02417: Time Series Analysis

Week 1 - Introduction and overview

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DTU Compute

Based on material previous material from the course

February 24, 2025

Material in the course

- The course webpage `02417.compute.dtu.dk`
- Learn for messages and projects
- Book
- Slides
- Exercises
- Assignments

What to use Time Series Models for

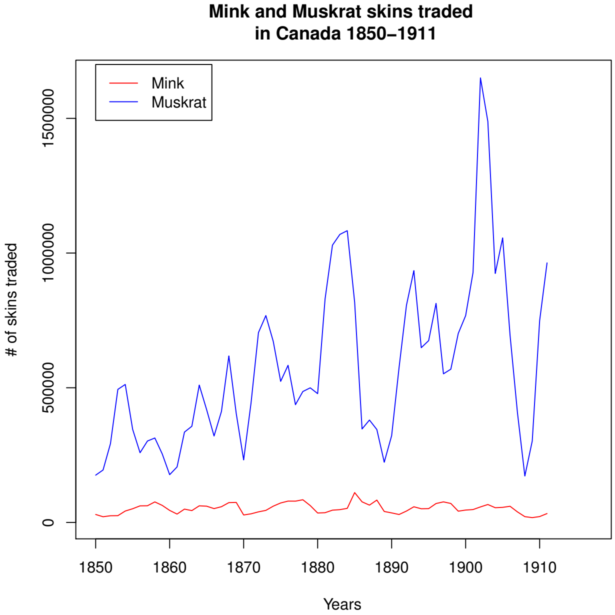
Applications:

- Prediction
- Estimation and hypothesis testing
- Control and decision making

We want a good model!

- Use data to fit a model
- Basically, any modelling technique can be used, there are no rules!
- Pros and cons: robustness, complexity, computation time, man hours to set up, ...
- WE ONLY DO LINEAR MODELS in this course (multiply and add using matrices)! Very fast and reliable, always a good starting points when developing models, can be tweaked later to include non-linear effects...

What you should be able to do



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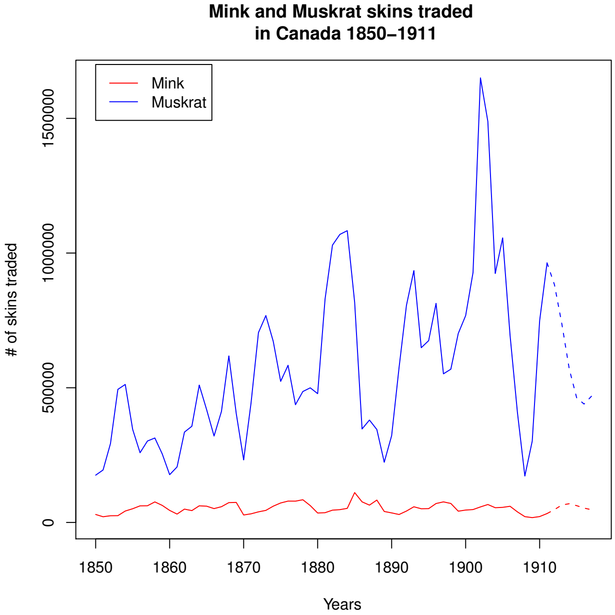

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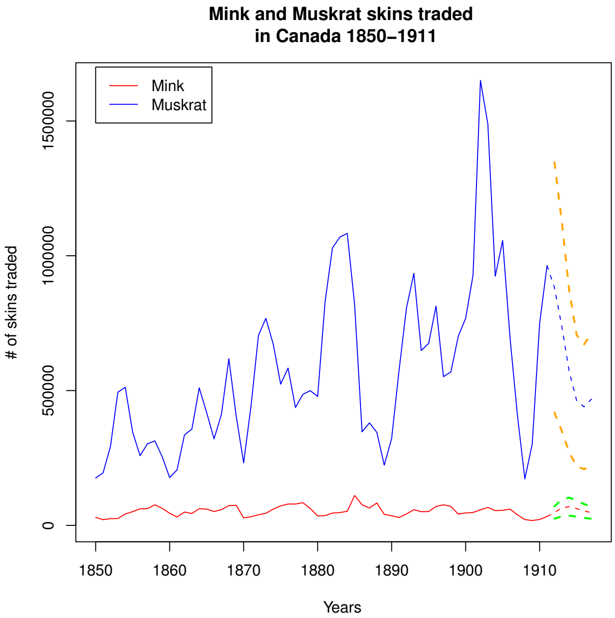


Introductory example – shares (COLO B 1 month)

What do think about the trend here? what would you buy or sell?

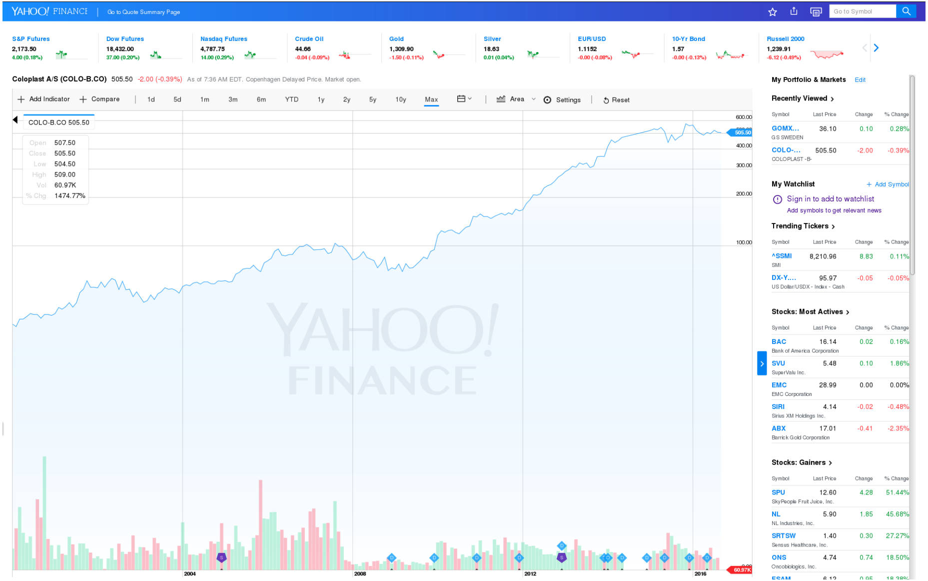
Introductory example – shares (COLO B 1 year)

Would you do the same?

Introductory example – shares (COLO B all)

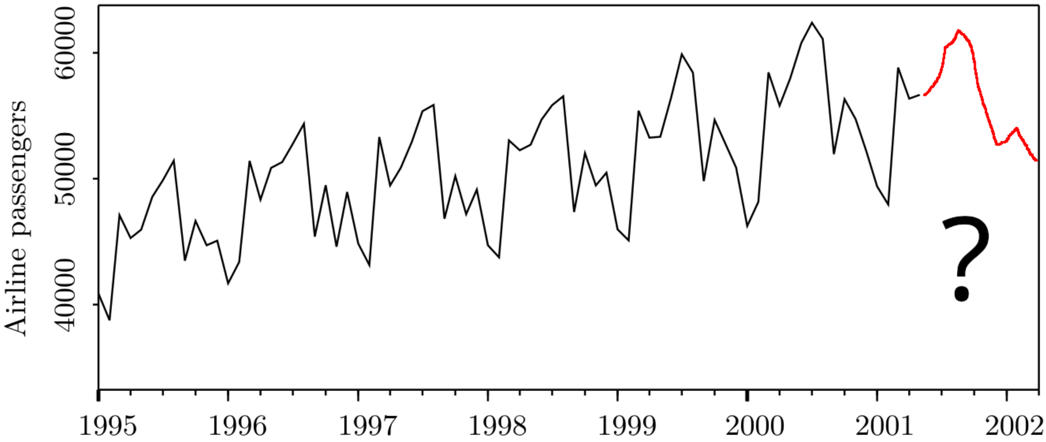
Can we use a linear trend model here?

Introductory example – shares (COLO B log(all))



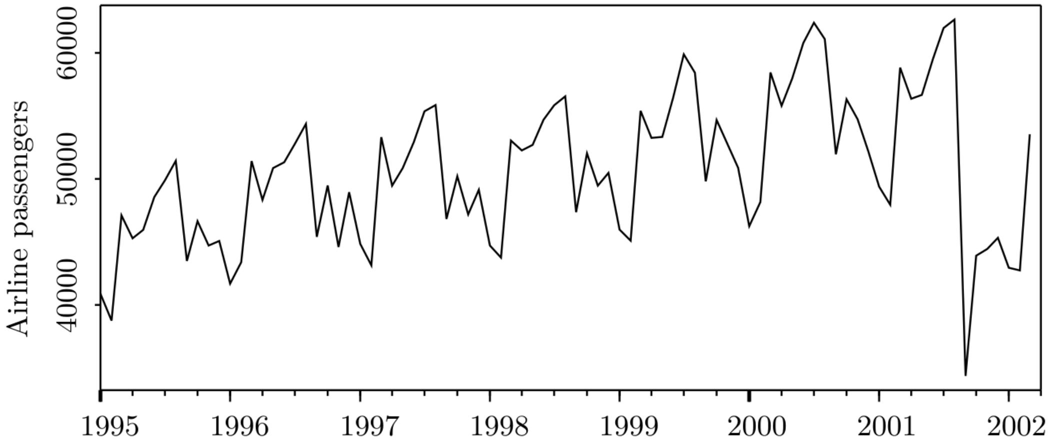
Take log(y): Often we can do non-linear transformations, resolve in cos and sine or splines,...

Number of Monthly Airline Passengers in the US

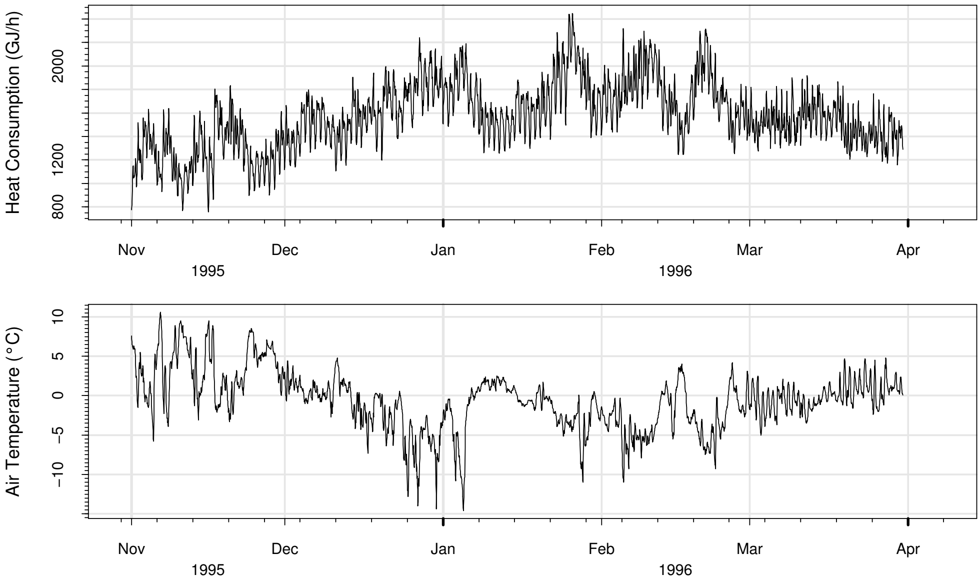


Is this a good prediction?

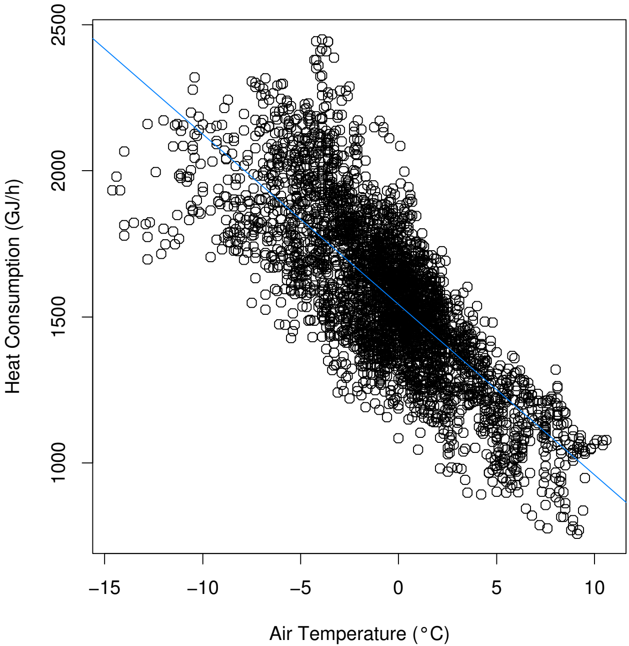
Number of Monthly Airline Passengers in the US



Consumption of District Heating (VEKS) – data

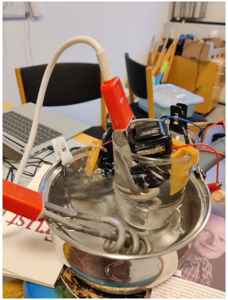


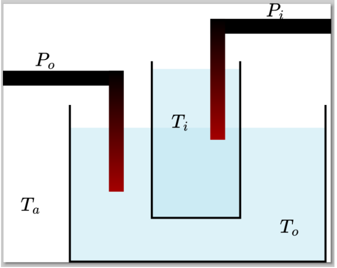
Consumption of DH – simple model



Discussion: What is a dynamical system?

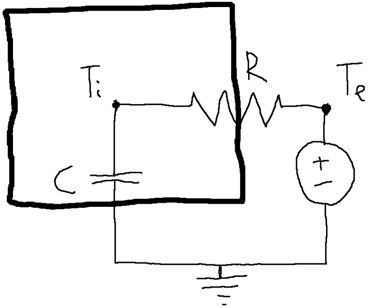
Last year!





Simplest first order RC-system

Single state model of the temperature in a box:




Discretize the ODE

$$\frac{dT_\mathrm{i}}{dt} = \frac{1}{RC}(T_\mathrm{e} - T_\mathrm{i})$$

It has the solution

$$T_\mathrm{i}(t + \Delta t) = T_\mathrm{e}(t) + e^{-\frac{\Delta t}{RC}}\big(T_\mathrm{i}(t) - T_\mathrm{e}(t)\big)$$

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if $\Delta t = 1$ and T_{e} is constant between the sample points then

$$T^{\mathrm{i}}_{t+1} = e^{-\frac{1}{RC}}\,T^{\mathrm{i}}_{t} + (1 - e^{-\frac{1}{RC}})\,T^{\mathrm{e}}_{t}$$

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since $e^{-\frac{1}{RC}}$ is between 0 and 1, then write it as

$$T^{\mathrm{i}}_{t+1} = \phi_1\, T^{\mathrm{i}}_{t} + \omega_1\, T^{\mathrm{e}}_{t}$$

where ϕ_1 and ω_1 are between 0 and 1.

Discretize the ODE

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Add a noise term and we have the ARX model

$$T^{\mathrm{i}}_{t+1} = \phi_1\,T^{\mathrm{i}}_{t} \quad + \omega_1\,T^{\mathrm{e}}_{t} \quad + \varepsilon_{t+1}$$

Discretize the ODE

$$\frac{dT_{\mathrm{i}}}{dt} = \frac{1}{RC}(T_{\mathrm{e}} - T_{\mathrm{i}})$$

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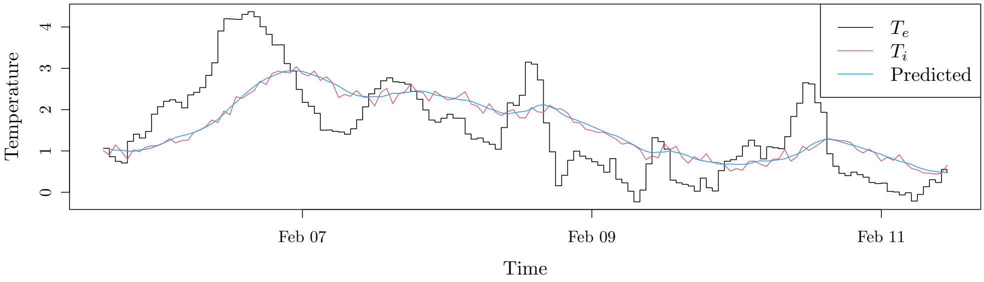
Add a noise term and we have the ARX model

$$T^{\mathrm{i}}_t \quad = \phi_1\, T^{\mathrm{i}}_{t-1} + \omega_1\, T^{\mathrm{e}}_{t-1} + \varepsilon_t$$

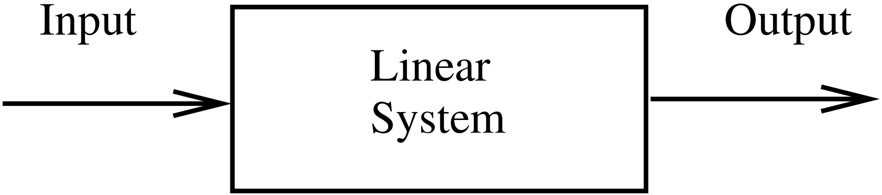
An ARMAX model

$$T^{\mathrm{i}}_t = \phi_1 T^{\mathrm{i}}_{t-1} + \omega_1 T^{\mathrm{e}}_t + \varepsilon_t + \theta_1 \varepsilon_{t-1}$$



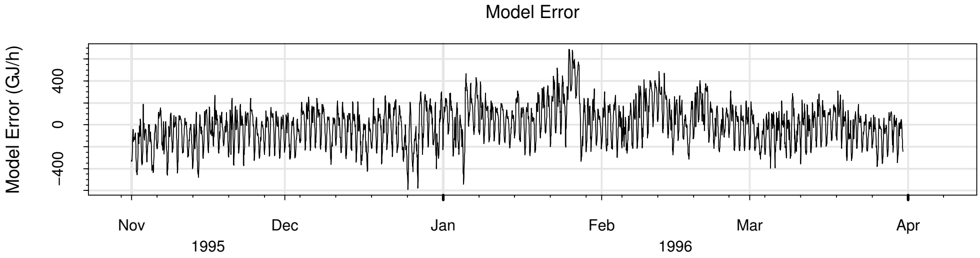
Linear Dynamic Systems – notation

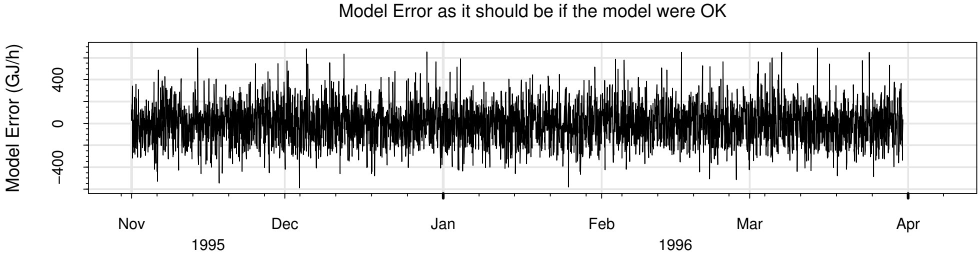
x	$\mathcal{F}[\cdot]$	y



$x(t)$	Differential eq., $h(u)$	$y(t)$
x_t	Difference eq., h_k, $h(B)$	y_t
$X(\omega)$	$\mathcal{H}(\omega)$	$Y(\omega)$
$X(z)$	$H(z)$	$Y(z)$

Consumption of DH – We use the model error to validate the model





A brief outline of the course

- General aspects of multivariate random variables
- Prediction using the general linear model
- Time series models
- Some theory on linear systems
- Time series models with external input

Some goals:

- Characterization of time series / signals; correlation functions, covariance functions, stationarity, linearity, ...
- Signal processing; filtering and smoothing
- Modelling; with or without external input
- Prediction with uncertainty