

# 02465: Introduction to reinforcement learning and control

The finite-horizon decision problem

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# DTU Compute

Department of Applied Mathematics and Computer Science

#### Lecture Schedule



#### Dynamical programming

- The finite-horizon decision problem
- 31 January
- 2 Dynamical Programming
  - 7 February
- 3 DP reformulations and introduction to Control
  - 14 February

#### Control

- Discretization and PID control 21 February
- 6 Direct methods and control by optimization
  - 28 February
- 6 Linear-quadratic problems in control 7 March
- Linearization and iterative LQR

14 March

#### Reinforcement learning

- 8 Exploration and Bandits 21 March
- Opening Policy and value iteration 4 April
- Monte-carlo methods and TD learning 11 April
- Model-Free Control with tabular and linear methods 18 April
- Eligibility traces and value-function approximations 25 April
- Q-learning and deep-Q learning 2 May

Syllabus: https://02465material.pages.compute.dtu.dk/02465public

Help improve lecture by giving feedback on DTU learn



### Reading material:

• [Her24, Chapter 4] Introduction

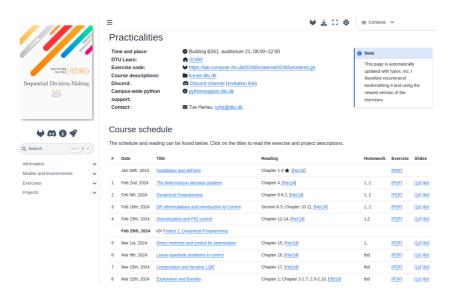
## **Learning Objectives**

- Introduction and key definitions
- Python and object-oriented programming

# Course practicalities

# Course webpage

02465material.pages.compute.dtu.dk/02465public/index.html



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#### Where and what



DTU Learn Announcements, assignment hand-ins, quizzes

Course homepage Exercises, projects, slides, documentation, installation, etc. https:

//02465material.pages.compute.dtu.dk/02465public

Off-hours QA Discord. See link on homepage.

- Exercises
  - (Not confirmed) Building B341, auditorium 21
- Ask project-related question online so that everyone has the same information (i.e. not in class)

# Project work









- Groups of 1, 2 or 3 students
  - Part 1 Dynamical programming (available now)
  - Part 2 Control
  - Part 3 Reinforcement Learning
- The projects are subject to DTUs rules of collaboration/Code of Conduct
  - This includes the individual programming.

#### Course practicalities

#### Exam

- The 4-hour written exam will contain:
  - Multiple-choice questions
  - Written-answer questions
  - Programming questions
- Test exams will be online later
- Exercises emphasize code-questions as I believe they test more skills
- Your evaluation is an overall assessment based on the written exam and project work
  - The project work is 20%.



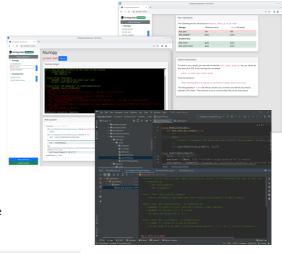


#### Course practicalities

# **Creating handins**

### See videos for week 0

- I hope this can help you debug code
- Example usage:
  - python -m irlc.project0.fruit\_project\_grade
  - Hand in your code/scores by uploading the .token file



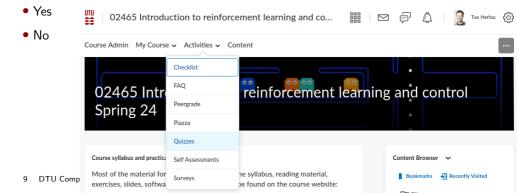
# Quiz 0: answer on DTU Learn





I will try to use quizzes this semester. You can find them under Quizzes on DTU I earn:

Do you use ChatGPT or a similar conversational AI tools in your studies?



#### ChatTutor



- ChatTutor allows you to ask questions to **both** TAs and an AI (ChatGPT)
- The platform will collect the data you put in (i.e., same as any other webpage!)
  - But please ask if you have questions!
- Optional offer:
  - Available from next week
  - Work in progress you will have other options if it is too janky :-).

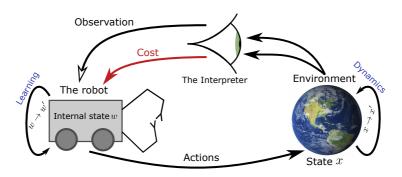
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# Types of machine learning

Supervised learning Learn a function  $f(x_i)\mapsto \hat{y}_i$  to minimize a loss Unsupervised learning Learn a structure to summarize data

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# Sequential decision making



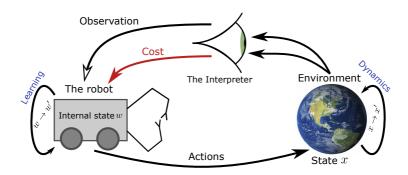
### Make decisions, one after another, to bring about a desired outcome

- Observe the world
- Take action
- Obtain cost

#### Minimize total cost

lecture\_01\_pacman.py





- Time is really important (sequential, non-i.i.d data)
- Must optimize behavior of dynamical systems using information that becomes progressively available as the systems evolve

• Future cost and state of the system will depend on current actions and state

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# Alpha-Go

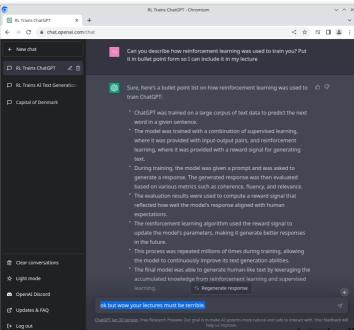


- Self-learning Go supercomputer
- Defeated world champion Lee Sedol in 2016
- Notable mentions: Atari/Dota/Starcraft II learners
- General approach: Reinforcement learning + Search trees

#### What is reinforcement learning and control

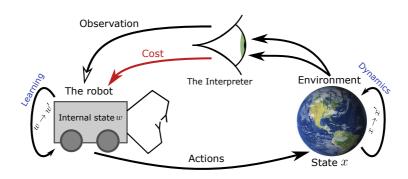
# **ChatGPT**





# The decision problem





State The configuration of the environment x

Action Either discrete or a vector u

 $\operatorname{\mathsf{Cost}/\mathsf{reward}}$  A number. Depends on state x and action u

# **Example: Mars landing**

Time Continuous

State/Actions x(t): (Position, velocity, fuel mass) u(t): thruster outputs

**Dynamics** Smooth differential equation

$$\dot{x}(t) = f(x(t), u(t))$$

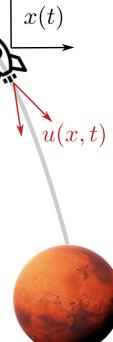
Cost Land the right place and use little fuel and don't kill anyone

Constraints Thrusters deliver limited force, ship cannot go into mars, etc.

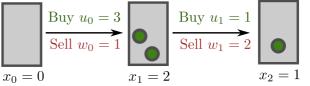
Objective Determine u(t) to minimize final cost

Really important constraints; no learning

lecture\_01\_car\_random.py



# Inventory control



ullet We order a quantity of an item at period  $k=0,\dots,N-1$  so as to meet a stochastic demand

 $x_k$  stock at the beginning of the kth period,  $u_k \geq 0$  stock ordered at the beginning of the kth period.  $w_k \geq 0$  Demand during the k'th period

- Dynamics:  $x_{k+1} = x_k + u_k w_k$
- Cost per new unit c; cost to hold  $x_k$  units is  $r(x_k)$

$$r\left(x_{k}\right)+cu_{k}$$

ullet Select actions  $u_0,\ldots,u_{N-1}$  to minimize cost

We want proven optimal rule for ordering

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#### **Examples**

# **Example: Atari**





States RAM memory state

Observations Pixel-based snapshots  $H \times W \times 3$ 

**Actions** Discrete joystick actions





Dynamics Discrete, stochastic (what the emulator does)

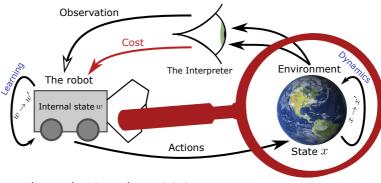
Cost High-score



Don't know dynamics; must learn from scratch

#### The environment

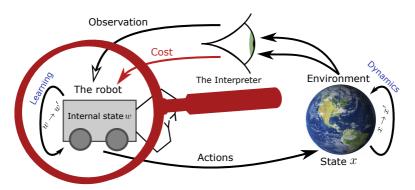




- Nature can be stochastic or deterministic
- The problem can be continuous-time or discrete-time
- We can know the dynamics or not

# The agent



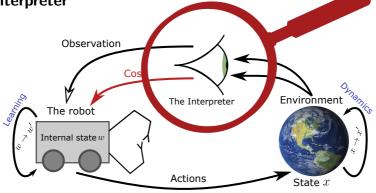


Policy How the robot chooses actions at given times/states

#### **Examples**







Reward The immediate evaluation of current step

Agents goal Maximize cumulative reward

# **Reward Hypothesis**

Every desired behavior of the agent can be described by the maximization of expected cumulative reward

DTU Compute 31 January, 2025 Lecture 1

# Making sense of these distinctions



- Why so many things in one course?
  - Study-line requirement
  - A single problem, and a single solution + tricks
  - A better overview (right tool for the job)
- Today, we will look at the problem

# DTU

# Basic control setup: Environment dynamics

Finite time Problem starts at time 0 and terminates at time N. Indexed as  $k=0,1,\ldots,N.$ 

State space The states  $x_k$  belong to the **state space**  $\mathcal{S}_k$ 

Control The available controls  $u_k$  belong to the **action space**  $\mathcal{A}_k(x_k)$ , which may depend on  $x_k$ 

**Dynamics** 

$$x_{k+1} = f_k(x_k, u_k, w_k), \quad k = 0, 1, \dots, N-1$$

Disturbance/noise A random quantity  $w_k$  with distribution

$$w_k \sim P_k(W_k|x_k,u_k)$$

#### Cost and control

Agent observe  $x_k$ , agent choose  $u_k$ , environment generates  $w_k$ Cost At each stage k we obtain cost

$$g_k(x_k,u_k,w_k), \quad k=0,\dots,N\!-\!1 \quad \text{ and } \quad g_N(x_k) \text{ for } k=N.$$

Action choice Chosen as  $u_k = \mu_k(x_k)$  using a function  $\mu_k : \mathcal{S}_k \to \mathcal{A}_k(x_k)$  $\mu_k(x_k) = \{ \text{Action to take in state } x_k \text{ in period } k \}$ 

Policy The collection  $\pi = \{\mu_0, \mu_1, \dots, \mu_{N-1}\}\$ Rollout of policy Given  $x_0$ , select  $u_k = \mu_k(x_k)$  to obtain a **trajectory**  $x_0, u_0, x_1, \ldots, x_N$  and accumulated cost

Cost-of-rollout = 
$$g_N(x_N) + \sum_{k=0}^{N-1} g_k(x_k, \mu_k(x_k), w_k)$$

Expected return (approximate) Generate T rollouts according to  $\pi$ 

$$J_{\pi}(x_0) \approx \frac{1}{T} \sum_{i=1}^{T} \{ \text{Cost-of-rollout } i \}$$

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#### Quiz 1: Discuss and answer on DTU Learn

# How do you feel about this argument? Justify your answer:

Decision-making is about determining the appropriate sequence of actions  $u_0, \ldots, u_{N-1}$ .

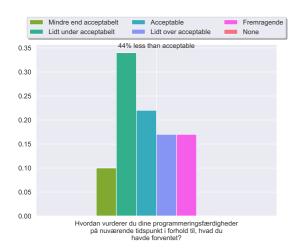
Once executed, we get a total cost. Let's say that on average this is  $c(\mathbf{u})$ . Thus, decision-making is ultimately an optimization problem: Find the sequence that on average minimize the cost:

$$u_0,\ldots,u_{N-1}=\operatorname*{arg\,min}_{\mathbf{u}}c(\mathbf{u}).$$

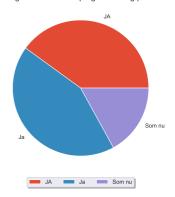
- **a.** It is computationally too complicated to solve such an optimization problem
- **b.** It is infeasible to derive or learn the function  $c(\mathbf{u})$
- **c.** Actually nothing is wrong: It is just not a theoretically interesting/fruitful way to approach decision-making
- d. Something else is wrong with the argument
- e. Don't know

# Programming: From KID study line evaluation





Vil du gerne have mere programmering på 1. år?



# Pre-semester quiz



This is new -- I have not used class inheritance before.

The code is mysterious.

I have seen code like this before, but it is not something I have used. I think I can pick it up.

I have written code that inherit from other classes (i.e., something like the second class). I am not an expert, but it is not something that worries me

This is easy. I have written code like this before and can reason about what it does.



(15.56 %)

THIS IS FINE.

(35.56 %)

#### **Initiatives**



#### What I have done:

- Re-structured the project work
- Simplification of exercises + videos
- Course notes on Python + online documentation
- This lecture
- Changed exam format
- Course responsible for the new mandatory programming course (02002/3)

### What I hope you will do:

- Decide to learn this you can!
- Set aside some time in the first block
- Don't give up:
  - Programming was not taught correctly − 100% valid criticism
  - You need to learn new programming techniques through your career

# Pacman game loop (without objects)



```
# chapter1/lecture1 code.py
     walls = np.ndarray( ) # Initialize a walls-variable
     food = np.ndarray( )
     pacman_x = 4
     pacman_y = 6
     for k in range(10):
         # Use the walls and pacman x, pacman y to figure out what actions are available.
         available actions = ... # compute using the walls-variable
         # Do some sort of planning (search?) by using the walls, pacman x, pacman y.
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         # select the best possible action
11
         # Compute the outcome of the action:
12
         pacman_x = pacman_x + action_x
13
14
         pacman y = pacman y + action y
         # Compute the reward
15
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         # Let the agent learn based on the outcome and reward
```

(about 500 lines total)

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# Same with two agents and two environments

```
# chapter1/lecture1 code.py
for k in range(10):
    if environment type == 2:
        available_actions = ... # compute using the walls-variable
    else:
        available actions = ... # This environment may differ
    if agent_type == 1: # Agent plan it's actions
        pass # do planning of first type
    elif agent_type == 2:
        pass # do planning of the second type
    if environment_type == 1: # Compute the outcome of the action:
        pacman_x = pacman_x + action_x
        pacman_y = pacman_y + action_y
        # Compute the cost-function
    else:
        pass # Updates relevant for second environment
        # Compute the cost function
    if agent_type == 2: # Allow the agent to learn based on cost
        pass # Learning for the second agent
    else:
        pass # Learning method for the first agent
```

# Using objects



```
# chapter1/lecture1_code.py
env = InventoryEnvironment() # Create an instance of the inventory environment
agent = RandomAgent(env) # Create an instance of a random-action agent
train(env, agent) # Train the agent
```

#### Training-function:

```
# chapter1/lecture1_code.py
def train(env, agent):
    s = env.reset()  # Reset and get first state, x_0
    for k in range(10):
        a = agent.pi(s) # The policy computes the action
        sp, r, done = env.step(a) # Environment computes next state, reward
        agent.train(s, a, sp, r, done) # Let the agent train
```

(this is a very rough sketch. Well get to the real training function soon)

## The simplest class



## The smallest and friendliest class

```
>>> class BasicClass: # Classnames are usually upper-case
... pass # `pass` is a special keyword which does nothing
...
```

#### Each class **instance** function like it's own little box of variables:

```
>>> a = BasicClass() # Create an instance of the class
>>> a.name = "My first class" # You can write data to the class like this
>>> b = BasicClass() # Another instance. a and b are not related and can store different data:
>>> b.name = "Another class"
>>>
>>> print("Class a:", a.name)
Class a: My first class
>>> print("Class b:", b.name)
Class b: Another class
```

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#### A class with a function



```
>>> class BasicDog:

... name = "Unnamed dog" # Each dog-instance will have the property name

def read_nametag(self):

# This is a class-function. Note we must pass it `self` as a first argument,

# instance of the class itself (i.e. the current object). This is how we can

print("This dog is named", self.name, "please give me treats!")

>>> dog = BasicDog()

>>> dog.name

'Unnamed dog'
```

#### self refers to the class instance

```
>>> dog.read_nametag() # Invoke the read_nametag() function. Note we don't pass the ob. This dog is named Pluto please give me treats!
```



#### def \_\_init\_\_ function is called when the class is created

```
>>> class BetterBasicDog:
...    def __init__(self, name):
...         self.name = name
...         self.age = 0
...         print(f"The __init__() function has been called with name='{name}'")
...         def birthday(self):
...         self.age = self.age + 1
...         print("Hurray for", self.name, "you are now", self.age, "years old")
...
```

#### Arguments can be passed along like this

```
>>> d1 = BetterBasicDog("Pluto")  # the __init__ function is now called
The __init__() function has been called with name='Pluto'
>>> d2 = BetterBasicDog(name="Lassie")  # Also support named arguments
The __init__() function has been called with name='Lassie'
```

## Functions can change the state of the class

```
>>> d1.birthday()
Hurray for Pluto you are now 1 years old
>>> d1.birthday()
Hurray for Pluto you are now 2 years old
```

# DTU

#### Quiz 2: What is the outcome of this code?

```
>>> class BetterBasicDog:
              def init (self. name):
                  self.name = name
                self.age = 0
                print(f"The init () function has been called with name='{name}'")
              def birthday(self):
                  self.age = self.age + 1
                  print("Hurray for", self.name, "you are now", self.age, "years old")
10
      >>> d1 = BetterBasicDog("Pluto")
11
      The init () function has been called with name='Pluto'
      # chapterOpythonC/quiz.py
      d1 = BetterBasicDog("Pluto")
      d1.birthday()
      d1.age = 5
      d1.name = "Lassie"
      d1.birthday()
```

- a. Ignore changes and prints out "Hurray for Pluto you are now 1 years old"
- b. Accept changes and prints out "Hurray for Lassie you are now 6 years old"
- **c.** It gives an error it is not possible to set the age.
- d. It uses name but ignores age, so we get:

```
"Hurray for Lassie you are now 1 years old"
```

# The parrot

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#### Inheritance

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ForgetfulParrot : Is like the regular Parrot , except the learn-function

```
>>> class ForgetfulParrot(Parrot):
... # The Parot class is used as a template.
... # All functions in the Parot-class are therefore 'imported' as default, including 'self.words'
def learn(self, word): # This function overwrite the 'actual' learn function in the Parot class
self.words = [word] # This parrot only know a single word
```

Inheritance: The functions are "copy-pasted" into the ForgetfulParrot

```
>>> old_parrot = ForgetfulParrot()
>>> old_parrot.learn("damn remote")
>>> old_parrot.learn("Jeopardy")
>>> print("Vocabulary", old_parrot.vocabulary())
Vocabulary ['Jeopardy']
```

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# DTU

#### Inheritance continued

More **inheritance**: Make a squeak before and after every word:

```
>>> class Parrot:
... def __init__(self):
... self.words = ["Squack!"]
... def learn(self, word):
... self.words.append(word)
... def speak(self):
... return random.choice(self.words) # Return a random word
... def vocabulary(self):
... return self.words
...
```

### Where is the bug?

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# Use super() to access functions in the parent class

```
>>> class SqueekyParrot(Parrot):
              def init (self, squeek="Quck!"):
                  super(). _init_() # Call the 'Parot' class init method to set up the words-variable.
                  self.squeek = squeek # save the squeek variable
              def speak(self):
                  word = super().speak() # Use the speak() function defined in the Parrot class.
                  return f"{self.squeek} {word} {self.squeek}"
      >>> squeeky = SqueekyParrot(squeek="Kvak-Kvak")
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      >>> squeeky.learn("Good night!")
      >>> squeeky.learn("Tell that damn bird to shut it's beak")
11
      >>> squeekv.learn("Sugar!")
12
      >>> squeeky.speak()
13
      'Kvak-Kvak Good night! Kvak-Kvak'
14
15
      >>> squeekv.speak()
16
      'Kvak-Kvak Sugar! Kvak-Kvak'
```

Consistency When we inherit from Parrot, we **know** the functions should be called speak, learn (and not talk, practice)

- Env: (reset, step, action\_space and a few other)
- Agent : (pi, train)

Functionality By using super().\_\_init\_\_ we saved a single line

 In control theory, we will use inheritance to add simulation-functionality to all models

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# The inventory environment

```
# inventory environment.py
class InventoryEnvironment(Env):
   def __init__(self, N=2):
        self.N = N
                                                 # planning horizon
        self.action_space = Discrete(3)
                                                 # Possible actions {0, 1, 2}
        self.observation_space = Discrete(3)
                                                 # Possible observations {0, 1, 2}
   def reset(self):
        self.s = 0
                                                 # reset initial state x0=0
        self.k = 0
                                                 # reset time step k=0
       return self.s, {}
                                                 # Return the state we reset to (and an
   def step(self, a):
        w = np.random.choice(3, p=(.1, .7, .2))
                                                      # Generate random disturbance
        s_next = max(0, min(2, self.s-w+a))
                                                      # next state; x \{k+1\} = f k(x k,
       reward = -(a + (self.s + a - w)**2)
                                                      \# reward = -cost = -q k(x k,
       terminated = self.k == self.N-1
                                                      # Have we terminated? (i.e. is k=
       self.s = s next
                                                      # update environment state
        self.k += 1
                                                      # update current time step
       return s_next, reward, terminated, False, {} # return transition information
```

Recall  $x_{k+1} = x_k - w_k + a_k$  (clipped at 0 and 2) and e.g.  $P(w=0) = \frac{1}{10}$ 

# The Agent:



```
# inventory_environment.py
class RandomAgent(Agent):
    def pi(self, s, k, info=None):
        """ Return action to take in state s at time step k """
        return np.random.choice(3) # Return a random action
```

- The policy  $\mu_k(x_k)$  corresponding to pi(x, k, info)
- A training function which is given  $x_k$ ,  $u_k$  and  $x_{k+1}$  plus obtained reward plus additional information
- In each exercise session, you will write at least one agent
- Look at the Agent -class
- truncated=False; info is 'extra information' (see documentation)

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# DTU

#### The train-function

The train-function computes an episode as follows:

```
# inventory_environment.py
def simplified_train(env: Env, agent: Agent) -> float:
    s, _ = env.reset()
    J = 0  # Accumulated reward for this rollout
    for k in range(1000):
        a = agent.pi(s, k)
        sp, r, terminated, truncated, metadata = env.step(a)
        agent.train(s, a, sp, r, terminated)
        s = sp
        J += r
        if terminated or truncated:
            break
    return J
```

Above computes the sum-of-reward for one episode:

```
# inventory_environment.py
env = InventoryEnvironment()
agent = RandomAgent(env)
stats, _ = train(env,agent,num_episodes=1,verbose=False) # Perform one rollout.
print("Accumulated reward of first episode", stats[0]['Accumulated Reward'])
```

# Approximate value function



### Approximate

$$J_{\pi}(x_0) = \mathbb{E}\left[g_N(x_N) + \sum_{k=0}^{N-1} g_k(x_k, \mu_k(x_k), w_k)\right]$$
 (1)

#### As average over 1000 trajectories

```
# inventory_environment.py
stats, _ = train(env, agent, num_episodes=1000,verbose=False) # do 1000 rollouts
avg_reward = np.mean([stat['Accumulated Reward'] for stat in stats])
print("[RandomAgent class] Average cost of random policy J_pi_random(0)=", -avg_reward)
```

# Quiz 3: Bobs friend



Bob has  $x_0 = 20$  kroner. He can either:

- Action u=0: Put them in the bank at a 10% interest, thereby ending up with 22 kroner.
- Action u = 1: Lend them to a friend.
  - With probability  $\frac{1}{4}$  he looses everything
  - With probability  $\frac{3}{4}$  his friend gives him 12 kroner (aka one beer) as a thank you, and thus he will have 20+12=32 kroner total.

Bobs goal is to decide whether to put his money in the bank, or lend them to his friend. Which one of the following statements are correct:

- **a.** The state spaces are  $S_k = \{1, 2, \dots, 32\}$ .
- **b.** The dynamics is  $f_0(x_0, u_0, w_0) = 1.1x_0 + \frac{3}{4}(x_0 + 12u_0)$ .
- **c.** The action space is  $A_0(x_0) = \{0, 1\}$
- **d.** It is not possible to determine an optimal policy since we don't know what Bobs friend will do.

#### **Exercises**



Let's try it -- I will probably try to prepare solutions at home and be willing to present them

Let's try it -- But I am not going to volunteer to present anything.

The format is okay, but I don't want other students to present solutions. It should just be the TA who present the solution.

I prefer a format where we just work on the exercises and raise our hand if we have questions: I will be in the first room if this happens.



- IT015: Passive exercises; installation problems
- Aud.21 + IT019: Interactive exercises.
   Try to prepare and present homework exercises.

#### 1 Bobs financially challenged friend

Bob has  $x_0 = 20$  kroner. He can either:

- Action u=0: Put them in the bank at a 10% interest, thereby ending up with 22 kroner.
- Action u = 1: Lend them to a friend.
  - With probability <sup>1</sup>/<sub>4</sub> he looses everything
  - With probability  $\frac{3}{4}$  his friend gives him 12 kroner (aka one beer) as a thank you, and thus he will have 20+12=32 kroner total.





Tue Herlau.

Sequential decision making.

(Freely available online), 2024.