Solutions for Exercises, Week 2

1. Solution for Trans.1

(1) $a_1\ a_2\ a_3\ b_1\ b_2$
(2) $a_1\ a_2\ b_1\ a_3\ b_2$
(3) $a_1\ a_2\ b_1\ b_2\ a_3$
(4) $a_1\ b_1\ a_2\ a_3\ b_2$
(5) $a_1\ b_1\ a_2\ b_2\ a_3$
(6) $a_1\ b_1\ b_2\ a_2\ a_3$
(7) $b_1\ a_1\ a_2\ a_3\ b_2$
(8) $b_1\ a_1\ a_2\ b_2\ a_3$
(9) $b_1\ a_1\ b_2\ a_2\ a_3$
(10) $b_1\ b_2\ a_1\ a_2\ a_3$

2. Solution for Trans.2

The number of interleavings is given by:

$$\begin{pmatrix} n_1 + n_2 \\ n_1 \end{pmatrix} = \begin{pmatrix} n_1 + n_2 \\ n_2 \end{pmatrix} = \frac{(n_1 + n_2)!}{n_1! * n_2!}$$

An argument:

> Each interleaving must contain n_1 actions from P_1 and n_2 actions from P_2, ie. in total $n_1 + n_2$ actions. We may say that each interleaving has $n_1 + n_2$ places and an interleaving is the uniquely given by selecting n_1 of these for the actions of P_1 (the actions are supposed to come in the given order). As known from combinatorics, the number of ways n_1 elements can be selected out of $n_1 + n_2$ elements is given by the above expression.

An other argument:

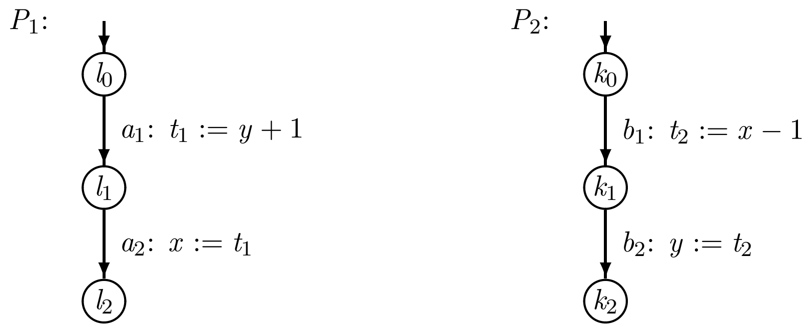
> For any interleaving, the actions from P_1 can be permuted in $n_1!$ ways and the actions from P_2 in $n_2!$ ways. From any interleaving, we may thus generate $n_1! * n_2!$ permuations of the total of $(n_1 + n_2)!$ permutations of all actions of P_1 and P_2. From this, the expression follows.

2. Solution for Trans.5

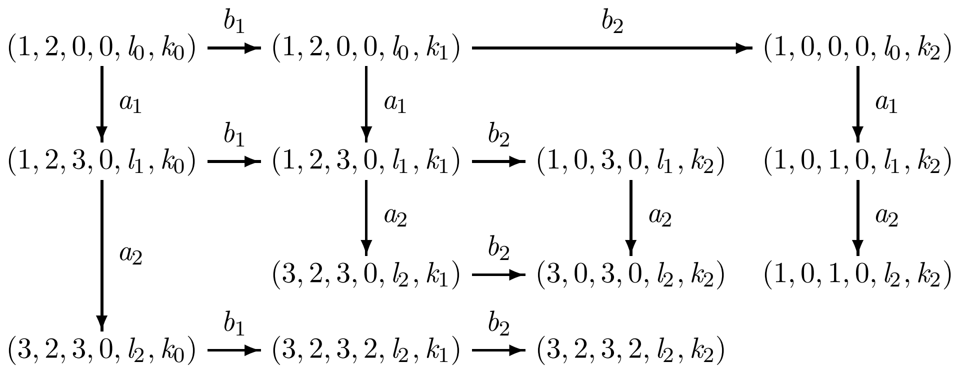
Rewriting to atomic assignment statements:

$x := 1;\ y := 2;$
co
  $\langle t_1 := y + 1 \rangle;\ \langle x := t_1 \rangle \parallel \langle t_2 := x - 1 \rangle;\ \langle y := t_2 \rangle$
oc

Or equivalently as transition diagrams:



Let the global state be given by a vector $(x, y, t_1, t_2, \pi_1, \pi_2)$ where π_i is the control pointer for process P_i. Assuming arbitrarily t_1 and t_2 to be initially 0, the transition system for the concurrent systems is given by the transition graph:



By inspection of the final states ($\pi_1 = l_2 \wedge \pi_2 = k_2$) we find the possibilities for (x, y):

$$\{(1,0), (3,0), (3,2)\}$$

3. Solution for Trans.6

Transition diagrams of the processes in Andrews Fig. 2.2: