

1 Insertions in balanced search trees (w) Show the 2-3 tree that results from inserting the keys 41, 38, 31, 12, 19, 8, 11 in this order into an initially empty tree.

2 Range queries Explain how to report all items with a key between a and b (both included) in a balanced binary search tree in $O(\log n + k)$ time, where k is the number of items that are returned.

3 Databases You are working as a consultant for the company "Boxes, Boxes and Boxes", that sells boxes. They want a database containing information about all their boxes. Each box has an id, a size, a type, and a price. They want to be able to update the database with insertions and deletions of boxes. The database should support the following updates and queries efficiently:

- $\text{Insert}(i, s, t, p)$: Insert a box with id i , size s , type t and price p into the database.
- $\text{Delete}(i)$: Delete the box with id i from the database.
- $\text{Report-Price}(a, b)$: Return the id of all boxes with a price between a and b .
- $\text{Find-Size}(s)$: Return the id of the box with size closest to s .

3.1 Give a data structure supporting the required updates and queries. Analyse the space and the update and query times of your data structure.

You may assume that the prices and sizes of the boxes are unique.

3.2 Change your solution to handle the case where the sizes and prices are not unique.

4 Counting range queries Explain how to modify a balanced binary search tree to make it possible to report the number of items with a key between a and b (both included) in $O(\log n)$ time.

5 Height of 2-3 trees Show that the height of a 2-3 tree has height at most $\lg(n + 1)$, where n is the number of elements in the tree (not the number of nodes). *Hint*: First show by induction on h that $n \geq 2^{h+1} - 1$, where h is the height of the tree.

6 Deletion in 2-3 trees Give an efficient algorithm for deleting elements from 2-3 trees.

Use rotations as explained in the lecture.

You can use the tree constructed in Exercise 1 as a starting point.

What is the running time of a deletion?

What kind of scenario can make the height of the tree shrink by one?

Puzzle: The Blind Man A blind man was handed a deck of 52 cards with exactly 10 cards facing up. How can he divide it into two piles, each of which having the same number of cards facing up?