ☐ $z_{\text{obs}} = \frac{2/51}{\sqrt{\frac{19\cdot 32}{51^3}+\frac{17\cdot 34}{51^3}}}$

3 ☐ $z_{\text{obs}} = \frac{2}{51\sqrt{\frac{6}{17}\frac{11}{17}\frac{2}{51}}}$

4 ☐ $z_{\text{obs}} = \frac{2/51}{\sqrt{\frac{6}{17}\frac{6}{19}\frac{2}{51}}}$

5 ☐ $z_{\text{obs}} = \frac{2/51}{\sqrt{\frac{19\cdot 34}{51^3}+\frac{17\cdot 32}{51^3}}}$

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Exercise VI

A sample with the following 10 observations is taken:

```
x <- c(-1.63, -1.37, -1.21, -0.60, -0.36, -0.26, -0.18, 0.02, 0.29, 0.39)
```

Notice that the observations have been sorted in the code above.

The sample mean and sample standard deviation are calculated:

```
mean(x)

## [1] -0.491

sd(x)

## [1] 0.7003
```

Question VI.1 (19)

What is the sample variance?

1 ☐ $s^2 = 0.21$

2 ☐ $s^2 = 0.49$

3 ☐ $s^2 = 1.46$

4 ☐ $s^2 = 1.70$

5 ☐ $s^2 = 2.36$

Question VI.2 (20)

What is the first quartile of the sample?

1 ☐ $Q_1 = -1.37$

2 ☐ $Q_1 = -1.29$

3 ☐ $Q_1 = -1.21$

4 ☐ $Q_1 = -0.91$

5 ☐ $Q_1 = -0.60$

Question VI.3 (21)

Which of the following is a correct 95% confidence interval for the mean of the population from which the sample is taken?

1 ☐ $-0.491 \pm t_{0.975}\frac{0.490}{\sqrt{10}} = [-0.84, -0.14]$ where $t_{0.975} = 2.26$ is a quantile in t-distribution with 9 degrees of freedom

2 ☐ $-0.491 \pm t_{0.95}\frac{0.700}{\sqrt{9}} = [-0.92, -0.64]$ where $t_{0.95} = 1.83$ is a quantile in t-distribution with 9 degrees of freedom

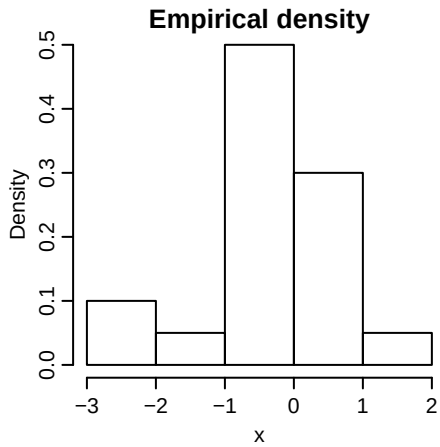
3 ☐ $-0.491 \pm t_{0.95}\frac{0.490}{\sqrt{9}} = [-0.79, -0.19]$ where $t_{0.95} = 1.83$ is a quantile in t-distribution with 9 degrees of freedom

4 ☐ $-0.491 \pm t_{0.975}\frac{0.700}{10} = [-0.65, -0.33]$ where $t_{0.975} = 2.26$ is a quantile in t-distribution with 9 degrees of freedom

5 ☐ $-0.491 \pm t_{0.975}\frac{0.700}{\sqrt{10}} = [-0.99, 0.01]$ where $t_{0.975} = 2.26$ is a quantile in t-distribution with 9 degrees of freedom

Question VI.4 (22)

Another sample is taken and its empirical density is:



What is the size of the sample, i.e. how many observations n are in the sample?

1 ☐ 20

2 ☐ 30

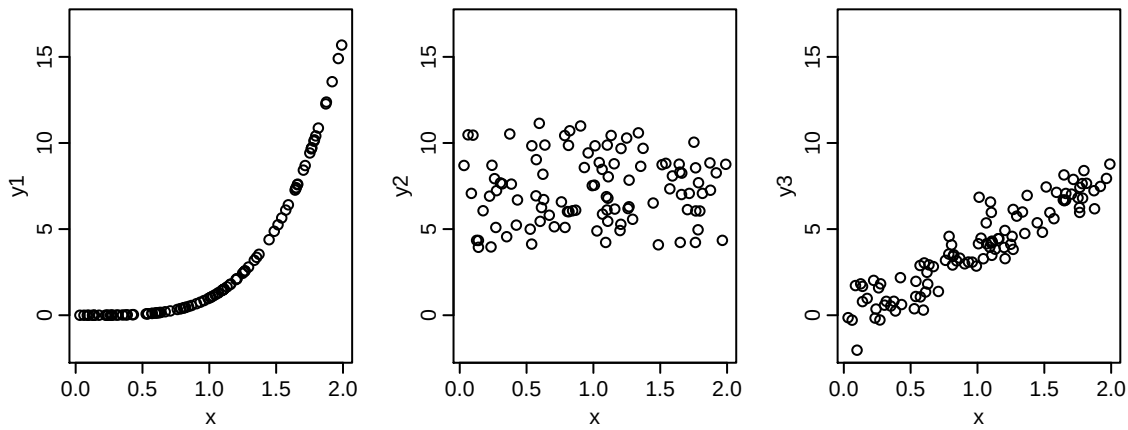
3 ☐ 100

4 ☐ 300

5 ☐ This question cannot be answered with the given information

Question VI.5 (23)

The following three plots are of coherent values of x and y for samples from three different populations:



The following statements are about the correlations of the three populations from which the samples were taken. Which of the statements is not very unlikely?

1 ☐ $\rho_{XY_1} = 0$, $\rho_{XY_2} = 0$ and $\rho_{XY_3} = 0.33$

2 ☐ $\rho_{XY_1} = 0$, $\rho_{XY_2} = 0$ and $\rho_{XY_3} = -0.89$

3 ☐ $\rho_{XY_1} = 0$, $\rho_{XY_2} = 0.61$ and $\rho_{XY_3} = 0.91$

4 ☐ $\rho_{XY_1} = 0.87$, $\rho_{XY_2} = 0$ and $\rho_{XY_3} = 0.92$

5 ☐ $\rho_{XY_1} = 0.22$, $\rho_{XY_2} = 0$ and $\rho_{XY_3} = -0.34$

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Exercise VII

In a finite population of N units with mean $\mathrm{E}[Y] = \mu$ and variance $\mathrm{V}[Y] = \sigma^2$ we are considering a sample with n units Y_i, $i = 1, \ldots, n$. If the sample is taken randomly and without replacement, then the sample mean is $\bar{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i$ and the variance is $\mathrm{V}(\bar{Y}) = \left(\frac{N-n}{N}\right)\frac{\sigma^2}{n}$. The interest is now on the sum of the sample $\tau = \sum_{i=1}^{N} Y_i$, which can be estimated by $\hat{\tau} = \frac{N}{n}\sum_{i=1}^{n} Y_i$.

Question VII.1 (24)

What is the variance of the estimator $\hat{\tau}$ i.e. $\mathrm{V}(\hat{\tau})$?

1 ☐ $\mathrm{V}(\hat{\tau}) = \frac{N^2}{n}\sigma^2$

2 ☐ $\mathrm{V}(\hat{\tau}) = N(N-n)\frac{\sigma^2}{n}$

3 ☐ $\mathrm{V}(\hat{\tau}) = \frac{N^2}{n^3}\sigma^2$

4 ☐ $\mathrm{V}(\hat{\tau}) = N^2(1-n)\sigma^2$

5 ☐ $\mathrm{V}(\hat{\tau}) = \frac{N}{n}\sigma^2$

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Exercise VIII

Up until the 1970s in Finland, it was only allowed to sell and serve alcoholic beverages in towns and not in rural areas. When it was wanted to ease the restrictions on alcohol sale in rural areas it raised concerns if this would lead to an increased rate of road accidents. Ahead of easing the restrictions a project was carried out in which: four rural municipalities were granted extraordinary permission to sell alcohol in shops, and four other municipalities were granted permission to, besides selling alcohol in shops, serve alcohol in restaurants and others serving places. Finally, four other rural municipalities without extraordinary permits acted as control. Data on the number of traffic accidents from the 12 selected municipalities over the year the project ran is presented in the following table:

Name	Control	Sale	SaleAndServing
	177	226	226
	225	196	229
	167	198	215
	176	206	188
Sum	745	826	858

and the chosen analyses is an ANOVA. The result is:

```
## Analysis of Variance Table
##
## Response: Accidents
##           Df Sum Sq Mean Sq F value Pr(>F)
## Treatment  A 1696.2  848.08     C     D
## Residuals  B 3670.7  407.86
```

Where `Treatment` is a factor dividing the municipalities into the three groups and `Accidents` is the number of accidents.

Question VIII.1 (25)

To investigate whether the permission to sell alcohol has an effect on the rate of traffic accidents, the average number of traffic accidents in the 3 groups are compared. Assuming that the variance in the number of traffic accidents is constant between the groups, what is then the result of the test for a difference in the mean number of traffic accidents between the 3 groups on significance level $\alpha = 0.05$?

1 ☐ The test statistic $F_{\text{obs}} = 1.232$ which under H_0 follows an F-distribution with 3 and 8 degrees of freedom, gives a p-value of 0.360 and the study therefore gives no reason to believe that an easing of alcohol restrictions will increase number of traffic accidents

2 ☐ The test statistic $F_{\text{obs}} = 2.079$ which under H_0 follows an F-distribution with 2 and 9 degrees of freedom, gives a p-value of 0.181 and the study therefore showes that easing of alcohol restrictions will certainly lead to an increase in the number of traffic accidents

3 ☐ The test statistic $F_{\text{obs}} = 2.079$ which under H_0 follows an F-distribution with 2 and 9 degrees of freedom, gives a p-value of 0.181 and the study therefore gives no reason to believe that an easing of alcohol restrictions will increase number of traffic accidents

4 ☐ The test statistic $F_{\text{obs}} = 4.324$ which under H_0 follows an F-distribution with 2 and 9 degrees of freedom, gives a p-value of 0.0434 and the study therefore shows that easing of alcohol restrictions will lead to a change of the number of traffic accidents

5 ☐ The test statistic $F_{\text{obs}} = 4.324$ which under H_0 follows an F-distribution with 3 and 8 degrees of freedom, gives a p-value of 0.0434 and the study therefore shows that easing of alcohol restrictions will lead to a change of the number of traffic accidents

Question VIII.2 (26)

What is the estimate of the standard deviation of the errors?

1 ☐ $\hat{\sigma} = 1696.2/(12-1) = 154$

2 ☐ $\hat{\sigma} = \sqrt{1696.2/(3-1)} = 29.1$

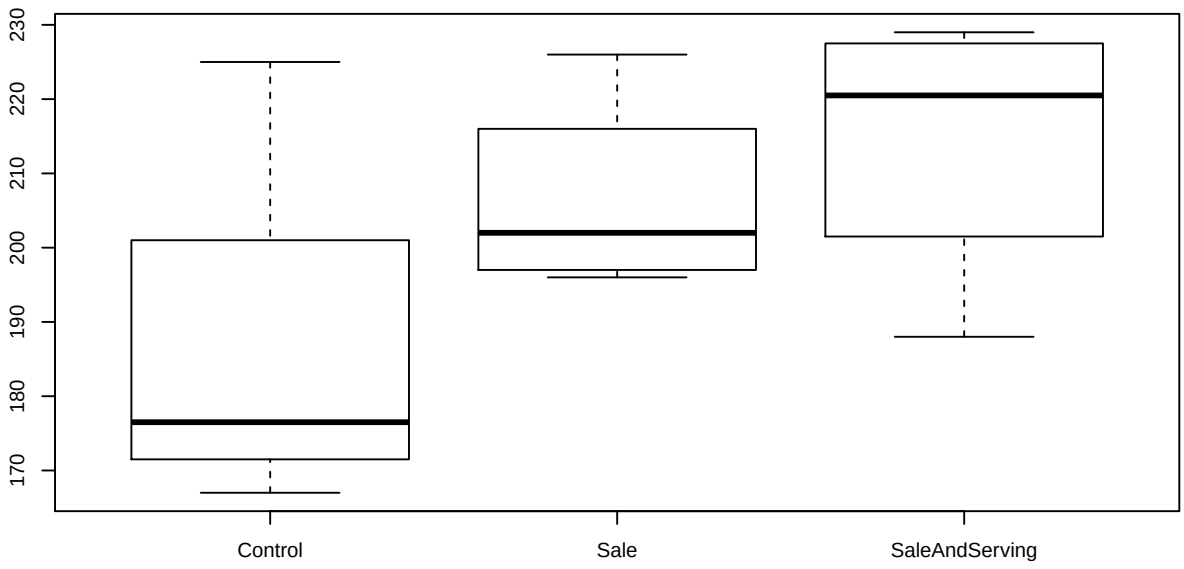
3 ☐ $\hat{\sigma} = \sqrt{1696.2/(12-1)} = 11.0$

4 ☐ $\hat{\sigma} = \sqrt{3670.7/(12-3)} = 20.2$

5 ☐ $\hat{\sigma} = 5367.1/(12-3)^2 = 66.3$

Question VIII.3 (27)

The assumption of homogeneous variance is validated with the following box plots:



Which of the following statements is the most correct conclusion based on this plot and the informations given (not all the statements are necessarily meaningful)?

1 ☐ Taking the high number of observation into account there is no evidence that the assumption of homogeneous variance is not fulfilled

2 ☐ Taking the high number of observation into account there is evidence that the assumption of homogeneous variance is not fulfilled

3 ☐ Taking the low number of observation into account there is no evidence that the assumption of homogeneous variance is not fulfilled

4 ☐ Taking the low number of observation into account there is evidence that the assumption of homogeneous variance is not fulfilled

5 ☐ Based on the information provided there cannot be drawn any conclusions about the assumption of homogeneous variance

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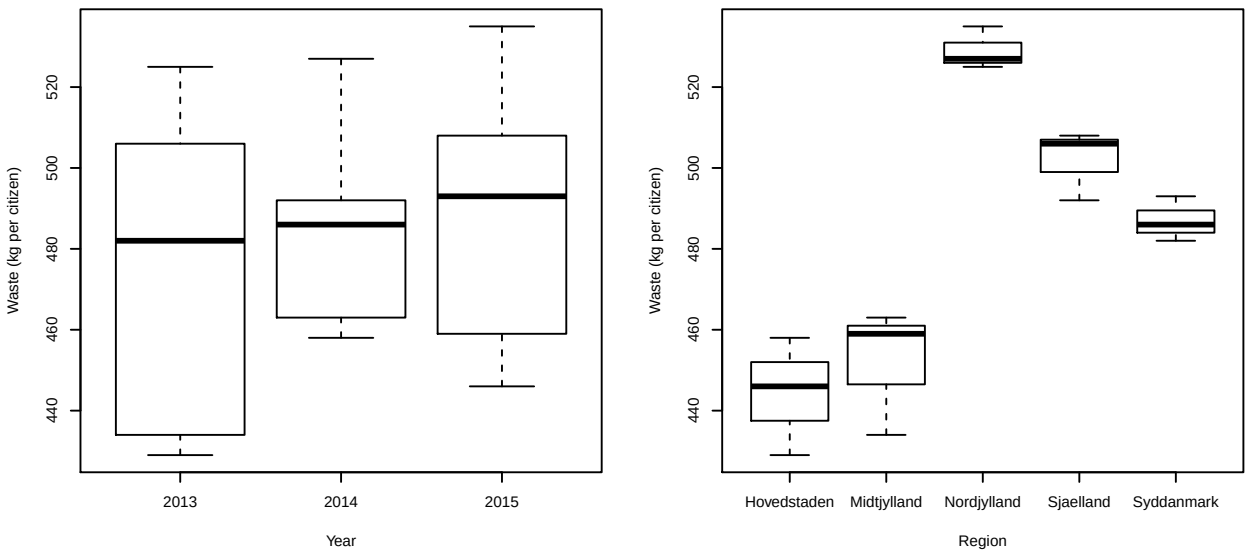
Exercise IX

The Environmental and Food Agency collects data on waste in Denmark every year and publishes a report with data and analyses. The report is named "Affaldsstatistik 2015"([1]) and in it one can find the amount of waste (kg) per citizen for the years 2013 to 2015 grouped on regions:

	Hovedstaden	Midtjylland	Nordjylland	Sjaelland	Syddanmark
2013	429	434	525	506	482
2014	458	463	527	492	486
2015	446	459	535	508	493

The following box plot shows waste per citizen grouped on year and on region:



A 2-way ANOVA is carried out and the result is:

```
## Analysis of Variance Table
##
## Response: Waste
##           Df  Sum Sq Mean Sq F value    Pr(>F)
## Year       2   463.3   231.7  2.5551    0.1386
## Region     4 14847.1  3711.8 40.9386 2.266e-05 ***
## Residuals  8   725.3    90.7
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

[1] http://www2.mst.dk/Udgiv/publikationer/2017/05/978-87-93614-01-7.pdf

Question IX.1 (28)

Which of the following statements is correct when using a significance level of $\alpha = 5\%$?

1 ☐ From the box plot it can be seen that there is no significant difference in waste between the years, which is also the conclusion from the ANOVA test

2 ☐ Anwers taken out of the exam.

3 ☐ From the box plot it is not possible to conclude if there is a significant difference in waste between the years, but from the ANOVA test no significant difference in waste between the years can be concluded

4 ☐ From the box plot it is not possible no conclude if there is a significant difference in waste between the years, but from the ANOVA test a significant difference in waste between the years can be concluded

5 ☐ None of the statements above are correct

Question IX.2 (29)

Further, in the report it is listed how large a proportion of the waste is sorted in the five regions and the proportion of waste that is sorted is calculated for each year and each region. A 2-way ANOVA has been carried out on this data with the following result:

```
## Analysis of Variance Table
##
## Response: Proportion
##           Df    Sum Sq   Mean Sq F value   Pr(>F)
## Year       2 0.0109878 0.0054939  13.054 0.003026 **
## Region     4 0.0173773 0.0043443  10.323 0.003019 **
## Residuals  8 0.0033668 0.0004208
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Which one of the following conclusions is correct using a significance level of 5% (both argument and conclusion must be correct)?

1 ☐ Since the p-value > 0.05 for the relevant test, a significant change in the sorted proportion over the years is not detected

2 ☐ Since $P(F > 13.054) < 0.05$ where F follows the relevant F-distribution, a significant change in the sorted proportion over the years is detected

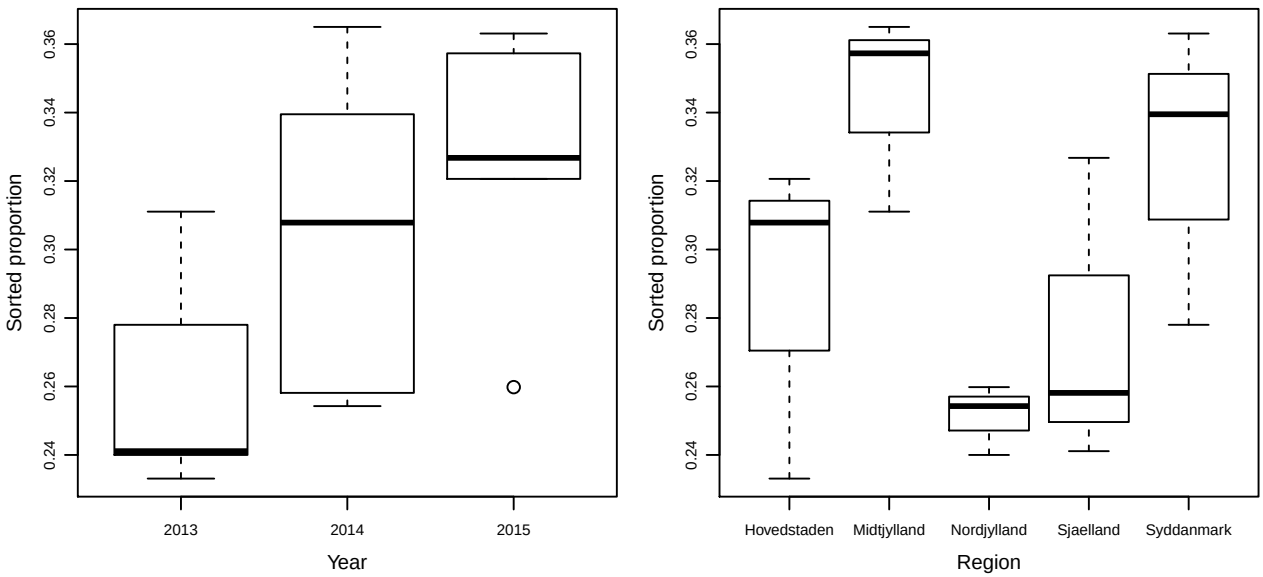
3 ☐ Since $P(T > 0.003) > 0.05$ where T follows the relevant t-distribution, a significant change in the sorted proportion over the years is not detected

4 ☐ Since $P(T < 10.323) < 0.05$ where T follows the relevant t-distribution, a significant change in the sorted proportion over the years is detected

5 ☐ Since $1 - P(T > 10.323) > 0.05$ where T follows the relevant t-distribution, a significant change in the sorted proportion over the years is not detected

Question IX.3 (30)

The box plots showing the proportion of sorted waste by year and by region are:



It is seen that in 2015 there is an observation which is low compared to the others, and it is identified as an outlier according to the modified box plot.

Which of the following statements is not correct (Tip: Remember that there is only one observation for each year in each region)?

1 ☐ The lowest observation in 2013 is from Hovedstaden

2 ☐ The lowest observation in 2015 (i.e. the outlier) is from Nordjylland

3 ☐ Each year Sjaelland has had a higher observation than Hovedstaden

4 ☐ Nordjylland has the lowest median

5 ☐ The 75% quantile for 2014 is higher than the 25% quantile for 2015

Continues on page 27

THE EXAM IS FINISHED. Enjoy the late summer!