External Memory II

- Searching with Fast Updates
- Searching Strings

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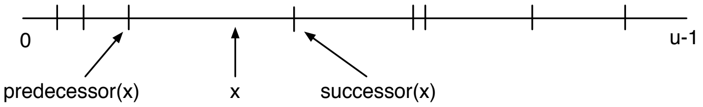
Searching

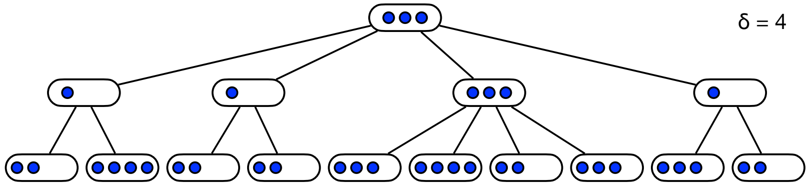
- Searching. Maintain a set S ⊆ U = {0, ..., u-1} supporting
  - member(x): determine if x ∈ S
  - predecessor(x): return largest element in S ≤ x.
  - successor(x): return smallest element in S ≥ x.
  - insert(x): set S = S ∪ {x}
  - delete(x): set S = S - {x}



Searching

- Applications.
  - Relational data bases.
  - File systems.

B-tree

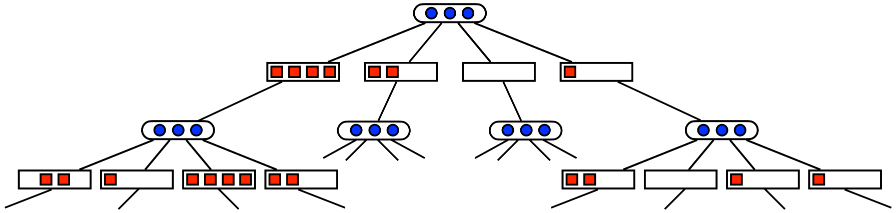


- B-tree of order δ = Θ(B) with N keys.
  - Keys in leaves. Routing elements in internal nodes.
  - Degree between δ/2 and δ.
  - Root degree between 2 and δ.
  - Leaves store between δ/2 and δ keys.
  - All leaves have the same depth.
- Height. $\Theta(\log_\delta (N/B)) = \Theta(\log_B N)$
- Search and update. $O(\log_B N)$ I/Os.

B^ε-tree

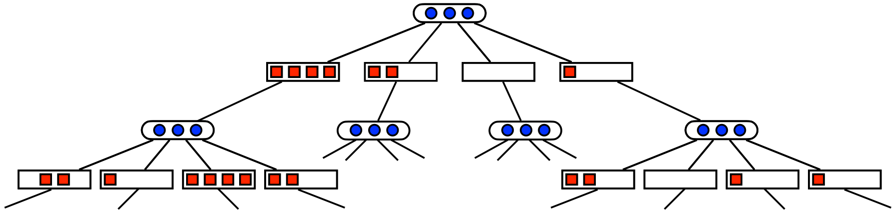
- Idea.
  - Speed up updates by buffering them at each node along the path to a leaf.
  - Move many updates together in each I/O.
  - Search (almost) as before.
  - $\varepsilon \in (0, 1]$ is a parameter.
- Solution in 2 steps.
  - Focus on $\sqrt{B}$-tree (ε = 1/2).
    - Searching in $O(\log_B N)$ I/Os.
    - Updates in $O((\log_B N)/\sqrt{B})$ amortized.
  - Generalize to any ε.

$\sqrt{B}$-tree



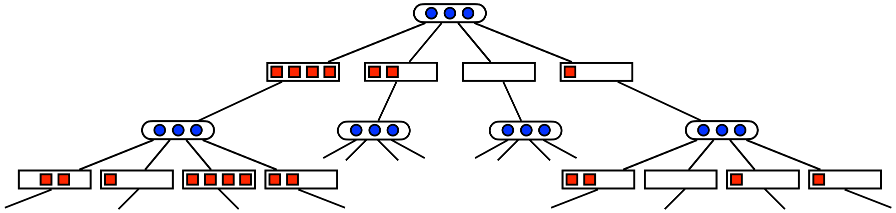
- $\sqrt{B}$-tree with N keys.
  - B-tree of degree $\Theta(\sqrt{B})$ with buffers of size $\Theta(\sqrt{B})$ at each edge.
  - Buffer stores delayed updates in subtree.
  - Nodes and child buffers stored together in O(1) blocks.
- Height. $\Theta(\log_{\sqrt{B}} N) = \Theta(\log_B N)$

$\sqrt{B}$-tree



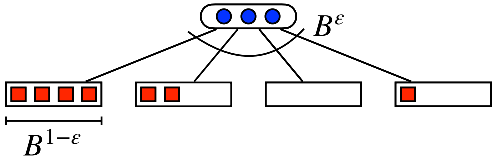
- Searching.
  - Find leaf using routing elements. Check buffers along path.
- I/Os. $O(\log_B N)$.

$\sqrt{B}$-tree



- Updates.
  - Insert update message into buffer at child.
  - If buffer full, flush and recurse at child.
  - If we fill leaf, rebalance tree as B-tree.
- I/O intuition. A flush moves $\sqrt{B}$ messages together ⇒ O((log$_B$ N)/$\sqrt{B}$) amortized I/Os.
- I/Os.
  - Assign (ch)/$\sqrt{B}$ credits to each update, where h = O(log$_B$ N) is height and c > 1 is appropiate constant.
  - Put c/$\sqrt{B}$ credits each node on path.
  - ⇒ We can pay for buffer overflows and rebalancing.

B^{ε}-tree



- B^{ε}-tree with N keys.
  - B-tree of degree Θ(B^{ε}) with buffers of size Θ($B^{1-\varepsilon}$) at each edge.
- Searching. $O\left(\frac{\log_B N}{\varepsilon}\right)$ I/Os.
- Updates. $O\left(\frac{\log_B N}{\varepsilon B^{1-\varepsilon}}\right)$ I/Os.

B^{ε}-tree

	Search	Update
B-tree	$O(\log_B N)$	$O(\log_B N)$
$\sqrt{B}$-tree	$O(\log_B N)$	$O\left(\frac{\log_B N}{\sqrt{B}}\right)$
B^{ε}-tree	$O\left(\frac{\log_B N}{\varepsilon}\right)$	$O\left(\frac{\log_B N}{\varepsilon B^{1-\varepsilon}}\right)$

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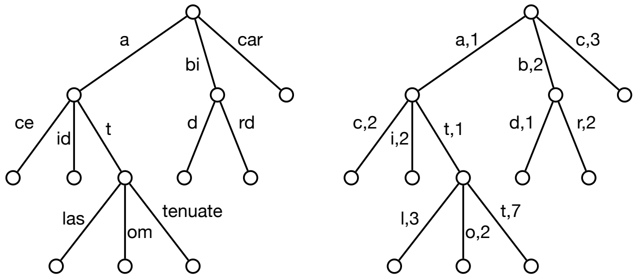
String Searching

- String searching. Maintain a set S = {S_1, S_2, ..., S_K} of K strings of total length N supporting the following operations:
  - search(P): return string in S with longest common prefix with P.
  - insert(P): set S = S ∪ {P}
  - delete(P): set S = S - {P}

String Searching

- Goal.
  - Searching in O(|P|/B + $\log_B$ K) I/Os.
  - Ignore insert and delete.
- Solution in 3 steps.
  - Blind tries.
  - String B-trees.
  - String B-trees with fast searches.

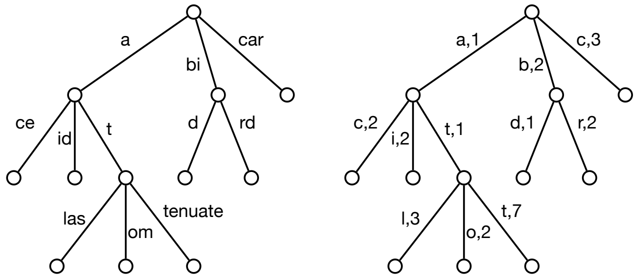
Blind Trie



1. ace
2. aid
3. atlas
4. atom
5. attenuate
6. bid
7. bird
8. car

- Data structure.
  - Sorted set of strings.
  - Compact trie for S. Edges store first char + string length.
- Space. Strings + trie: O(N + K) = O(N).

Blind Trie



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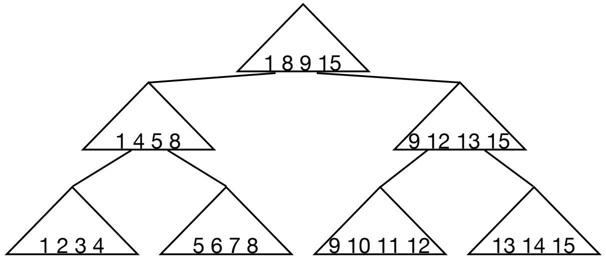
- Search. Traverse and verify candidate.
- Time. O(|P|)

String B-tree



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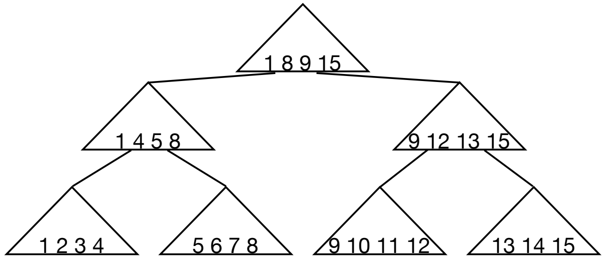
9. cod
10. dog
11. fit
12. lid
13. patent
14. sun
15. zoo

- Data structure. Combination of B-tree and blind tries.
  - Sorted set of strings.
  - Nodes store blind trie over B strings.
  - Leftmost and rightmost string in subtree stored for each child.
- Space. Strings + O(B) per node: O(N + N) = O(N)

String B-tree



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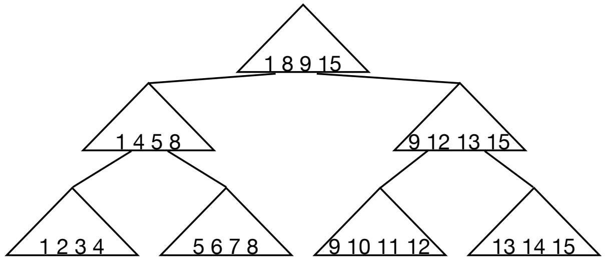
- Searching.
  - Traverse and verify at each node.
- I/Os.
  - O(|P|/B + 1) I/Os at each node.
  - ⇒ $O((|P|/B)\log_B K)$ I/Os total.

String B-tree



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- Fast searching.
  - Traverse and verify at each node.
  - But: remember longest prefix matched at each node.
- I/Os.
  - O(|P|/B) I/Os in total for string verification. O(1) I/Os at each node.
  - ⇒ $O((|P|/B) + \log_B K)$ I/Os.

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