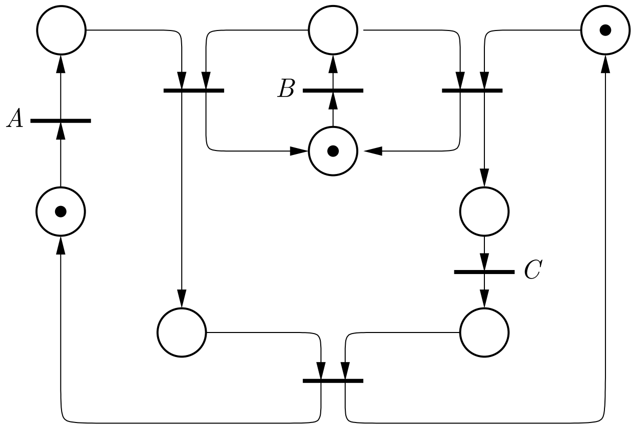
Suggested Solutions for

Written Exam, December 11, 2018

PROBLEM 1

Question 1.1

A direct translation of the CSP program to a Petri Net yields:



[This net cannot be further reduced.]

Question 1.2

As A and C are seen to synchronize with each other for each round and as A and C may actually be executed concurrently (if P_2 executes B and then synchronizes with P_3), the following predicate is a characteristic invariant of the program:

$$I \triangleq |a - c| \leq 1$$

Question 1.3

The selective synchronization by P_2 with P_1 and P_3 can standardly be realized with a pair of semaphores. Likewise, the synchronization of A and C can be obtained by cross-signalling:

```
var $S_A, S_B, S_C, S_{AC}$ : semaphore;
$S_A$ := 0;  $S_B$ := 0;  $S_C$ := 0;  $S_{AC}$ := 0;

process $P_A$;          process $P_B$;          process $P_C$;
  repeat                  repeat                  repeat
    $A$;                    $B$;                    P($S_{AC}$);
    P($S_{AC}$);            V($S_{AC}$);            V($S_B$);
    V($S_B$)                P($S_B$)                $C$;
    V($S_C$);             forever                   V($S_A$);
    P($S_A$)                                        P($S_C$)
  forever                                         forever
```

PROBLEM 2

Question 2.1

(a) The statement pairs are checked for critical references with respect to each other:

Pair	Mutually atomic	Rationale [not required]
a, b	NO	Two critical references in a.
a, c	YES	Only one critical reference in each statement.
a, d	NO	Both reading and writing of x in a are critical.
b, c	NO	In c, writing y and reading x are both critical.
b, d	YES	Only one critical reference in d.
c, d	YES	In c, only the reading of x is critical.

[Being executed indivisibly, all reads and writes in statement b together only count as a single critical reference.]

(b) Transition diagrams:

b:

$$\begin{tikzcd} {} \arrow[d] \\ k_0 \arrow[d, "b_1:\ x := y + 2"] \\ k_1 \end{tikzcd}$$

c:

$$\begin{tikzcd} {} \arrow[d] \\ l_0 \arrow[d, "c_1:\ t := x"] \\ l_1 \arrow[d, "c_2:\ y := t + 3"] \\ l_2 \end{tikzcd}$$

[Location and action labels not required.]

(c) Going through the three possible interleavings of the atomic actions, the possible final values of (x, y) are found to be:

$$(2,5), (2,3), (5,3)$$

Question 2.2

(a) I holds initially since $x = 0 \land y = 0 \Rightarrow x = y$.

Checking all atomic actions:

a_1: By cases of I:
Case $y = x$: In terms of the old values the new sum is $y + (3 - x)$, but as $x = y$, this equals 3. Thus I is preserved.
Case $x + y = 3$: The new value of y is 3 minus the old value of x, but this must then be equal to the old value of y. Thus, both variables are assigned the old value of y and hence I holds.

a_2: Executed only if $x = y$ and since both variables are incremented by 1, this property is preserved and therefore I holds.

a_3: This action always sets (x, y) to $(3, 0)$ and thus the $x + y = 3$ case of I holds.

Since I holds initially and is preserved by all atomic actions, I is an invariant of the program.

(b) Transition graph:

$$\begin{tikzcd}
 & (0,3) \arrow[rrr, "a_1"] & & & (3,3) \arrow[ddd, "a_1"] \\
 & & (1,2) \arrow[r, "a_1"] & (2,2) \arrow[d, "a_1"] \arrow[ur, "a_2"] & \\
 & & (1,1) \arrow[u, "a_1"] \arrow[ur, "a_2"] & (2,1) \arrow[l, "a_1"] \arrow[dr, "a_3"] & \\
\bullet \arrow[r] & (0,0) \arrow[uuu, "a_1"] \arrow[urr, "a_2"] & & & (3,0) \arrow[llll, "a_1"] \arrow[loop right, "a_3"]
\end{tikzcd}$$

(c) Defining

$$H \triangleq 0 \leq x \leq 3 \wedge 0 \leq y \leq 3$$

it is seen that $H \wedge I$ becomes a characteristic invariant. [In fact, $H \triangleq 0 \leq x \leq 3$ suffices.]

(d) **Assuming weak fairness**

- F **does not** hold. [From the state $(0,0)$ the execution may go to state $(1,1)$ and from there follow any combination of the two inner cycles:

$$(1,1) \xrightarrow{a_1} (1,2) \xrightarrow{a_1} (2,2) \xrightarrow{a_1} (2,1) \xrightarrow{a_1} (1,1) \longrightarrow \cdots \qquad (*)$$

  and

$$(1,1) \xrightarrow{a_2} (2,2) \xrightarrow{a_1} (2,1) \xrightarrow{a_1} (1,1) \longrightarrow \cdots \qquad (**)$$

  Since any such execution path does not remain at a single state, it satisfies weak fairness. In all cases, $x = 3$ will never occur.]

- G **holds**. [From $\Diamond\Box\,(x \neq 1)$ it follows that at some point the inner cycles are left and then the execution must follow the outer cycle:

$$(0,0) \xrightarrow{a_1} (0,3) \xrightarrow{a_1} (3,3) \xrightarrow{a_1} (3,0) \xrightarrow{a_1} (0,0) \xrightarrow{a_1} \cdots \qquad (***)$$

  where x will become 0. (Actually, infinitely often.)]

- H **does not** hold. [The only reachable state for which $x + y \geq 5$ is $(3,3)$ and by the argument for F, $x = 3$ might never occur under weak fairness.]

- J **does not** hold. [From the states with $x = 2$, $(2,2)$ and $(2,1)$, the outer cycle $(***)$ may be reached immediately and then followed forever, avoiding $x = 1$.]

Assuming strong fairness

- F **holds**. [The execution cannot any longer stay in the inner cycles $(*)$ and $(**)$ as a_3 will then be enabled infinitely often and hence must be taken leading to the state $(3,0)$.]

- G **holds**. [By weak fairness.]

- H **does not** hold. [In the cycle

$$(0,0) \xrightarrow{a_2} (1,1) \xrightarrow{a_2} (2,2) \xrightarrow{a_1} (2,1) \xrightarrow{a_3} (3,0) \xrightarrow{a_1} (0,0) \longrightarrow \cdots$$

  all actions are executed and hence strong fairness is satified. However, the state $(3,3)$ is never reached.]
- J **holds**. [From $(2,2)$ and $(2,1)$, the only way to avoid the state $(1,1)$ would be to enter and remain in the outer loop $(***)$. However, in this, a_2 would be infinitely often enabled without being taken violating strong fairness.]

PROBLEM 3

Question 3.1

(a) [The monitor will let the first writer start writing and then queue up the first reader on the *head* condition queue and the second reader on the *tail* condition queue where it will be followed by the second writer and the third reader. When the first writer is done, the first two readers are allowed to read while the second writer is moved to the *head* queue. The third reader is still in the *tail* queue where it is joined by the fourth reader. Thus:]

$$r = 2 \wedge w = 0 \wedge \mathit{waiting}(\mathit{head}) = 1 \wedge \mathit{waiting}(\mathit{tail}) = 2$$

(b) Initially, I holds trivially because *tail* is empty. In *lock*, calls only wait on *tail* if *heading* is true. Also, *heading* is set to false only when *tail* is empty. Hence I is preserved in *lock* and as it is trivially preserved by *unlock*, I is an invariant of the monitor.

(c) The following should be an invariant of the monitor:

For any (single) call *lock*(*write*) waiting in *head* either $w > 0$ or *write* is true while $r > 0$.

Question 3.2

In the declaration part of the monitor, a limit variable *lim* is added:

```
$lim : posinteger := L$;
```

The operation *setLimit* is defined as:

```
**procedure** $setLimit(k : posinteger)$ {
  **if** $lim \leq r < k$ **then** $signal(head)$;
  $lim := k$;
}
```

In *lock*, the given wait condition must be weakened to take the limit into account:

```
  **while** $w > 0 \vee (write \wedge r > 0) \vee r \geq lim$ **do** { ... };
```

Finally, the signalling in *unlock* must be extended:

```
  **if** $r = 0 \vee r = lim - 1$ **then** $signal(head)$
```

[The conditions constrain the signalling such that superflous wakeups will take place only in the very special situation that a writer is waiting in *head* and the number of active readers has exactly reached the current limit.]

Question 3.3

(a) In basic read/write locking, *lock* calls are accepted whenever the conditions are legal:

```
module RWLock
  op lock(write : boolean);
  op unlock();
body

  process BasicControl;
    var r, w : integer := 0;
    repeat
      in lock(write) and $w = 0 \wedge (write \Rightarrow r = 0) \rightarrow$
                       if write then $w := w + 1$ else $r := r + 1$
      [] unlock() $\rightarrow$ if $w > 0$ then $w := w - 1$ else $r := r - 1$
      ni
    forever;

end RWLock;
```

(b) To handle *lock* calls in FCFS order these must be accepted without filtering on the *write* parameter. If actually a write, all reader activity must cease before ending the rendezvous.

```
module RWLock
  op lock(write : boolean);
  op unlock();
body

  process FCFS_Control;
    var r, w : integer := 0;
    repeat
      in lock(write) and $w = 0 \rightarrow$ while $write \wedge r > 0$ do
                                  in unlock() $\rightarrow r := r - 1$ ni;
                              if write then $w := w + 1$
                                       else $r := r + 1$
      [] unlock() $\rightarrow$ if $w > 0$ then $w := w - 1$ else $r := r - 1$
    ni
  forever;

end RWLock;
```