02105 - Algorithms and Data Structures May 2022 (MC part)

Der anvendes en scoringsalgoritme, som er baseret på "One best answer"

Dette betyder følgende:
Der er altid netop ét svar som er mere rigtigt end de andre
Studerende kan kun vælge ét svar per spørgsmål

The following approach to scoring responses is implemented and is based on "One best answer"
There is always only one correct answer – a response that is more correct than the rest
Students are only able to select one answer per question

For each question below, mark whether or not the statement is correct.

Select the correct answers

	Yes	No
$42n + \frac{42n}{3} + \frac{50n}{7} = \Theta(n)$	✗	○
$n^2(n^2 + 8n^3) = O(n^5 \log n)$	✗	○
$5\sqrt{n} + \log^2 n = \Omega(\log^3 n)$	✗	○
$\log n + \log^2(\sqrt{n}) = O(\log n)$	○	✗
$2^n \cdot 2^n = O(2^{n+7})$	○	✗

Mark the running time in O-notation of each of the following algorithms as a function of n. Mark the best bound.

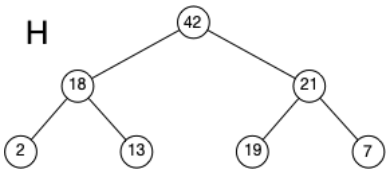
```
ALG1(n)
s = 1
for i = 1 to 2⌈√n⌉ + 42⌈log n⌉ do
    for j = i to ⌈√n⌉ do
        s = i + j
    end for
end for
```

```
ALG2(n)
c = 0
j = n
while j ≥ ⌈log n⌉ do
    j = j − 1
end while
```

```
ALG3(n)
if n ≤ 1 then
    return 1
else
    return ALG3(⌈n/2⌉) + ALG3(⌈n/2⌉)
end if
```

Select the correct answers	$O(1)$	$O(\log n)$	$O(\sqrt{n})$	$O(n)$	$O(n \log n)$	$O(n^2)$	$O(n^2 \log n)$	$O(n^3)$	$O(2^n)$
Alg1	○	○	○	○	○	○	○	○	○
Alg2	○	○	○	○	○	○	○	○	○
Alg3	○	○	○	○	○	○	○	○	○

Consider the following binary heap H.



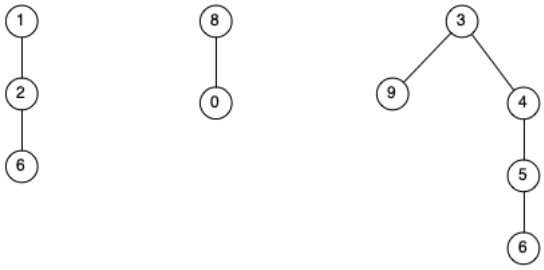
Let H_1 be the result of applying an Extract-Max operation on H and let H_2 be the result of applying an Insert operation with key 20 on H. Construct the arrays representations of H, H_1, and H_2 and state the content of each of the following entries (recall that a heap with n items in the array representation consists of an array of length n+1 where index 0 is not used).

Select the correct answers

	2	7	13	18	19	20	21	42
H[3]	○	○	○	○	○	○	✗	○
H[4]	✗	○	○	○	○	○	○	○
H[5]	○	○	✗	○	○	○	○	○
H[6]	○	○	○	○	✗	○	○	○
H_1[3]	○	○	○	○	✗	○	○	○
H_1[4]	✗	○	○	○	○	○	○	○
H_1[5]	○	○	✗	○	○	○	○	○
H_1[6]	○	✗	○	○	○	○	○	○
H_2[3]	○	○	○	○	○	○	✗	○
H_2[4]	○	○	○	✗	○	○	○	○
H_2[5]	○	○	✗	○	○	○	○	○
H_2[6]	○	○	○	○	✗	○	○	○

Union Find

Consider the following forest of trees representing a family of sets in a union find data structure constructed using the quick union algorithm.



State the result of the following Find(.) operations.

Select the correct answers	0	1	2	3	4	5	6	7	8	9
Find(0)	○	○	○	○	○	○	○	○	✗	○
Find(1)	○	✗	○	○	○	○	○	○	○	○
Find(2)	○	✗	○	○	○	○	○	○	○	○
Find(3)	○	○	○	✗	○	○	○	○	○	○
Find(4)	○	○	○	✗	○	○	○	○	○	○
Find(5)	○	○	○	✗	○	○	○	○	○	○

Suppose we now use path compression on the forest above. Construct the forest of trees F after a Find(5) operation and answer the following question for F.

Select the correct answers

	1	2	3	4	5	6	7	8
The total number of leaves in all trees in F is	○	○	○	○	☒	○	○	○
The maximum depth of a tree in F is	○	☒	○	○	○	○	○	○
The minimum depth of a tree in F is	☒	○	○	○	○	○	○	○
The maximum degree of a root of a tree in F is	○	○	☒	○	○	○	○	○
The minimum degree of a root of a tree in F is	☒	○	○	○	○	○	○	○

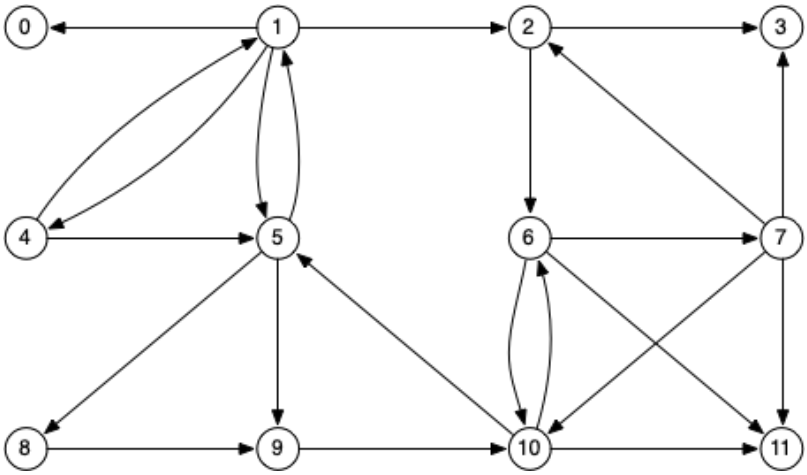
Consider the following data structures for representing a set of integers. We are interested in supporting the operation Max-Min-Sum(), that returns the sum of the smallest and largest integer in the set. For instance, if a data structure stores the set {32, 6, 18, 7, 2} then Max-Min-Sum() should return 34. For each data structure below mark the runtime of the Max-Min-Sum() operation in O-notation as a function of n, where n is the number of elements in the data structure. Mark the best bound

Select the correct answers

	$O(1)$	$O(\log n)$	$O(\sqrt{n})$	$O(n)$	$O(n \log n)$	$O(n^2)$	$O(2^n)$
Sorted array	☒	○	○	○	○	○	○
Max-heap	○	○	○	☒	○	○	○
Min-heap	○	○	○	☒	○	○	○
Array (not sorted)	○	○	○	☒	○	○	○
Binary search tree	○	○	○	☒	○	○	○

Graph Search

Consider the following graph G.



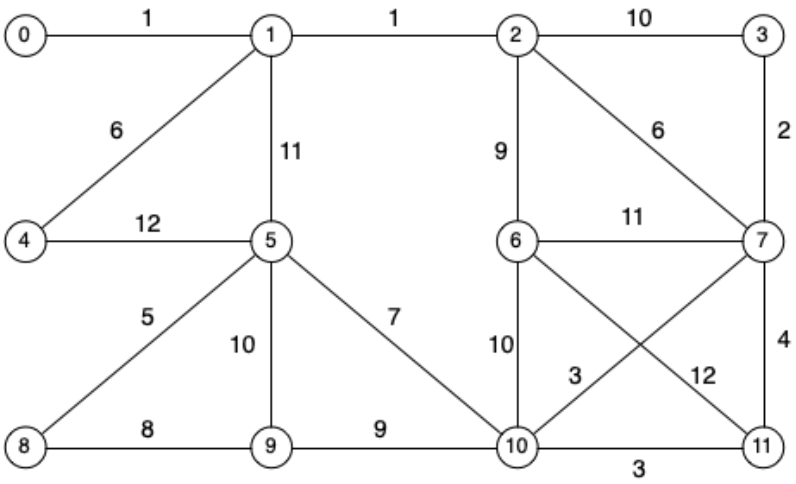
Construct the DFS tree T_1 for G when starting in vertex 4. Assume that each adjacency list is sorted in increasing order. Answer the following questions.

Select the correct answers	1	2	3	4	5	6	7	8	9
The depth of T_1 is	○	○	○	○	○	○	○	✗	○
The number of leaves of T_1 is	○	○	○	✗	○	○	○	○	○
The maximum number of children of a node in T_1 is	○	✗	○	○	○	○	○	○	○

Construct the BFS tree T_2 for G when starting in vertex 7. Assume that each adjacency list is sorted in increasing order. Answer the following questions.

Select the correct answers	1	2	3	4	5	6	7	8	9
The depth of T_2 is	○	○	○	⊗	○	○	○	○	○
The number of leaves of T_2 is	○	○	○	○	○	○	⊗	○	○
The maximum number of children of a node in T_2 is	○	○	○	⊗	○	○	○	○	○

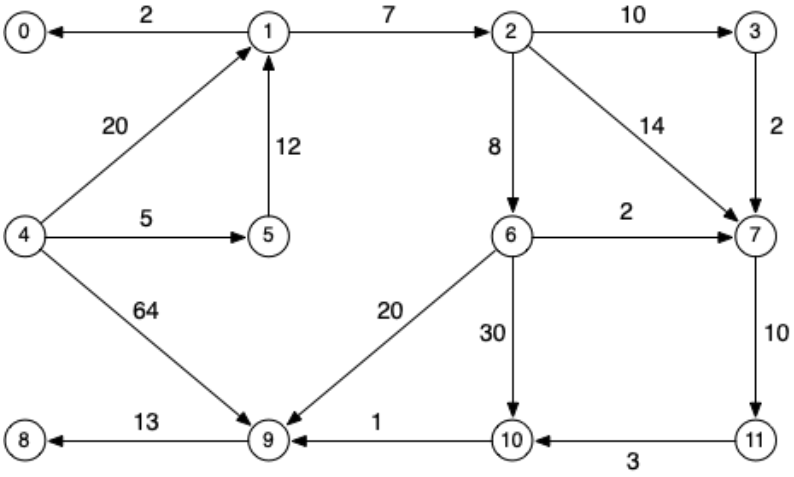
Consider the following graph.



Construct a minimum spanning tree T for the graph and answer the following questions.

Select the correct answers	1	2	3	4	5	6	7	8	9
The total number of edges on a longest path of T is	○	○	○	○	○	○	✗	○	○
The number of leaves of T is	○	○	○	○	○	✗	○	○	○
The maximum degree of a node in T is	○	○	✗	○	○	○	○	○	○

Consider the following graph



Construct a shortest path tree T for the graph when starting at vertex 4. Answer the following questions for T

Select the correct answers	1	2	3	4	5	6	7	8	9
The total number of edges on a longest path in T is	○	○	○	○	○	○	○	○	⊘
The number of leaves in T is	○	○	⊘	○	○	○	○	○	○
The maximum number of children of a node in T is	○	⊘	○	○	○	○	○	○	○