02417: Time Series Analysis

Model Examples

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Based on material previous material from the course

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Outline

In the global linear trend model the time is simply used as a regressor in the GLM model, e.g.

$$Y_t = \theta_1 + \theta_2 t + \varepsilon_t$$

where t is simply the time t in some units. The design matrix is for example

$$\boldsymbol{X} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ \vdots & \vdots \\ 1 & N \end{bmatrix}$$

if the sampling time is equidistant and the sampling period is normalized to 1.

In the global trend model the and transformations of time are used as regressors in the GLM model, e.g.

$$Y_t = \theta_1 f_1(t) + \theta_2 f_2(t) + \theta_3 f_3(t) + \ldots + \theta_p f_p(t) + \varepsilon_t$$

where x_t is simply the time t in some units. The design matrix is therefore

$$\boldsymbol{X} = \begin{bmatrix} f_1(1) & f_2(1) & \cdots & f_p(1) \\ f_1(2) & f_2(2) & \cdots & f_p(2) \\ \vdots & \vdots & \ddots & \vdots \\ f_1(N) & f_2(N) & \cdots & f_p(N) \end{bmatrix}$$

where $f_i(t)$ are deterministic functions. Typical functions are higher order polynomials and a classical example are seasonal components, which can be achieved with Fourier series

$$f_1(t) = \cos\left(\frac{2\pi t}{T}\right), \quad f_2(t) = \sin\left(\frac{2\pi t}{T}\right) \quad , \ldots, \quad f_{2n-1}(t) = \cos\left(\frac{2\pi n t}{T}\right) \quad , \quad f_{2n}(t) = \sin\left(\frac{2\pi n t}{T}\right)$$

where T is the seasonal period and n is the number of harmonics.
Naturally, constant, linear and seasonal parts can be combined with other regressors.