

Written examination: 13. December 2016

Course name and number: **Introduction to Statistics (02323 and 02402)**

Aids and facilities allowed: All

The questions were answered by

_____ (student number)

_____ (signature)

_____ (table number)

There are 30 questions of the "multiple choice" type included in this exam divided on 18 exercises. To answer the questions you need to fill in the prepared 30-question multiple choice form (on three separate pages) in CampusNet

5 points are given for a correct answer and -1 point is given for a wrong answer. ONLY the following 5 answer options are valid: 1, 2, 3, 4 or 5. If a question is left blank or another answer is given, then it does not count (i.e. "0 points"). Hence, if more than one answer option is given to a single question, which in fact is technically possible in the online system, it will not count (i.e. "0 points"). The number of points corresponding to specific marks or needed to pass the examination is ultimately determined during censoring.

The final answers should be given in the exam module in CampusNet. The table sheet here is ONLY to be used as an "emergency" alternative (remember to provide your study number if you hand in the sheet).

Exercise	I.1	II.1	III.1	III.2	IV.1	IV.2	V.1	VI.1	VII.1	VIII.1
Question	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Answer										

Exercise	VIII.2	IX.1	IX.2	IX.3	IX.4	X.1	XI.1	XI.2	XI.3	XI.4
Question	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
Answer										

Exercise	XI.5	XII.1	XII.2	XIII.1	XIII.2	XIV.1	XV.1	XVI.1	XVII.1	XVIII.1
Question	(21)	(22)	(23)	(24)	(25)	(26)	(27)	(28)	(29)	(30)
Answer										

The questionnaire contains 25 pages.

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Multiple choice questions: *Note that not all the suggested answers are necessarily meaningful. In fact, some of them are very wrong but under all circumstances there is one and only one correct answer to each question.*

Exercise I

Archaeopteryx is a genus of bird-like dinosaurs that is transitional between non-avian feathered dinosaurs and modern birds. Assume that we have data from 6 fossils of Archaeopteryx including measurements of the length of the thigh bone (femur) and the upper arm bone (humerus) as shown in the table below.

Femur	38	46	56	59	64	74
Humerus	41	50	63	70	71	76

Data can be loaded into R by:

```
femur = c(38,46,56,59,64,74)
humerus = c(41,50,63,70,71,76)
```

Question I.1 (1)

Archaeologists have long believed that there should be a linear relationship between the length of the femur and length of humerus of extinct animals such as Archaeopteryx. What conclusion can be made by analyzing the above data when the significance level $\alpha = 0.05$ is used?

- 1 There is reason to assume a linear relationship, as the length of bones in animals are always positively correlated.
- 2 There is reason to assume a linear relationship, with p -value for the relevant test being 0.0013.
- 3 There is reason to assume a linear relationship, with p -value for the relevant test being 0.0780.
- 4 There is no reason to assume a linear relationship, with p -value for the relevant test being 0.0033.
- 5 There is no reason to assume a linear relationship, with p -value for the relevant test being 0.0780.

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Exercise II

A study aims to investigate whether intake of a natural product affects weight. The study should include 10 subjects (men with similar weight). The weight change D_i ($i = 1, \dots, 10$) after one month of use of the natural product is recorded. It is of interest to test if the weight change can be assumed to be zero, i.e. to test the hypothesis $H_0 : \mu_D = 0$ against the alternative $H_1 : \mu_D \neq 0$. It is decided to apply the significance level $\alpha = 0.05$.

Question II.1 (2)

Assuming that the standard deviation of weight change is $\sigma = 1$ kg, what is the power for detecting an actual weight change of at least 1 kg? (Hint: the function `power.t.test` in R can be useful here.)

- 1 50.0%
- 2 69.3%
- 3 80.3%
- 4 89.7%
- 5 99.3%

Continues on page 4

Exercise III

It is believed that the amount of cholesterol in chicken eggs, X , is normally distributed with mean $\mu = 200$ mg and standard deviation $\sigma = 15$ mg, i.e. $X \sim N(200, 15^2)$.

Question III.1 (3)

What is the proportion of chicken eggs having an amount of cholesterol higher than 205 mg?

- 1 $P(X > 205) = 0.631$
- 2 $P(X > 205) = 0.369$
- 3 $P(X > 205) = 0.491$
- 4 $P(X > 205) = 0.394$
- 5 $P(X > 205) = 0.605$

Question III.2 (4)

Industrial kitchens may buy cartons of eggs, where the content, Y , in a carton corresponds to the combined content of 100 eggs, i.e. the total content of cholesterol in a carton is $Y = \sum_{i=1}^{100} X_i$. The content of the 100 eggs can be assumed independent from each other.

You buy a carton of eggs, corresponding to buying 100 eggs. Which of the following R commands gives the probability that the total cholesterol, Y , is higher than 20.5 g (note that 200 mg is 0.2 g)?

- 1 `pnorm(q=100*0.205, mean=100*0.200, sd=100*0.015)`
- 2 `1-pnorm(q=100*0.200, mean=100*0.205, sd=sqrt(100*0.015*0.015))`
- 3 `pnorm(q=100*0.205, mean=100*0.200, sd=100*100*0.015)`
- 4 `1-pnorm(q=100*0.205, mean=100*0.200, sd=sqrt(100*0.015*0.015))`
- 5 `pnorm(q=100*0.205, mean=100*0.200, sd=sqrt(100*0.015*0.015))`

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Exercise IV

We consider a binomial random variable Y where $n = 100$ and $p = 0.45$.

Question IV.1 (5)

Calculate $P(Y > 40)$:

1 0.183

2 0.971

3 0.420

4 0.817

5 0.866

Question IV.2 (6)

We define a new random variable X so that $X = k \cdot Y$, where the constant k is given by $k = 2$ and Y is binomial distributed random variable with $n = 100$ and $p = 0.45$. Please state the variance of the random variable X :

1 $\text{Var}(X) = \text{Var}(k \cdot Y) = k + n \cdot p(1 - p) = 26.75$

2 $\text{Var}(X) = \text{Var}(k \cdot Y) = k^2 \cdot n^2 \cdot p^2(1 - p)^2 = 49.50^2$

3 $\text{Var}(X) = \text{Var}(k \cdot Y) = k^2 \cdot n^2 \cdot p(1 - p) = 9900$

4 $\text{Var}(X) = \text{Var}(k \cdot Y) = k^2 \cdot n \cdot p(1 - p) = 99.00$

5 $\text{Var}(X) = \text{Var}(k \cdot Y) = k \cdot n \cdot p(1 - p) = 49.50$

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Exercise V

We consider an exponentially distributed random variable X with parameter β . The distribution function is given by $F(X \leq x) = 1 - e^{-x/\beta}$, where $x > 0$ and $\beta > 0$. Please note that the mean value of X equals β .

Question V.1 (7)

Please state the median of X :

- 1 The median of X becomes $0.5 \cdot 2 \cdot \beta$
- 2 The median of X becomes $0.5^2 \cdot \beta$.
- 3 The median of X becomes $\log(2) \cdot \beta$ (where \log is the natural logarithm)
- 4 The median of X becomes $\log(\frac{1}{2}) \cdot \beta^2$ (where \log is the natural logarithm)
- 5 The median of X becomes $2 \cdot \beta$

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Exercise VI

A biologist is interested in examining the effects of three different diets (A, B, C) for cultivating tiger shrimps. She purchases 24 uniform larvae from a hatchery for the experiment. Each larva is placed in its own container, and it is determined by random which diet that should be given, such that each diet is tested on 8 different larvae. The larvae grow to become tiger shrimps, and after completing the study period the weight of the shrimps is measured, Y_{ij} (in grams). Since the weight can be assumed to follow a normal distribution, the following model is applied

$$Y_{ij} = \mu + \alpha_i + \varepsilon_{ij}.$$

In the model α_i describes the effect of diet i ($i = 1, 2, 3$). Finally, μ is the average and ε_{ij} is the model residuals which are assumed normally distributed with mean 0 and standard deviation σ_ε . An ANOVA of the above model is given below, and it is seen that diet is statistically significant.

Analysis of Variance Table

Response: Y

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
Diet	2	44.67	22.3350	8.9221	0.001568 **
Residuals	21	52.57	2.5033		

Question VI.1 (8)

Beforehand there was an interest in comparing the mean value of diet A and diet C. Their estimated mean values are $\hat{\mu}_A = 12.7251$ and $\hat{\mu}_C = 15.7251$, respectively. Please provide a 95% confidence interval for the mean difference in weight between diet A and diet C.

1 $-3.000 \pm 2.119\sqrt{52.488^2(\frac{1}{12} + \frac{1}{12})}$

2 $-3.000 \pm 1.960\sqrt{64.624(\frac{1}{12} + \frac{1}{12})}$

3 $-3.000 \pm 1.960\sqrt{44.670(\frac{1}{3} + \frac{1}{3})}$

4 $-3.000 \pm 2.080\sqrt{2.503(\frac{1}{8} + \frac{1}{8})}$

5 $-3.000 \pm 2.080\sqrt{52.570(\frac{1}{4} + \frac{1}{4})}$

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Exercise VII

The exercise is no longer a part of the curriculum

Question VII.1 (9)

The question is no longer a part of the curriculum

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Exercise VIII

A study aims at comparing the wear resistance of two different kinds of rubber (A and B) used as material for shoe soles. The study includes 100 school children aged 8-10 years. Each child receives a pair of shoes where the sole of one shoe is made of material A, while the sole of the other shoe is made of material B. For each pair of shoes, it is decided by randomization whether material A should be on shoe to the right or to the left. The children use the shoes every day for 3 months, and after the experiment has been completed the wear (in mm) on each shoe is measured.

Question VIII.1 (10)

If it can be assumed that the measured wear is continuous and normally distributed for each kind of rubber, please specify which of the following statistical tests should be applied, if you want to test whether the materials A and B are equal with respect to wear:

- 1 A contingency table analysis
- 2 An F test comparing two variances
- 3 A usual (non-paired) t-test
- 4 A paired t-test
- 5 A one-way ANOVA

Question VIII.2 (11)

It turns out that the null hypothesis is accepted, i.e. it is concluded that the two materials wear out equally. Instead the researchers calculate for each child in the study the average wear for the pair of shoes. It is of interest to analyze, using a standard t-test, if boys and girls wear the shoes equally, or alternatively, if there is a difference in wear between gender (two-sided test). A total of 50 girls and 50 boys were included in the experiment.

As the wear can be assumed normally distributed within each gender with equal variance, we get the usual test statistics $t_{obs} = 2.23$ with 98 degrees of freedom. The p -value becomes:

- 1 The p -value becomes 0.014
- 2 The p -value becomes $2 \cdot 0.014$
- 3 The p -value becomes $2 \cdot 0.05$
- 4 The p -value becomes 0.23
- 5 The p -value becomes $1 - 0.23$

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Exercise IX

A course at a university is offered each semester typically with more than 300 students taking the exam. Examination results for 280 students who have passed the course at the previous exam is given in the table below. For example, the tables shows that 24 students got the grade 12. The distribution of the 280 grades is considered in the next 4 questions.

Grade	02	4	7	10	12	In total
Count	22	78	84	72	24	280

The data (grades) can be loaded into R by:

```
grades = rep(x=c(2,4,7,10,12), times=c(22,78,84,72,24))
```

Question IX.1 (12)

Use the central limit theorem to determine a 95% confidence interval for the mean grade based on the students who have passed the exam. (It is important in this question that the grades are perceived numerically, eg. 02 corresponds to the number 2, etc.).

- 1 [6.51 ; 7.43]
- 2 [6.62 ; 7.32]
- 3 [4 ; 10]
- 4 [5.12 ; 8.67]
- 5 [5.99 ; 8.72]

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Question IX.2 (13)

You now want to test whether the proportion of students who have passed the exam with a grade of '7' or higher can be assumed to be 65%, which has been an objective in designing the grading scale. We denote this proportion p_{7+} . From the table in the previous question it is seen that 180 students out of 280 got the grade '7' or higher.

Determine the p -value when we wish to test $H_0 : p_{7+} = 0.65$ against $H_1 : p_{7+} \neq 0.65$:

- 1 0.8021
- 2 $1.745 \cdot 10^{-6}$
- 3 $8.725 \cdot 10^{-7}$
- 4 0.5989
- 5 0.4011

Question IX.3 (14)

A student is interested in analyzing the data in more detail, and run the following code using the 280 grades stored in the vector `grades`

```
k = 100000
samples = replicate(k, sample(grades, replace = TRUE))
simval = apply(samples, 2, sd)
resultater = quantile(simval, c(0.025,0.975))
```

Please state which numerical result that has been calculated in the vector `resultater`:

- 1 A 95% confidence interval for the standard deviation of the grades (non-parametric bootstrap)
- 2 A 95% confidence interval for the distribution of the grades (parametric bootstrap)
- 3 A 95% prediction of the median of the grades (non-parametric bootstrap)
- 4 A 95% prediction for the standard error of the grades (parametric bootstrap)
- 5 A 95% confidence interval for 75% percentile of the grades (parametric bootstrap)

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Question IX.4 (15)

You now want to examine if the distribution of the grades is the same for men and women. The distribution of grades by sex is shown in the table below.

Grades	02	4	7	10	12	In total
Men	14	47	59	47	18	185
Women	8	31	25	25	6	95

Please calculate the expected number of men with the grade '7' in the case where the grade distribution is assumed equal for men and women (i.e. assuming the null hypothesis):

1 55.5

2 59

3 47.57

4 28.5

5 42

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Exercise X

A discrete random variable X , is used to describe the number of events during a time interval. X has the density function on the familiar form: $P(X = x) = \frac{2^x}{x!}e^{-2}$, for $x \geq 0$.

Question X.1 (16)

What is the mean of X ?

1 $\frac{1}{2}$

2 $\log(2)$ (where \log is the natural logarithm)

3 2

4 π

5 2^2

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Exercise XI

A study aims at comparing cognitive abilities of 3 groups of children. The groups consist of a) children with Tourette's Syndrome (TS), b) children with ADHD and c) children without any of these diagnoses (Control).

In the study, each child is asked to solve a sequence of tasks on a computer and the average reaction time, R_i (milliseconds) is recorded for each child. The study included 17 children with TS, 13 with ADHD and 20 controls, i.e. a total of $n = 50$ children.

When analyzing the data from the experiment it has been assumed that the reaction time R_i is normally distributed for each group with constant variance, σ_E^2 . In order to compare if the mean reaction time is the same for the three groups (TS, ADHD and controls) the following ANOVA table is provided

Analysis of Variance Table

```
Response: reactiontime
          Df Sum Sq Mean Sq F value    Pr(>F)
group     A 485848  242924 D      .976e-07 ***
Residuals B 542563    C
---
```

It is seen, however, that not all numbers are given in the ANOVA table, but some are only shown by the symbols A, B, C and D. These 4 symbols are part of the solution to the next question.

Question XI.1 (17)

Which distribution does the value D follow if the mean reaction time is the same for all three groups (TS, ADHD and Control)?

- 1 $F(A, A+B)$ i.e. an F -distribution with degrees of freedom A and $A+B$ from the ANOVA table
- 2 $F(C, B)$ i.e. an F -distribution with degrees of freedom C and B from the ANOVA table
- 3 $F(A, B)$ i.e. an F -distribution with degrees of freedom A and B from the ANOVA table
- 4 $F(A, C)$ i.e. an F -distribution with degrees of freedom A and C from the ANOVA table
- 5 $F(B, A)$ i.e. an F -distribution with degrees of freedom B and A from the ANOVA table

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Question XI.2 (18)

What is the conclusion from the ANOVA table if the significance level $\alpha = 0.05$ is applied?

- 1 We must reject the hypothesis that the mean reaction time of the control children equals the mean reaction time for children with TS or ADHD.
- 2 We must reject the hypothesis that the variance of the reaction time of the control children equals the variance of the reaction time for children with TS or ADHD.
- 3 We can prove that the variance of the reaction time is the same for all three groups since the p -value equals $0.976 \cdot 10^{-7}$.
- 4 We must reject the hypothesis that the average reaction time is the same for all three groups since the p -value equals $0.976 \cdot 10^{-7}$.
- 5 We can not reject the hypothesis that the average reaction time is the same for all three groups.

Question XI.3 (19)

Subsequently it is decided to examine whether the age of the children $x_{1,i}$ influences the reaction time, Y_i . Another experiment was conducted in which $n = 12$ children with no diagnosis (control), but with different ages solved the sequence of tasks and the average reaction time was measured. The model $Y_i = \beta_0 + \beta_1 \cdot x_{1,i} + \varepsilon_i$ is applied in order to examine the association between age and reaction time. In the model the residuals ε_i are assumed i.i.d. normally distributed with constant variance, hence $\varepsilon_i \sim N(0, \sigma^2)$. You get the following output for the new experiment:

Call:

```
lm(Reactiontime ~ Age)
```

Residuals:

Min	1Q	Median	3Q	Max
-54.520	-35.522	4.268	27.160	51.949

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	933.15	153.23	6.090	0.000117 ***
Age	-41.05	15.36	-2.672	0.023400 *

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 37.72 on 10 degrees of freedom

Multiple R-squared: 0.4166, Adjusted R-squared: 0.3583

F-statistic: 7.141 on 1 and 10 DF, p-value: 0.0234

Using the numbers in the output from the model please provide the test statistics related to the hypothesis $H_0 : \beta_1 = 0$:

1 -2.672

2 153.23

3 -41.05

4 37.72

5 0.4166

Question XI.4 (20)

Using the analysis result from the previous question find the estimate of the correlation coefficient, $\hat{\rho}$, between Reaction time (Y_i) and Age ($x_{1,i}$):

1 $\hat{\rho} = -\sqrt{0.3583}$

2 $\hat{\rho} = -\sqrt{0.4166}$

3 $\hat{\rho} = \sqrt{0.3583}$

4 $\hat{\rho} = 0.4166$

5 $\hat{\rho} = -\sqrt{0.3583/0.4166}$

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Question XI.5 (21)

A critique of the model $Y_i = \beta_0 + \beta_1 \cdot x_{1,i} + \varepsilon_i$ used in the previous question is that it does not account for whether the answer is correct or not on the individual questions, but simply the reaction time is recorded.

It is decided to expanded model to the following: $Y_i = \beta_0 + \beta_1 \cdot x_{1,i} + \beta_2 \cdot x_{2,i} + \varepsilon_i$, where $x_{2,i}$ is the number of correct answers in the sequence of questions (the remaining variables are defined as they were in the previous question). Based on a new study including 12 different children we obtain the results:

Call:

```
lm(Reactiontime ~ Age + Correct)
```

Residuals:

Min	1Q	Median	3Q	Max
-39.958	-24.407	6.917	12.897	42.297

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)	
(Intercept)	892.633	123.203	7.245	4.84e-05	***
Age	-48.104	15.081	-3.190	0.0110	*
Correct	5.310	1.765	3.009	0.0147	*

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 28.73 on 9 degrees of freedom
Multiple R-squared: 0.5908, Adjusted R-squared: 0.4999
F-statistic: 6.498 on 2 and 9 DF, p-value: 0.01793

Provide the estimates $\hat{\beta}_1$, $\hat{\beta}_2$ and $\hat{\sigma}^2$:

- 1 $\hat{\beta}_1 = -48.104$, $\hat{\beta}_2 = -5.310$ and $\hat{\sigma}^2 = 28.73$
- 2 $\hat{\beta}_1 = 892.633$, $\hat{\beta}_2 = -48.104$ and $\hat{\sigma}^2 = 28.73$
- 3 $\hat{\beta}_1 = 892.633$, $\hat{\beta}_2 = -48.104$ and $\hat{\sigma}^2 = 5.310$
- 4 $\hat{\beta}_1 = -48.104$, $\hat{\beta}_2 = 5.310$ and $\hat{\sigma}^2 = 892.633$
- 5 $\hat{\beta}_1 = -48.104$, $\hat{\beta}_2 = 5.310$ and $\hat{\sigma}^2 = 28.73^2$

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Exercise XII

An engineer is studying a process Y which can be expressed by $Y = U/B$. It can be assumed that U and B are independent random variables. The engineer has 20 pairwise measurements of U and B stored as vectors in the statistical program R and these are referred to as `uobs` and `bobs`, respectively.

Question XII.1 (22)

The engineer would like to calculate a 95% confidence interval for the variance of Y , i.e. σ_Y^2 using non-parametric bootstrap. Which of the following chunks of code in R is most appropriate?

- 1

```
samples = replicate(10000, sample(uobs/bobs, replace=FALSE))
results = apply(samples, 1, var)
quantile(results, c(0.025, 0.975))
```
- 2

```
samples = replicate(10000, sample(uobs/bobs, replace=TRUE))
results = apply(samples, 2, var)
quantile(results, c(0.025, 0.975))
```
- 3

```
samples = replicate(10000, sample(var(uobs)/var(bobs), replace=TRUE))
results = apply(samples, 2, var)
quantile(results, c(0.025, 0.975))
```
- 4

```
samples = replicate(10000, sample(uobs/bobs, replace=TRUE))
results = apply(samples, 1, var)
quantile(results, c(0.95))
```
- 5

```
samples = replicate(10000, sample(uobs/bobs, replace=FALSE))
results = apply(samples, 2, var)
quantile(results, c(0.025, 0.975))
```

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Question XII.2 (23)

We continue with the problem from the previous question, i.e. we analyze a process Y that can be expressed by $Y = U/B$.

If we assume that $U \sim N(\mu = 35, \sigma^2 = 10^2)$ and $B \sim N(\mu = 50, \sigma^2 = 10^2)$, what is the probability that Y exceeds 1, i.e. please calculate the probability $P(Y > 1)$:

1 < 0.001

2 0.1444

3 0.4701

4 0.5298

5 0.8556

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Exercise XIII

A research institute wants to estimate a 95% confidence interval for the true proportion p of consumers who are consciously trying to purchase organic foods when shopping. The research institute plans to ask n consumers the question: "Do you consciously chose to buy organic foods when you do your grocery shopping?". Possible answers to this must be "Yes" or "No".

Question XIII.1 (24)

How many independent consumers n must respond to the survey for at 95% confidence interval for the true proportion, p , to not be wider than 0.04 (Hint: As a starting point for the calculations it can be assumed that 50% of consumers will answer 'Yes' to the question)?

1 $n = \frac{1.96 \cdot \frac{1}{2} \cdot \frac{1}{2}}{0.01} = 49$

2 $n = \left(\frac{1.96 \cdot \frac{1}{2} \cdot \frac{1}{2}}{0.02}\right)^2 = 600.25$ i.e. at least 601

3 $n = \left(\frac{1.96^2 \cdot \frac{1}{2} \cdot \frac{1}{2}}{0.02^2}\right) = 2401$

4 $n = \left(\frac{2 \cdot 1.96 \cdot \frac{1}{2} \cdot \frac{1}{2}}{0.02}\right)^2 = 1250.50$ i.e. at least 1251

5 $n = \left(\frac{1.96 \cdot \sqrt{\frac{1}{2} \cdot \frac{1}{2}}}{0.01}\right)^2 = 9604$

Question XIII.2 (25)

The question is no longer a part of the curriculum

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Exercise XIV

Assume that the number of attempts for the driving test (before it is passed) in a particular municipality can be described by the model $Y = X + 1$, where X is a Poisson distributed random variable with mean $\lambda = 0.4$, i.e. $X \sim Pois(\lambda = 0.4)$.

Question XIV.1 (26)

We now consider the number of attempts among 100 randomly selected individuals who must pass the driving test. What will be the mean μ and variance σ^2 of the total number of attempts $\sum_{i=1}^{100} Y_i$ for everyone passing the exam?

- 1 $\mu = 140$ and $\sigma^2 = \sqrt{40}$
- 2 $\mu = 140$ and $\sigma^2 = \sqrt{140}$
- 3 $\mu = 140$ and $\sigma^2 = 140$
- 4 $\mu = 140$ and $\sigma^2 = 40$
- 5 $\mu = 140$ and $\sigma^2 = 40^2$

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Exercise XV

Assume that the number of traffic accidents per day, X , follows a Poisson distribution. From 200 independent observations, the rate λ has been estimated to $\hat{\lambda} = 1.2$.

Question XV.1 (27)

Please provide a 95% confidence interval for the true rate λ :

1 $[1.2; 4.8]$

2 $[0; 4]$

3 $1.2 \pm 1.96 \cdot \sqrt{\frac{1.2}{200}}$

4 $1.2 \pm 1.96 \cdot \frac{1.2}{200}$

5 $1.2 \pm 1.96 \cdot \frac{1.2^2}{200^2}$

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Exercise XVI

Two types of prescription drugs (A and B) to lower blood cholesterol, are compared in a clinical study. In analyzing the data, it was estimated how much drug A reduces cholesterol, denoted Δ_A , and correspondingly how much drug B is reducing cholesterol, denoted Δ_B (both drugs were found to reduce cholesterol and in the following positive values of Δ indicate reduction).

A 95% confidence interval for the difference in reduction ($\Delta_A - \Delta_B$) has been estimated. This interval is $[0.24; 0.50]$ mmol/L.

Question XVI.1 (28)

Which of the following is a reasonable conclusion to the survey?

- 1 Drug A reduces cholesterol by 0.24 mmol/L while drug B reduces cholesterol by 0.50 mmol/L
- 2 There is 95% probability that drug A is better to lower the cholesterol than drug B for any person
- 3 There is 95% probability that drug A will lower cholesterol with at least 39 mmol/L compared to drug B for any person
- 4 There is at least 95% confidence that drug A reduces cholesterol better than drug B
- 5 None of the above

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Exercise XVII

An engineer is planning to take a sample from a population. We consider the following three statements:

- I. If the sample has variance zero, then the variance in the population is also zero.
- II. If the population has variance zero, then the variance in the sample is also zero.
- III. If the sample has zero variance, then the mean and median is the same in the sample.

Question XVII.1 (29)

Which of the three above statements are correct?

- 1 Only I. and II.
- 2 Only I. and III.
- 3 Only II. and III.
- 4 I., II., and III. are all correct
- 5 None of the above

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Exercise XVIII

It is well known that cuckoos lay their eggs in another bird species nests, and thus leaves the task to raise their offspring to the host bird. Furthermore, it is a theory that cuckoos are able to adapt the size of their eggs depending on the size of the host bird.

To investigate this theory an ornithologists has over a period measured the size (length of the egg) of 10 eggs in each of two different host bird nests, here called the host bird A and B , that is, a total 20 eggs are measured. She gets the estimates 2 mm for the standard deviation of the size for both the host bird A and B .

Question XVIII.1 (30)

It now turns out that the observed difference in size, $\bar{x}_A - \bar{x}_B$, of the eggs is 1 mm. What conclusion does one arrive at when you want to test the hypothesis $H_0 : \mu_A = \mu_B$ against $H_1 : \mu_A \neq \mu_B$, using an ordinary t-test and significance level $\alpha = 5\%$?

- 1 The difference in the size of the eggs is statistically significant
- 2 The difference in the size of the eggs is statistically non-significant
- 3 One can not conclude anything without stating the actual size of the eggs for A and B
- 4 One can not conclude anything without the knowledge of the population size
- 5 It is not appropriate to use an ordinary t-test for this analysis

The exam is over. Have a good Christmas vacation!