## Reading Material

At the lecture we will talk more about network flow. We will consider two applications: bipartite matching and finding disjoint paths. We will also discuss how to find good augmenting paths. For reading material see the webpage.

Exercises

1   The Edmonds-Karp algorithm and the scaling algorithm   Use both the Edmonds-Karp's algorithm and the scaling algorithm to compute a maximum flow and minimum cut on the two graphs below. For each augmenting path write the nodes on the path and the value you augment the path with.
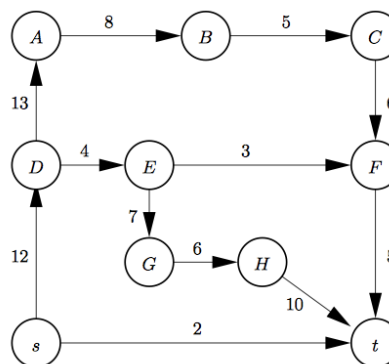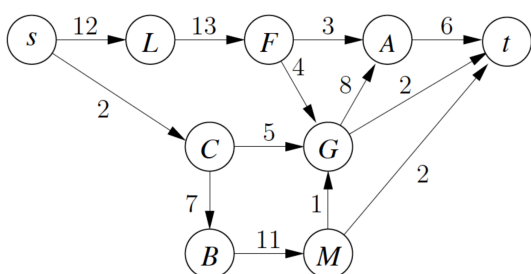
2   Blood Donations   Solve exercise KT 7.8.

3   Christmas Trees (from the Exam E15)   The Dean has asked you to arrange the annual Christmas party for the students at DTU. You have to make a plan for how to place the tables in the hall. The local fire department has divided the hall up into an $n \times m$ grid of subsquares and declared that you can place at most two tables in each row and at most one in each column. Unfortunately, the Dean who loves Christmas has put up Christmas trees in many of the subsquares. You cannot place a table in a subsquare with a Christmas tree.

Example   Here $n = 4$ and $m = 8$. The $*$ are Christmas trees and T are tables. In the example the maximum number of tables that can be placed is 7.

$*$	T						T
T	$*$	$*$	T	$*$	$*$		$*$
	$*$	$*$		$*$	$*$	T	$*$
	$*$	T	$*$		T	$*$	

3.1 Model the problem as a graph problem. Explain how you model the problem as a graph problem and draw the graph corresponding to the example above.

3.2 Describe an algorithm that given n, m, and the placement of the Christmas trees computes the maximum number of tables you can place in the hall. Analyze the asymptotic running time of your algorithm. Remember to argue that your algorithm is correct.

3.3 [†] Implement your algorithm on CodeJudge.

4   Escape   Solve exercise KT 7.14.

5 [∗] **Euler tours in mixed graphs** It is well-known that a strongly connected graph has an Euler tour (a cycle that contains each edge exactly once) if and only if any vertex have the same number of ingoing and outgoing edges.

In the following we consider mixed graphs, which are graphs where some edges are directed and some are not. We assume that the graphs are connected, i.e., if we ignore the orientation on all directed edges, then the graph is connected.

We want an algorithm to decide wether it is possible to assign directions to all undirected edges, such that the graph has an Euler tour. In the example below the mixed graph on the left can be assigned directions (graph on right) such that there is an Euler tour (the numbers on the edges denotes the order the edges can be visited in).
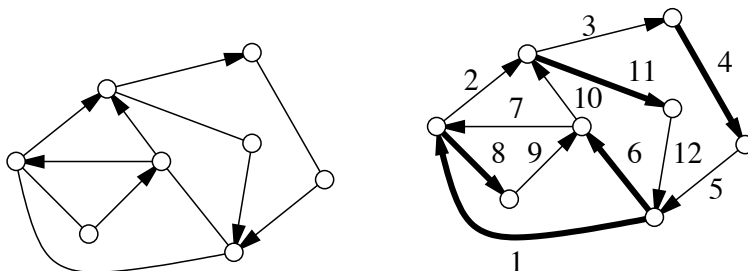
Give an algorithm to decide wether it is possible to assign directions to all undirected edges, such that a graph G has an Euler tour.

Hint: Consider a bipartite graph, where the nodes on one side are edges in G and the nodes on the other side are the vertices in G. Use your algorithm from the previous exercise.

Puzzle of the week: The twelve-coin problem There are twelve coins, eleven of which are identical and one of which is different, but it is not known whether it is heavier or lighter than the others. You have a traditional balance scale with two pans. To use such a scale, you place a coin into each pan and the scale will determine which coin is heavier. The balance or scale may be used three times to isolate the unique coin and determine wether it is heavier or lighter than the others.