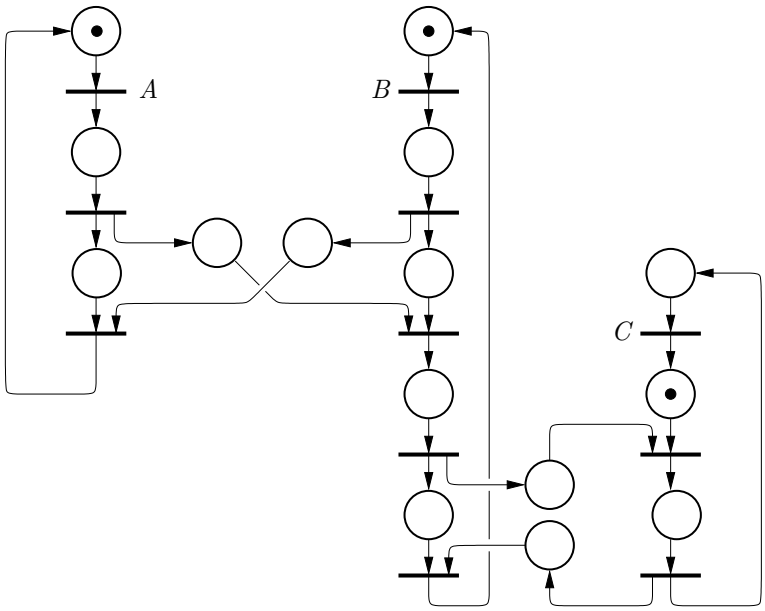
Suggested Solutions for

Written Exam, December 10, 2019

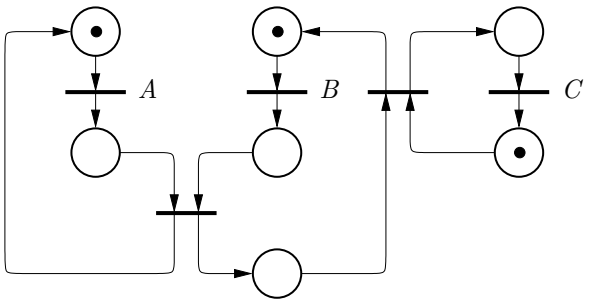
PROBLEM 1

Question 1.1

A direct translation yields:



[By recognizing the cross-signalling between P_A and P_B as well as the back-and-forth signalling between P_B and P_C to be true synchronizations, this may also be expressed as:]



Question 1.2

The following inequalities may be seen, e.g. by applying the semaphore invariant:

$$I \triangleq c \leq a \leq c + 3$$

Question 1.3

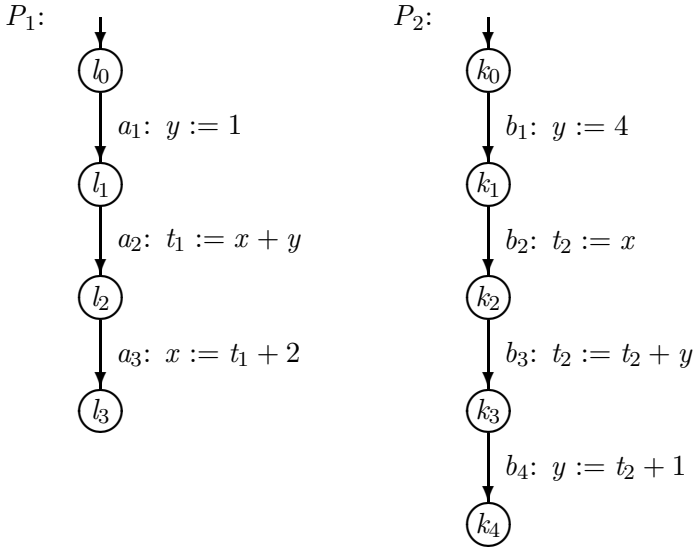
The above Petri Net is readily implemented using synchronous communications:

```
process P1;          process P2;          process P3;
  repeat               repeat               repeat
    A;                   B;                   P2 ? ();
    P2 ! ()              P1 ? ();             C
  forever                P3 ! ()            forever
                       forever
```

PROBLEM 2

Question 2.1

(a) Transition diagrams:



[Location and action labels not required. Note that a_2 can be considered atomic.]

(b) [Rather than working through the 35 possible interleavings, we observe that the final value of y may be set by either a_1 (to 1) or b_4. In the latter case, the value depends on readings of x and y. If x has not been changed, it is 0 and y may be read as either 1 or 4 yielding values 2 and 5 to y. If x has been changed, P_1 may have read y as 1 giving $x = 3$. As P_2 may read y as 1 or 4, this yiels final values 5 and 8. Finally P_1 may read $y = 4$ giving $x = 6$ and P_2 will then also read $y = 4$ yielding a final y-value of 11.]

It is found that the final value of y may be either of

$$1, 2, 5, 8, 11$$

Question 2.2

(a) I holds initially since $x = 0 \wedge y = 0$.

Checking all atomic actions:

a_1: Since both x and y are non-negative before, so are the expressions $x + y$ and $y + 1$ and hence $0 \leq x \wedge 0 \leq y$ holds after the action.

Before the action $x \leq 1$ and $y \leq x + 1$ implies $y \leq 2$ and therefore $x + y \leq 3$ why $x \leq 3$ holds after the action.

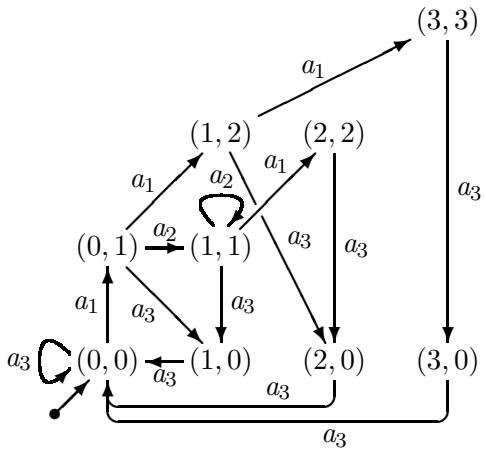
If $y = x + 1$, a_1 increments x by at least 1 and y by 1 preserving the inequality $y \leq x + 1$. If $y < x + 1$, incrementing y by 1 and not decrementing x ensures $y \leq x + 1$ after the action.

a_2: After the action, $(x, y) = (1, 1)$ which satisfies I.

a_3: By I and the condition, $0 \leq y \leq 3$ before the action and hence $0 \leq x \leq 3$ holds after the action. As y becomes 0, also $0 \leq y \leq x + 1$ is satisfied after the action.

Since I holds initially and is preserved by all atomic actions, I is an invariant of the program.

(b) Transition graph:



(c) The predicate $I \wedge y \leq 3$ is seen to encompass all the reachable states and is thus an invariant of the program. However, as it allows states which is are not reachable (e.g. $(2,1)$), the predicate is **not** a characteristic invariant of the program.

(d) **Assuming weak fairness**

- F **holds**. [As control cannot remain at a state with outgoing transitions, any execution will lead to $(0,0)$ from where a_1 takes the program to $(0,1)$.]
- G **does not** hold. [The only reachable states satisfying $y = 2(x-1)$ are $(1,0)$ and $(2,2)$. These may be avoided by an execution following an outer cycle:

$$(0,0) \xrightarrow{a_1} (0,1) \xrightarrow{a_1} (1,2) \xrightarrow{a_1} (3,3) \xrightarrow{a_3} (3,0) \xrightarrow{a_3} (0,0) \xrightarrow{a_1} \cdots \qquad (*)$$

  Since this execution path does not remain at a single state, it satisfies weak fairness.]
- H **does not** hold. [The cycle $(*)$ also avoids $x = 2$.]
- J **does not** hold. [From $(0,1)$ or $(1,1)$, the execution may follow the inner cycle

$$(0,1) \xrightarrow{a_2} (1,1) \xrightarrow{a_3} (1,0) \xrightarrow{a_3} (0,0) \xrightarrow{a_1} (0,1) \xrightarrow{a_2} \cdots \qquad (**)$$

  avoiding states where $x \geq 2$.]

Assuming strong fairness

- F **holds**. [By weak fairness.]
- G **holds**. [As $(0,1)$ is reached infinitely often (cf. F), a_2 is enabled infinitely often and hence $(1,1)$ is reached infinitely often. From $(1,1)$ either $(1,0)$ or $(2,2)$ must be reached.]
- H **holds**. [As $(1,1)$ is reached infinitely often, if $(1,0)$ is to be avoided forever, the execution must continue to $(2,2)$.]
- J **does not** hold. [In the sequence $(**)$ all actions are executed infinitely often and hence also strong fairness is satisfied.]

PROBLEM 3

Question 3.1

(a) The **if**-statement ensures that all pending calls of *pass*() will be accepted when a multiple is reached.

(b) In the module header the declaration **op** $set(l : integer)$; is inserted. The operation may be implemented by adding the following branch to the outermost **in**-construct:

$$
\begin{array}{l}
[] \;\; set(l : integer) \textbf{ and } l \geq 2 \rightarrow \\
\quad \textbf{while } count \neq 0 \textbf{ do} \\
\quad\quad \textbf{in } incr() \rightarrow count := (count + 1) \bmod k \textbf{ ni}; \\
\quad k := l
\end{array}
$$

[It is not specified what calling $set(l)$ with $l < 2$ should do. Here such calls just block.

As an alternative solution, *set* may be accepted only when $count = 0$ and then preventing calls of *incr*() from being accepted when there are pending calls of of *set* when $count = 0$.]

Question 3.2

(a) Synchronization code for each process P_i $[i : 1..n]$:

$$
\begin{array}{l}
\vdots \\
ModCount.incr(); \\
ModCount.pass(); \\
\vdots
\end{array}
$$

The constant K_0 must be defined to n.

(b) If the above synchronizaiton code is used for more than one synchronization point, it cannot be guaranteed that all processes have called *pass*(), before *incr*() is called for the next synchronization point rendering the processes stuck with a non-zero value of *count*.

This issue is solved by using two of these one-time barriers in each round:

$$
\begin{array}{l}
\vdots \\
ModCount_1.incr(); \\
ModCount_1.pass(); \\
ModCount_2.incr(); \\
ModCount_2.pass(); \\
\vdots
\end{array}
$$

For both module instances, K_0 must be defined to n.

Question 3.3

(a) The *ModCount* module is readily implemented as a monitor:

```
monitor ModCount

  var count : integer := 0;
      Zero : condition;

  procedure incr() {
    count := (count + 1) mod K0;
    if count = 0 then signal_all(Zero)
  }

  procedure pass() {
    if count ≠ 0 then wait(Zero)
  }

end
```

(b) Calls of *pass*() should be waiting only if *count* is not zero:

$$I \triangleq waiting(Zero) > 0 \Rightarrow count \neq 0$$

I holds initially as *Zero* is empty. The queue size is only incremented when *count* is non-zero and whenever *count* becomes zero, the condition queue is completely emptied. Hence I is a monitor invariant.

(c) The solution is obviously not robust towards spurious wakeups, as the wait condition in *pass*() is not rechecked. Changing the **if**-statement to a **while**-statement does remedy this, but then the behaviour of *ModCount* is changed, as not all pending calls of *pass*() are guaranteed to be released when the next multiple is reached.

A robust solution can be obtained by holding back calls of *incr*() and *pass*() while *Zero* is being emptied:

```
monitor ModCount

  var count, rem : integer := 0;
      Zero, Hold : condition;

  procedure incr() {
    while rem > 0 do wait(Hold);
    count := (count + 1) mod K0;
    if count = 0 then { rem := length(Zero);
                        signal_all(Zero)      }
  }

  procedure pass() {
    while rem > 0 do wait(Hold);
    while count ≠ 0 do wait(Zero);
    if rem > 0 then { rem := rem − 1;
                      if rem = 0 then signal_all(Hold) }
  }

end
```

A slightly simpler solution utilizes the round-counting idea for robustness:

```
monitor ModCount

  var count, round : integer := 0;
      Zero : condition;

  procedure incr() {
    count := (count + 1) mod K0;
    if count = 0 then { round := round + 1;
                        signal_all(Zero)      }
  }

  procedure pass() {
    var myround : integer := round;
    if count ≠ 0 then
      while myround = round do wait(Zero);
  }

end
```