Mandatory Exercise: Hashing

Philip Bille

1   Set Intersection   Let $\mathscr{S} = S_1, \ldots, S_k$ be non-empty sets of integers, $S_i \subseteq U = \{0, \ldots, u-1\}$, $1 \le i \le k$. The total size of the sets is n. We want a data structure for $\mathscr{S}$ that supports the following operation:

- intersect(i, j): Return yes if $S_i \cap S_j \neq \emptyset$ and no otherwise.

Solve the following exercises.

1.1 Give a data structure that uses $O(k^2)$ space and supports fast intersect queries. The query time for an intersect query should be $o(n)$, i.e., asymptotically faster than n.

1.2 Give a data structure that uses $O(n)$ space and supports fast queries. *Hint:* A good solution has a query time that depends on the size of one of the input sets to the intersect query.

1.3 [∗] (Bonus exercise, not part of the assesment) Give a data structure with $O(\sqrt{n})$ query time and as little space as you can achieve.

In each of the exercises only consider space and query time. Ignore preprocessing time.