Logic and Learning in Serious Games

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Recent projects on games and learning

Games for learning logical reasoning

Collective learning in games through social networks

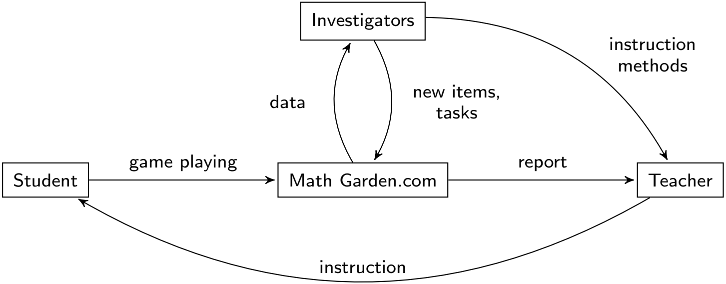
Outline

Games for learning logical reasoning

Collective learning in games through social networks

Math Garden (Rekentuin.nl or MathsGarden.com)

- adaptive training environment with educational games
- abstract thinking development
- daily practice as input for monitoring



Math Garden (Rekentuin.nl or MathsGarden.com)

- In 4 yrs the number of schools has grown from 8 to 1000.
- Over 80,000 active users, in 4 yrs over 250 billion answers (1mln/day).

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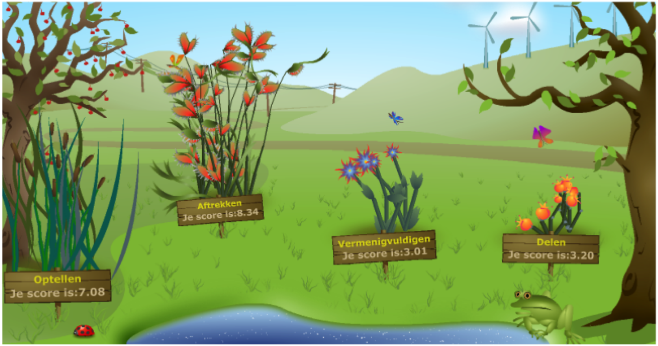

- arithmetic games
- complex reasoning games
- different abilities different flowerbeds
- unplayed wither

Practical Advantages

Mix of practice and measurement allows:

- automatic correction of tests;
- pupils can practice at their level.

Difficulty Levels

- students play game-items suited for their level (75% correctly)
- the tasks' difficulty and the students' level are continuously estimated
- via the Elo rating system (used for ranking chess players, Elo 1978)
- i.e.: students are ranked by playing, and items are rated by getting played
- ratings depend on accuracy and speed of item solving (Klinkenberg 2011)

By-products:

1) rating of all items (item difficulty parameters)
2) rating of children (reflecting the reasoning ability)

The Goal

introduce a dedicated **logical reasoning training** in primary schools

understand the empirically established item **difficulty parameters**

by means of a **logical analysis** of the items

computational paradigms for cognition

Mastermind Game

- Meirowitz 1970, but similar to the earlier Bulls and Cows
- an **inductive inquiry** game, trials of experimentation and evaluation

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Previous Mathematical Analysis

- question of the underlying logical reasoning and its difficulty
- mathematical results only on existence of efficient strategies (Knuth 1976, Irving 1978, Koyama 1993, Kooi 2005)

Static Mastermind is a version of the Mastermind game

- the goal is to find out the minimum number of guesses
- the code-breaker can make all at once
- at the beginning of the game
- without waiting for the individual feedbacks
- and upon receiving them all at once
- completely determine the code in the next guess

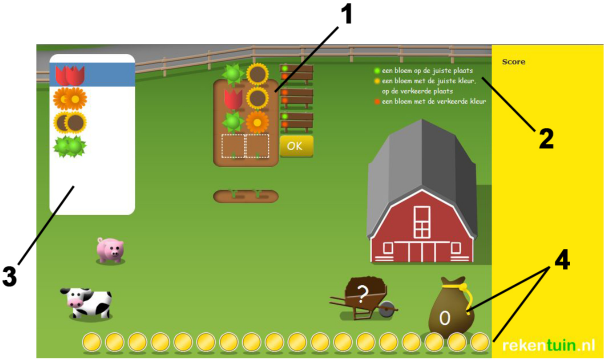
Complexity of Static Mastermind (Stuckman 2006)

Definition (Mastermind Satisfiability Decision Problem)

Input	A set of guesses G and their corresponding scores.
Question	Is there at least one valid solution?

Theorem
Mastermind Satisfiability Decision Problem is NP-complete wrt ℓ (positions).

Deductive Mastermind: Flowercode in Math Garden



1) decoding board
2) short feedback instruction
3) domain of flowers to choose from
4) timer in the form of disappearing coins

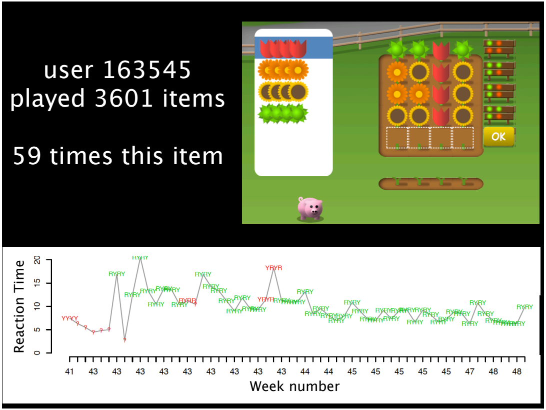
Some Facts about Flowercode

- running since November 2010
- 321 game-items, 1-5 flowers, 2-5 colors
- by December 2012, **4,895,648** items had been played
- by **37,339** primary school students (grades 1-6, age: 6-12 years)
- in over **700** locations (schools and family homes)

We can access:

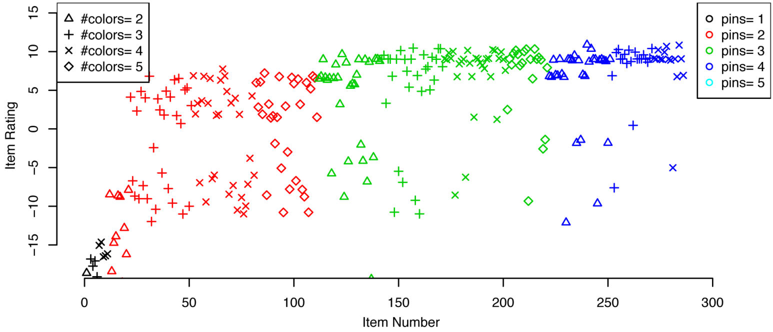
- the individual progress of individual players on a single game
- the most frequent mistakes with respect to a game-item
- the relative difficulty of game-items
- correlations with different, mathematical games
- etc.

Some Facts about Flowercode

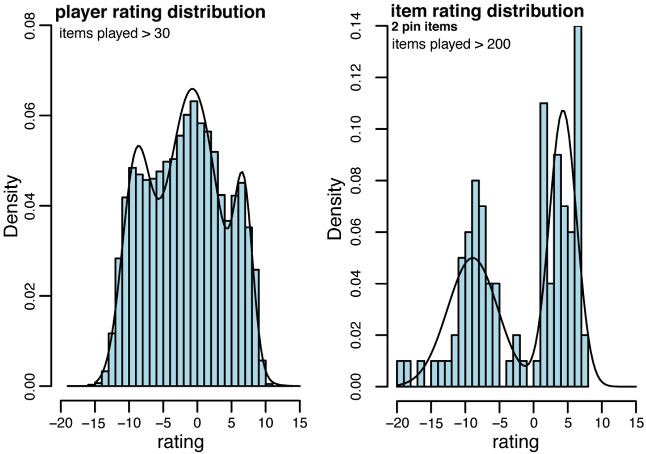


The Necessity of Prior Difficulty Assessment

initial difficulty estimation in terms of non-logical aspects
(number of flowers, colors, lines, the rate of the hypotheses elimination, etc.)



The Necessity of Prior Difficulty Assessment



how to fix this to facilitate the training effect?

Logical Analysis

- each (premiss,feedback) standardly gives rise to a boolean formula
- each task is then a set of boolean formulas
- the set is satisfiable because each game has (exactly one) solution
- proof-theory for propositional logic can be used to find this solution
- difficulty of a task is reflected in the structure and length of the proof

An Example of the Boolean Translation

Let us take the following premiss with feedback: 

Call the sunflower s, and the tulip t. The corresponding formula is:

$$(goal(1){\neq}s \wedge goal(2){\neq}t) \wedge ((goal(1){=}t \wedge goal(2){\neq}s) \vee (goal(2){=}s \wedge goal(1){\neq}t))$$

- analytic tableau is a decision procedure for propositional logic
- it solves satisfiability of finite sets of formulas of propositional logic
- by giving an adequate valuation

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- unfolding breaks them into smaller formulae
- until contradiction is found or no further reduction is possible

Analytic Tableaux for Deductive Mastermind

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- it solves satisfiability of finite sets of formulas of propositional logic
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$$\begin{tikzcd}[arrows=-] \varphi \wedge \psi \arrow[d, "\wedge"] \\ \varphi,\ \psi \end{tikzcd} \qquad \begin{tikzcd}[arrows=-] & \varphi \vee \psi \arrow[dl] \arrow[dr] \arrow[d, phantom, "\vee"] & \\ \varphi & & \psi \end{tikzcd}$$

Analytic Tableau and DM: An Observation

By construction of DM

Applying the analytic tableaux method to the Boolean translation of a Deductive Mastermind game-item will give the unique missing assignment *goal*.

2-placed Deductive Mastermind Game-items

$$gg, go, oo, rr, gr, or$$

c_i, c_j

oo

$goal(1) \neq c_i$
$goal(2) \neq c_j$
$goal(1) = c_j$
$goal(2) = c_i$

c_i, c_j

rr

$goal(1) \neq c_i$ $goal(2) \neq c_j$
$goal(1) \neq c_j$ $goal(2) \neq c_i$

c_i, c_j

gr

$goal(1) = c_i$
$goal(2) \neq c_j$

$goal(2) = c_j$
$goal(1) \neq c_i$

c_i, c_j

or

$goal(1) \neq c_1$
$goal(2) = c_1$
$goal(1) \neq c_2$
$goal(2) \neq c_2$

$goal(2) \neq c_2$
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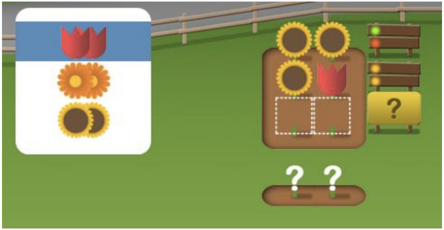
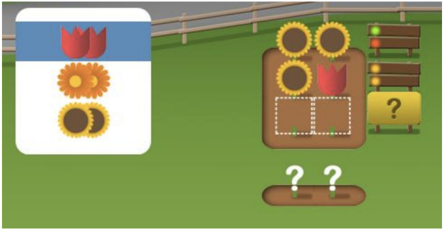
$goal(2) \neq c_1$
$goal(2) \neq c_1$

$$oo < rr < gr < or$$

An Example



An Example

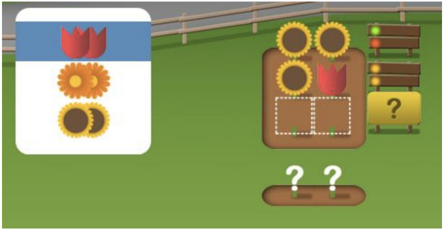


$$Bt(h_1, f_1)$$
$$Bt(h_2, f_2)$$

gr

$goal(1) = c_1$	$goal(2) = c_1$
$goal(2) \neq c_1$	$goal(1) \neq c_1$
$Bt(h_2, f_2)$	$Bt(h_2, f_2)$
oo	oo
$goal(1) = c_2$	$goal(1) = c_2$
$goal(2) = c_1$	$goal(2) = c_1$

An Example



$$Bt(h_1, f_1) \quad Bt(h_2, f_2)$$

gr

$$goal(1)=c_1 \quad goal(2)\neq c_1 \quad Bt(h_2, f_2)$$

oo

$$goal(1)=c_2 \quad goal(2)=c_1$$

$$goal(2)=c_1 \quad goal(1)\neq c_1 \quad Bt(h_2, f_2)$$

oo

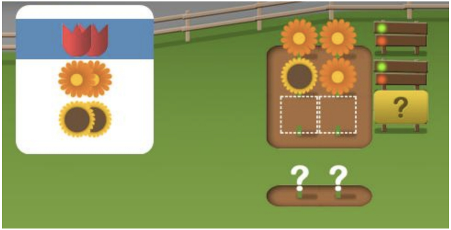
$$goal(1)=c_2 \quad goal(2)=c_1$$

$$Bt(h_2, f_2) \quad Bt(h_1, f_1)$$

oo

$$goal(1)=c_2 \quad goal(2)=c_1 \quad Bt(h_1, f_1)$$

Another Example



$Bt(h_1, f_1)$
$Bt(h_2, f_2)$

gr

$goal(1){=}c_1$
$goal(2){\neq}c_1$
$Bt(h_2, f_2)$

$goal(2){=}c_1$
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Hypotheses and Preliminary Results

- tableau give ‘ideal’ reasoning scheme
- abstract complexity measure (tree size)
- shape and size of the tree depends on what goes first (minimal size)
- reasoning optimization

items’ initial difficulty corresponds to the size of top-botom trees

items’ logical difficulty corresponds to the size of the minimal trees

the reasoning is optimized according to feedback complexity

Results

all factors but one (*gr*) were significant in predicting item difficulties

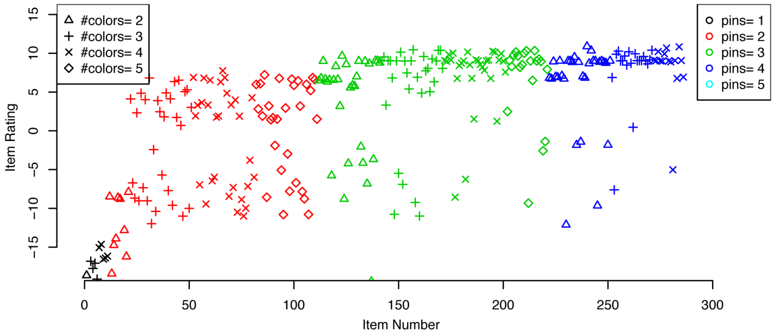
two difficulty clusters

1) **easy**

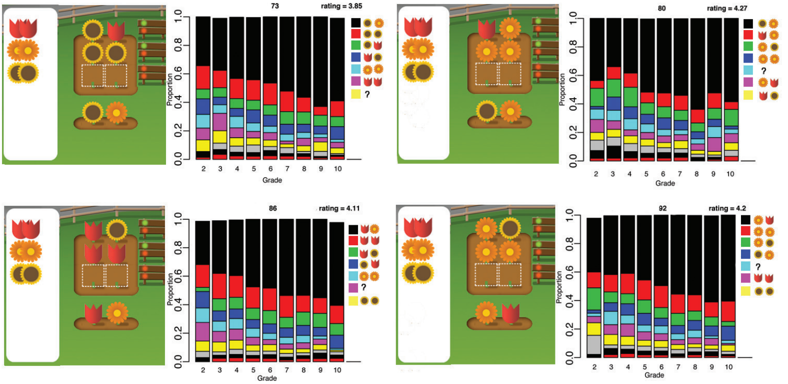
   no *or* feedback and no *gr* feedback

   no *or* feedback, at least one *gr* feedback, and all colors are included

2) **difficult**: otherwise

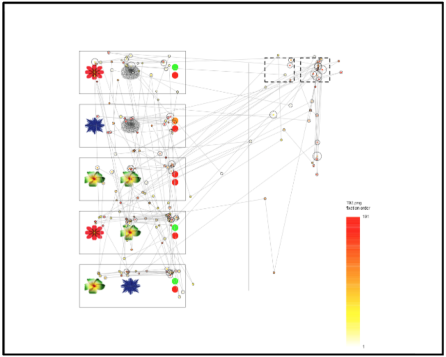


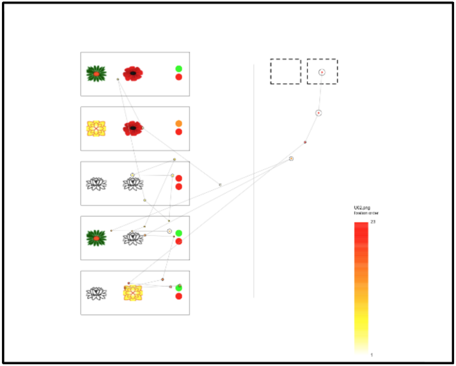
Error Analysis



Learning and Explanation

Pattern Recognition and Eye-tracking

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Conclusions Part 1

- computer adaptive practice online
- using proof-theory to analyze the cognitive difficulty of logical reasoning
- good prediction of item-difficulties (75% of variance explained)

Nina Gierasimczuk, Han van der Maas, and Maartje Raijmakers, An analytic tableaux model for Deductive Mastermind empirically tested with a massively used online learning system, Journal of Logic, Language and Information 2013.

Outline

Games for learning logical reasoning

Collective learning in games through social networks

Serious Games: Examples of Teams Learning in Games

Military Training: US Army

Practical Motivation: Serious Games and Social Networks

Merging techniques of serious games and social networks.

Constructing an **active** (games) and **social** (networks) learning environments.

Formal analysis can reveal hidden dependencies and inform design.

Decisions one needs to make

1. **Game Structure**:

   players, actions, individual payoffs.

2. **Learning Goal**:

   what the players should eventually learn.

3. **Social Network Learning**:

   agents update their private beliefs through network communication.

4. **Gameplay**:

   agents determine what strategy to play and play the game.

5. **Game Learning**:

   agents use rewards to reinforce played strategies.

The Game-Network Learning Model: Main Idea

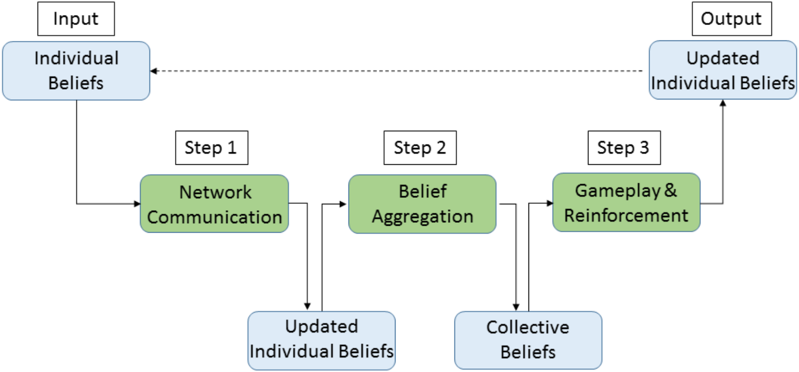
Cooperative games: grand coalition, group-rational players

Collective learning: toward social optimum, beliefs about joint strategies

Two belief updates:

- After network communication (feedback from neighbors)
- After gameplay (feedback from payoffs)

The Game-Network Learning Model: Main Idea



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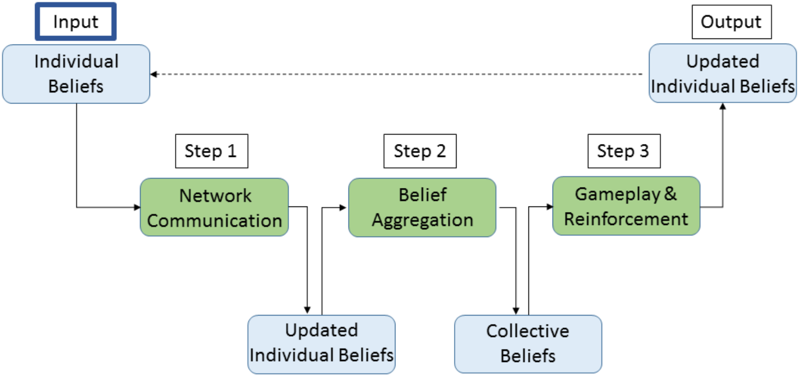

Input: Individual Beliefs

Stochastic belief matrix:

$$B = \begin{pmatrix} b_{11} & \dots & b_{1k} \\ \vdots & & \vdots \\ b_{n1} & \dots & b_{nk} \end{pmatrix}$$

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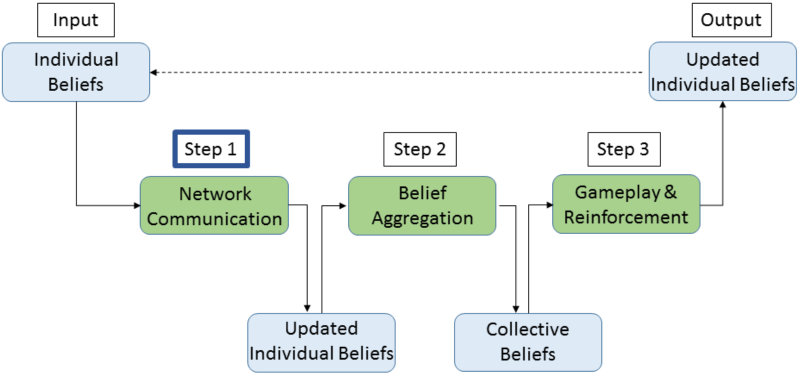
For example:

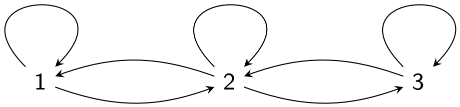
$$B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Rows are agents, columns are joint strategies.
The entries are strengths of belief in a strategy being the optimal one.


Input
Individual Beliefs
Output
Updated Individual Beliefs
Step 1
Network Communication
Step 2
Belief Aggregation
Step 3
Gameplay & Reinforcement
Updated Individual Beliefs
Collective Beliefs

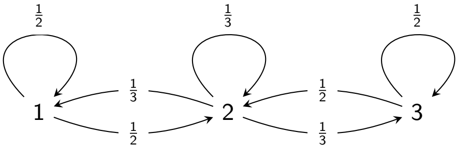
Step 1: Network Communication

Social network:



Step 1: Network Communication

Weights of trust in network communication:



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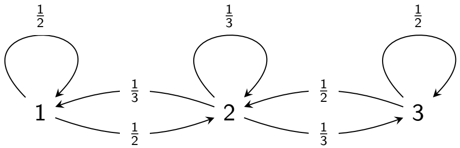
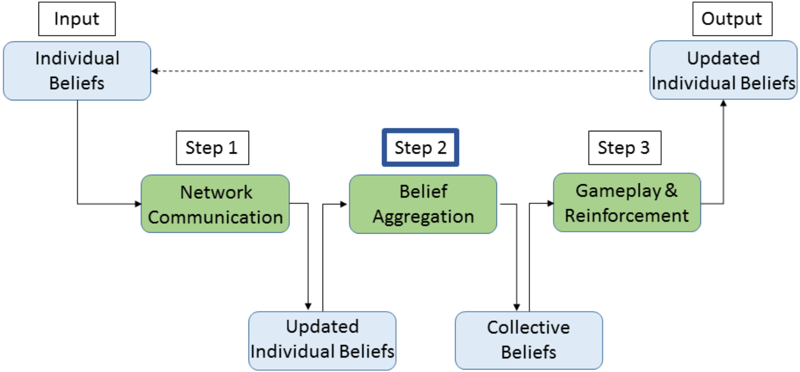


Belief update in communication:
Player's perspective: **weighted average** of beliefs of neighbors.

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \rightsquigarrow \quad \begin{pmatrix} 1/2 & 1/2 & 0 \\ 1/3 & 1/3 & 1/3 \\ 0 & 1/2 & 1/2 \end{pmatrix}$$

The Game-Network Learning Model



Step 2: Belief Aggregation

- Purpose: deciding collectively which joint strategy to play
- Method: probabilistic social choice function (PSCF)

$$B = \begin{pmatrix} b_{11} & \dots & b_{1k} \\ \vdots & & \vdots \\ b_{n1} & \dots & b_{nk} \end{pmatrix} \quad \mapsto \quad \vec{b} = \begin{pmatrix} b_1 & \dots & b_k \end{pmatrix}$$

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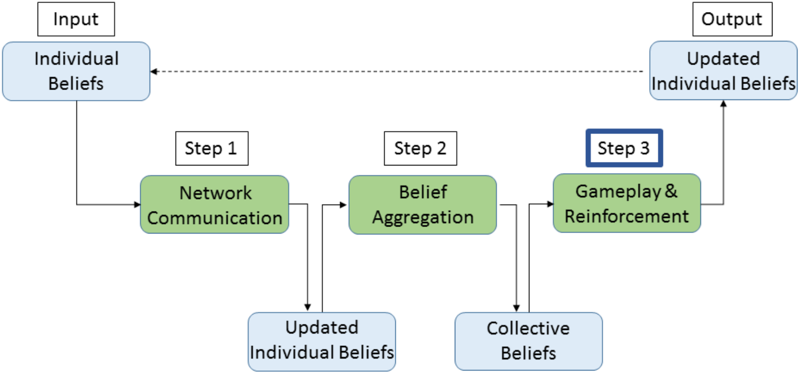
- input: belief matrix; output: societal probability distribution.

$$\begin{pmatrix} 1/2 & 1/2 & 0 \\ 1/3 & 1/3 & 1/3 \\ 0 & 1/2 & 1/2 \end{pmatrix} \quad \mapsto \quad \begin{pmatrix} 5/18 & 8/18 & 5/18 \end{pmatrix}$$

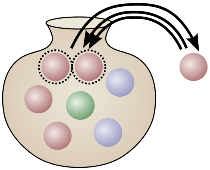
The Game-Network Learning Model



Step 3: Gameplay and Reinforcement

Reinforcement learning:

- Stochastic games
- Law of Effect
- Learn towards high reward

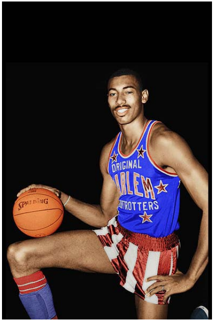


Collective reinforcement learning:

- Reinforce *joint* strategies
- With *social welfare* fraction

Motivation: Examples of Teams in Games

Team Sports: NBA League



In NBA stats earn you money.
Why would you fire the best players?
Wilt Chamberlain vs. Bill Russell

‘The secret of basketball is that it’s not about basketball.’
‘[Lakers and Celtics] teams were loaded with talented players, yes, but that’s not the only reason they won. They won because they liked each other, knew their roles, ignored statistics, and valued winning over everything else. They won because their best players sacrificed to make everyone else happy. They won as long as everyone remained on the same page.’

Bill Simons, *The Book of Basketball*

Reinforcement in the Game-Network model

Remember belief aggregation:

$$\begin{pmatrix} 1/2 & 1/2 & 0 \\ 1/3 & 1/3 & 1/3 \\ 0 & 1/2 & 1/2 \end{pmatrix} \quad \mapsto \quad \begin{pmatrix} 5/18 & 8/18 & 5/18 \end{pmatrix}$$

Reinforcement in the Game-Network model

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Now suppose after drawing a strategy players:

- Play joint strategy $s(2)$
- Receive average social welfare $U(s) = 1/6$

Reinforcement in the Game-Network model

Belief update about joint strategies:

$$\begin{pmatrix} 1/2 & 1/2 & 0 \\ 1/3 & 1/3 & 1/3 \\ 0 & 1/2 & 1/2 \end{pmatrix} \quad \rightsquigarrow \quad \begin{pmatrix} 5/12 & 7/12 & 0 \\ 5/18 & 8/18 & 5/18 \\ 0 & 7/12 & 5/12 \end{pmatrix}$$

Reinforcement in the Game-Network model

Belief update about joint strategies:

$$\begin{pmatrix} 1/2 & 1/2 & 0 \\ 1/3 & 1/3 & 1/3 \\ 0 & 1/2 & 1/2 \end{pmatrix} \quad \rightsquigarrow \quad \begin{pmatrix} 5/12 & 7/12 & 0 \\ 5/18 & 8/18 & 5/18 \\ 0 & 7/12 & 5/12 \end{pmatrix}$$

By **Bush-Mosteller** reinforcement:

$$\begin{aligned} s(1): &\quad b_{21} = 1/3 - 1/6 \cdot 1/3 = 5/18 \\ s(2): &\quad b_{22} = 1/3 + 1/6 \cdot 2/3 = 8/18 \\ s(3): &\quad b_{23} = 1/3 - 1/6 \cdot 1/3 = 5/18 \end{aligned}$$

More on reinforcement

- Probability of played strategy is increased by a payoff-dependent fraction of the distance between the original probability the max. probability 1.
- The probability of other strategies are decreased proportionally.

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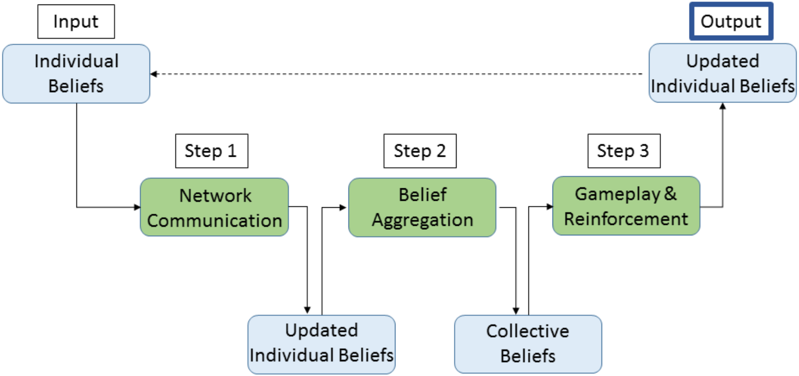
- The probability of other strategies are decreased proportionally.

Two advantages of Bush-Mosteller reinforcement:

- makes use of utility values that are scaled in the interval from 0 to 1.
- violates *Law of Practice*: learning does not slow down.

The Game-Network Learning Model



Learning Effect

Can network communication in a game have a **positive influence** on the learning outcome?

Can network communication **speed up learning** towards social optimum?

Can network communication **increase the probability** for playing the social optimum?

Learning Effect at a Given Round t

Dependent on the *network expertise* and *network structure*

Learning Effect at a Given Round t

Dependent on the **network expertise** and *network structure*

Definition (Expert for round t)

We say an agent is an **expert for round** t if his belief for the social optimum is higher than the average belief of society for the social optimum.

Learning Effect at a Given Round t

Dependent on the **network expertise** and *network structure*

Definition (Expert for round t)

We say an agent is an **expert for round** t if his belief for the social optimum is higher than the average belief of society for the social optimum.

Definition (Maximal Expert for round t)

We say an agent is a **maximal expert for round** t if his belief for the social optimum is maximal.

Learning Effect at a Given Round t

Dependent on the *network expertise* and **network structure**

Definition (Weight Centrality)

Let w_i be the total weight that agent i receives from his neighbours. The **weight centrality** of agent i is then given by the fraction $C_i^w = \frac{w_i}{n}$.

Learning Effect at a Given Round t

Dependent on the *network expertise* and *network structure*

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Let w_i be the total weight that agent i receives from his neighbours. The **weight centrality** of agent i is then given by the fraction $C_i^w = \frac{w_i}{n}$.

Theorem

If $C_{i_m}^w > \frac{1}{n} \geq C_i^w$ for all maximal experts i_m and other players i, then the probability for playing the social optimum at round t after network communication is higher than before network communication.

Learning Effect in the Long Run

Definition (Stable Expert)

We say an agent i is a **stable expert** if i is an expert for any round.

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We say an agent *i* is a **stable expert** if *i* is an expert for any round.

Theorem

Take the set of initial experts for round one. If

(i) *none of the experts 'listens' to any of the non-experts; and*

(ii) *they are in agreement in round one,*

then stable experts exist.

Learning Effect in the Long Run

Definition (Stable Expert)

We say an agent *i* is a **stable expert** if *i* is an expert for any round.

Theorem

Take the set of initial experts for round one. If

(i) *none of the experts 'listens' to any of the non-experts; and*

(ii) *they are in agreement in round one,*

(iii) *their weight centrality is above average, and the weight centrality of non-experts is below average,*

then the probability of playing the social optimum after communication is higher than before network communication at every round.

Conclusions, Part 2

Theoretical conclusions:

- Interdisciplinary computational model
- For learning in *cooperative games* through *social networks*
- Communication positively influences learning:
  - (i) Experts with high weight centrality: *given round t*
  - (ii) Stable experts with high weight centrality: *every round*
- Applications in Airline Safety Heros in Center for Applied Games

Nina Gierasimczuk and Sanne Kosterman, Collective Learning in Games through Social Networks, in: M.G. Armentano et al. (Eds.): Proceedings of the 1st IWSIA @ IJCAI 2015.

THANK YOU!

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Let us be interdisciplinary!