Mandatory Exercise: Approximation Algorithms 1

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1 Consultant You are a consultant for a company MultiServ that has a system that performs real-time scheduling of jobs on multiple servers. Once a batch of n jobs arrives they need to be allocated to servers. The company has two types of servers: m fast servers and k slow servers. Each job i takes time t_i to process on a slow server, and time $t_i/3$ to process on a fast server. The goal is to minimize the makespan of the schedule.

Currently, the company is using the following simple greedy algorithm to schedule a batch of n jobs: Process the jobs in any order. Assign the next job on list to machine with smallest current load.

1.1 Give an example showing that algorithm the company uses does not get the optimal result.

1.2 Prove that even though it is not optimal, this simple greedy algorithm is a 4-approximation algorithm.

1.3 Give an algorithm with a better approximation guarantee for the problem. That is, give an α-approximation algorithm for the problem, where $\alpha < 4$.

Remember to prove that your algorithm is an α-approximation algorithm. That is, prove that it: runs in polynomial time, returns a valid solution, and has an approximation factor less than 4.