

Weekplan: Union Find

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Reading

Introduction to Algorithms, 4th edition, Cormen, Rivest, Leisersons, and Stein (CLRS): Chapter 19.1-19.4 + *Algorithms*, 4th edition, Sedgewick and Wayne: Chapter 3.3.

Exercises

1 Run Union Find by Hand Look at the following sequence of operations: INIT(7), UNION(3, 4), UNION(5, 0), UNION(4, 5), UNION(4, 3), UNION(0, 1), UNION(2, 6), UNION(0, 4) and UNION(6, 0).

- 1.1 [w] Run the sequence of operations using quick find by hand. Show the contents of the id array after every step. Assume the UNION(i, j) operation always updates id for the set given by i .
- 1.2 [w] Run the sequence using quick union by hand. Show the trees after every step. Assume UNION(i, j) always sets the root of the tree given by i to be a child of the root of the tree given by j .
- 1.3 Run the sequence using weighted quick union by hand. Show the trees after every step. Assume UNION(i, j) sets the root of the tree given by i to be a child of the root of the tree given by j when the sizes of two trees are equal.
- 1.4 Show the result of path compression after a FIND(x) operation, where x is respectively a leaf, an internal node of depth 1, and an internal node of height 1, in one of the trees from the above exercises.
- 1.5 Give a sequence of operations that results in a tree of maximal depth using quick union.
- 1.6 Give a sequence of operations that results in a tree of maximal depth using weighted quick union.
- 1.7 Write pseudo code for a algorithm to do path compression. *Hint*: traverse the path twice.

2 Alternative to the Quick Find Algorithm One of your fellow students suggests the following intuitive variant of quick find UNION. Does it work?

```
UNION( $i, j$ )
  if FIND( $i$ )  $\neq$  FIND( $j$ ) then
    for  $k = 0$  to  $n - 1$  do
      if  $id[k] == id[i]$  then
         $id[k] = id[j]$ 
      end if
    end for
  end if
```

3 Dynamic Connected Components and Graph Search Using graph search (DFS or BFS) we can find the connected components of a graph. Give a simple solution for dynamic connected component using graph search and compare the complexity with the solutions based on union find.

4 [†] Computer Network Josefine is setting up a new computer network at The University of Algorithms. The network consists of N computers numbered from 0 to $N - 1$ that are initially not connected at all. She builds the network by adding network cables between pairs of computers one at a time. Two computers, A and B , are connected if there is at least one series of cables that leads from computer A to computer B . As other students need to use connected computers while setting up the network, she needs to be able to answer if two specific computers are connected at a specific time. More precisely, we want to support the following operations:

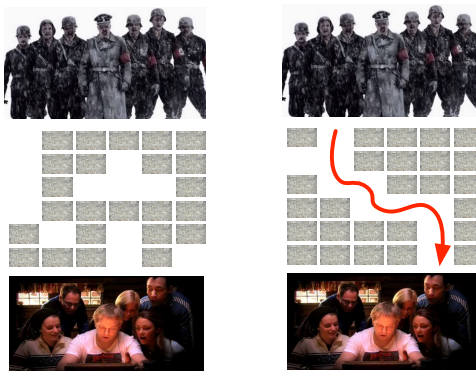
- `ADDCABLE(A, B)`: Add a cable between computers A and B .
- `CONNECTED(A, B)`: Return "Yes" if there is a series of cables that leads from computer A to computer B and "No" otherwise.

Give a data structure for this problem and implement it.

Input Line 1 contains integers N and M , where N is the number of computers and M is the total number of `ADDCABLE/CONNECTED` operations. Line 2... $M + 1$ can be either an `ADDCABLE` operation indicated by the letter A followed by two integers specifying the two computers or a `CONNECTED` operation indicated by the letter C also followed by two integers specifying two computers.

Output Line i is result of the i th `CONNECTED` operation.

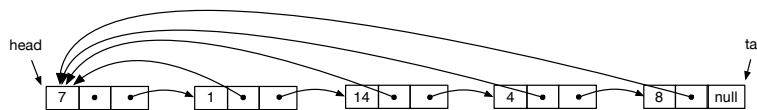
5 [*] Zombie Invasion In the post apocalyptic zombie world you and a small group of survivors have barricaded yourself in a small building. The only thing keeping the brutal zombies from eating you is a strong fortification. The fortification consists of a $k \times k$ grid of walls. Here illustrated by a 6×6 grid of walls.¹



In the top of the grid the zombies are waiting to come in, and you and your group is located in the bottom. Unfortunately, the walls are weak and collapse regularly. If a path of walls between the top and the bottom of the grid is collapsed the zombies can get in and eat you. In order to start evacuation you want to monitor if there currently is a path through the fortification (from top to bottom). Give a data structure that can efficiently keep track of this while the walls are collapsing one by one.

6 [*] Recursive Path Compression Write pseudo code for a *recursive* algorithm for path compression. *Hint*: it can be done with only few lines of code.

7 Union Find using Linked Lists and Weights We want to implement a variant of quick find using linked lists in the following way. Each set is represented by a singly linked list. The representative for a set is the first element in the list and each element in the list has a pointer to the representative. Furthermore we maintain a pointer to the tail of the list. For instance, the data structure for the set $\{1, 4, 7, 8, 14\}$ with representative 7 could look like this:



7.1 Using the representation, show how to implement `INIT(n)` in $O(n)$ time, `FIND(i)` in $O(1)$ time and `UNION(i, j)` in $O(|S(i)|)$ time, where $S(i)$ is the set containing i .

7.2 Show how to extend the solution such that `INIT` and `FIND` runs in the same time as the previous exercise, but the time for `UNION(i, j)` is $O(\min(|S(i)|, |S(j)|))$. *Hint*: maintain a little extra information.

7.3 [*] Show that for p `FIND` and m `UNION` operations on n elements the above solution gives the running time $O(p + m \log n)$.

¹Pictures from "Død snø", 2009.