Weekplan: Union Find

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Reading

Introduction to Algorithms, 4th edition, Cormen, Rivest, Leisersons, and Stein (CLRS): Chapter 19.1-19.4 + *Algorithms*, 4th edition, Sedgewick and Wayne: Chapter 3.3.

Exercises

1 Run Union Find by Hand Look at the following sequence of operations: INIT(7), UNION(3, 4), UNION(5, 0), UNION(4, 5), UNION(4, 3), UNION(0, 1), UNION(2, 6), UNION(0, 4) and UNION(6, 0).

1.1 [w] Run the sequence of operations using quick find by hand. Show the contents of the *id* array after every step. Assume the UNION(i, j) operation always updates *id* for the set given by i.

1.2 [w] Run the sequence using quick union by hand. Show the trees after every step. Assume UNION(i, j) always sets the root of the tree given by i to be a child of the root of the tree given by j.

1.3 Run the sequence using weighted quick union by hand. Show the trees after very step. Assume UNION(i, j) sets the root of the tree given by i to be a child of the root of the tree given by j when the sizes of two trees are equal.

1.4 Show the result of path compression after a FIND(x) operation, where x is respectively a leaf, an internal node of depth 1, and an internal node of height 1, in one of the trees from the above exercises.

1.5 Give a sequence of operations that results in a tree of maximal depth using quick union.

1.6 Give a sequence of operations that results in a tree of maximal depth using weighted quick union.

1.7 Write pseudo code for a algorithm to do path compression. *Hint:* traverse the path twice.

2 Alternative to the Quick Find Algorithm One of your fellow students suggests the following intuitive variant of quick find UNION. Does it work?

```
UNION($i$, $j$)
if FIND($i$) $\neq$ FIND($j$) then
  for $k = 0$ to $n - 1$ do
    if $id[k] == id[i]$ then
      $id[k] = id[j]$
    end if
  end for
end if
```

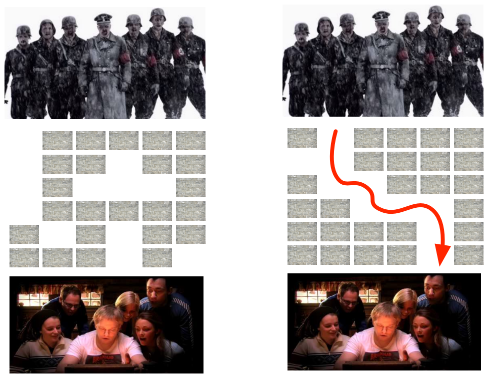
3 Dynamic Connected Components and Graph Search Using graph search (DFS or BFS) we can find the connected components of a graph. Give a simple solution for dynamic connected component using graph search and compare the complexity with the solutions based on union find.

4 Computer Network Josefine is currently setting up a new computer network at The University of Algorithms. The network consists of N computers numbered from 0 to $N - 1$ that are initially not connected at all. She builds the network by adding network cables between pairs of computers one at a time. Two computers A and B are connected if there exists at least one series of cables that leads from computer A to computer B. As other students need to use connected computers while she is setting up the network, she needs to be able to answer if two specific computers are connected at a specific time.

4.1 Give an algorithm that given a sequence of ADDCABLE(A, B) and CONNECTED(A, B) operations, answer all the CONNECTED queries with a "Yes" or "No".

4.2 [†] Implement your algorithm.

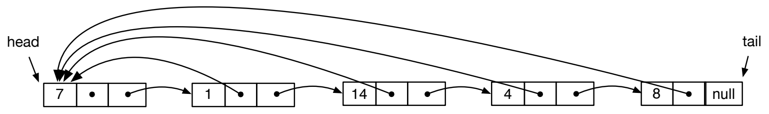
5 [$*$] **Zombie Invasion** In the post apocalyptic zombie world you and a small group of survivors have barricaded yourself in a small building. The only thing keeping the brutal zombies from eating you is a strong fortification. The fortification consists of a $k \times k$ grid of walls. Here illustrated by a 6×6 grid of walls.[1]



In the top of the grid the zombies are waiting to come in, and you and your group is located in the bottom. Unfortunately, the walls are weak and collapse regularly. If a path of walls between the top and the bottom of the grid is collapsed the zombies can get in and eat you. In order to start evacuation you want to monitor if there currently is a path through the fortification (from top to bottom). Give a data structure that can efficiently keep track of this while the walls are collapsing one by one.

6 [$*$] **Recursive Path Compression** Write pseudo code for a *recursive* algorithm for path compression. *Hint:* it can be done with only few lines of code.

7 **Union Find using Linked Lists and Weights** We want to implement a variant of quick find using linked lists in the following way. Each set is represented by a singly linked list. The representative for a set is the first element in the list and each element in the list has a pointer to the representative. Furthermore we maintain a pointer to the tail of the list. For instance, the data structure for the set $\{1, 4, 7, 8, 14\}$ with representative 7 could look like this:



7.1 Using the representation, show how to implement INIT(n) in $O(n)$ time, FIND(i) in $O(1)$ time and UNION(i, j) in $O(|S(i)|)$ time, where $S(i)$ is the set containing i.

7.2 Show how to extend the solution such that INIT and FIND runs in the same time as the previous exercise, but the time for UNION(i, j) is $O(\min(|S(i)|, |S(j)|))$. *Hint:* maintain a little extra information.

7.3 [$*$] Show that for p FIND and m UNION operations on n elements the above solution gives the running time $O(p + m \log n)$.

[1]Pictures from "Død snø", 2009.