Random Access to Grammar Compressed Strings

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Random Access to Compressed Strings

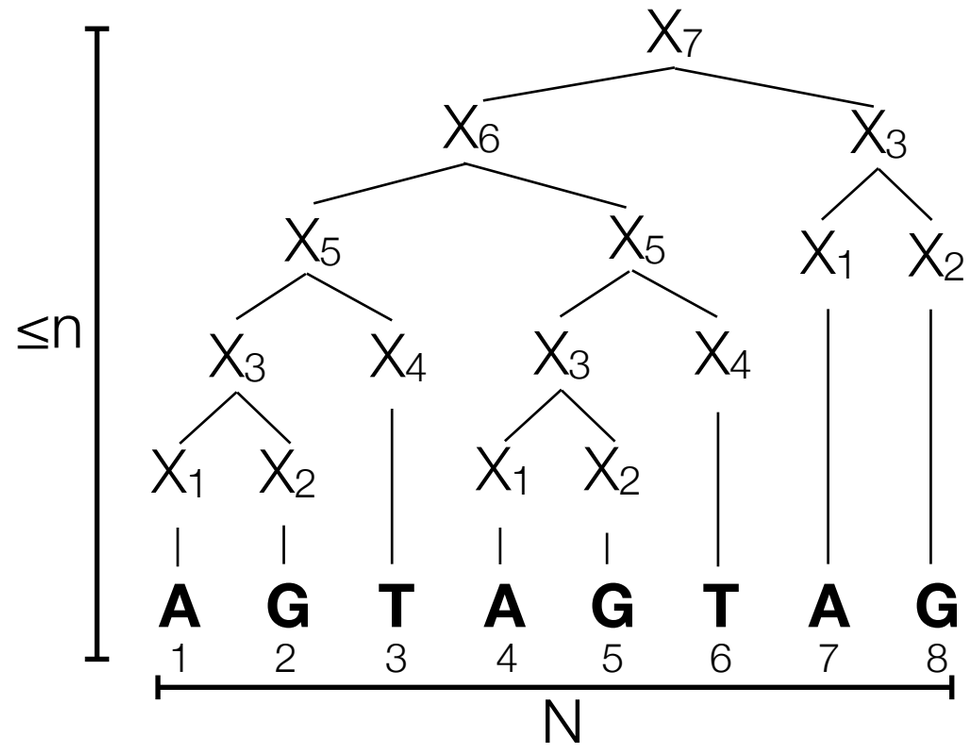
Random Access to Compressed Strings

- What is the ith character?
- What is the substring at [i,j]?
- Does pattern P appear in text? (perhaps with k errors?)

Random Access to Grammar Compressed Strings

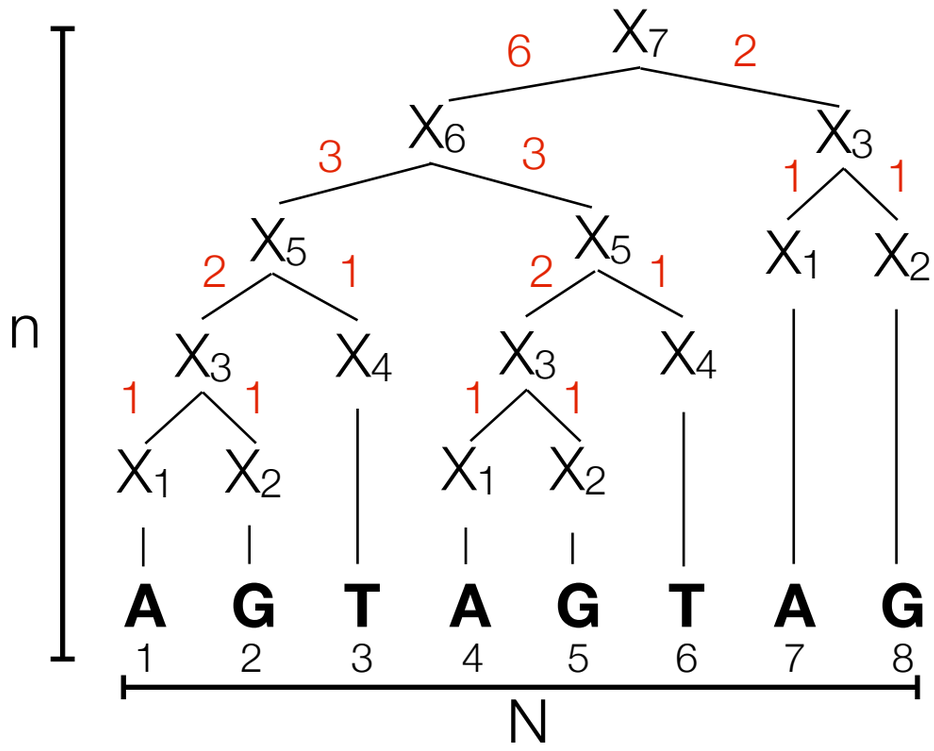
AGTAGTAG N = 8

$X_7 \rightarrow X_6X_3$
$X_6 \rightarrow X_5X_5$
$X_5 \rightarrow X_3X_4$
$X_4 \rightarrow \mathbf{T}$ n = 7
$X_3 \rightarrow X_1X_2$
$X_2 \rightarrow \mathbf{G}$
$X_1 \rightarrow \mathbf{A}$



- Grammar based compression captures many popular compression schemes with no or little blowup in space [Charikar et al. 2002, Rytter 2003].
- Lempel-Ziv family, Sequitur, Run-Length Encoding, Re-Pair, ...

Tradeoffs and Results



- What is the ith character?

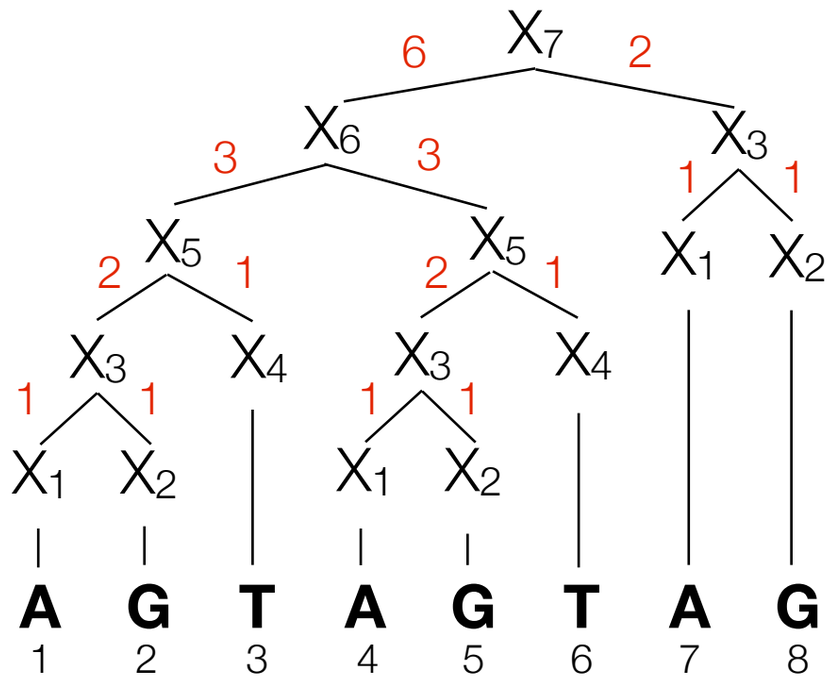
O(N) space	O(n) space	O(n) space
O(1) query	O(n) query	O(log N) query

- What is the substring at [i,j]?

O(n) space

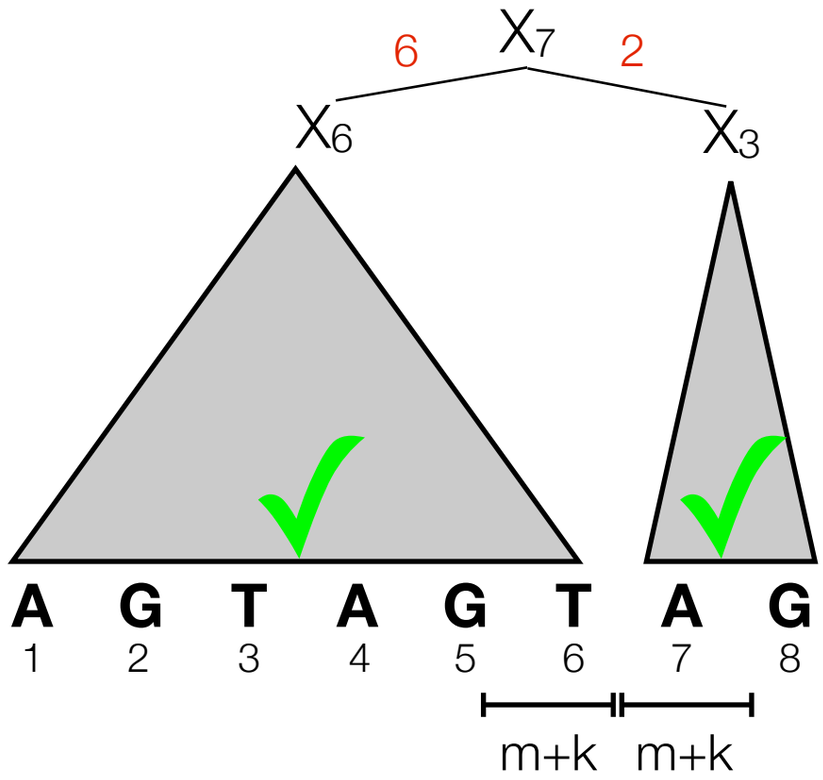
O(log N + j - i) query

Application: Black-Box Compressed String Matching



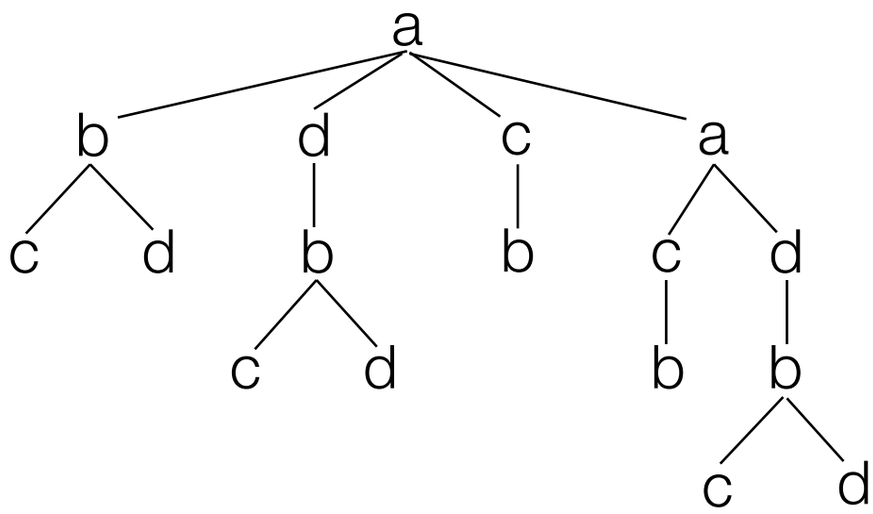
- Does “**AGGA**” appear in the text (perhaps with k errors)?

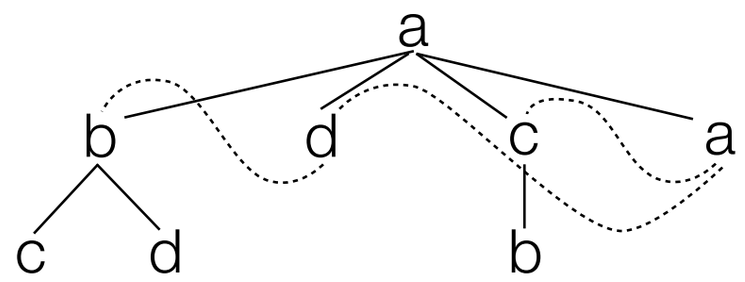
Application: Black-Box Compressed String Matching



- Does “**AGGA**” appear in the text (perhaps with k errors)?
- Total time O(n (log N + m + Blackbox(m))).

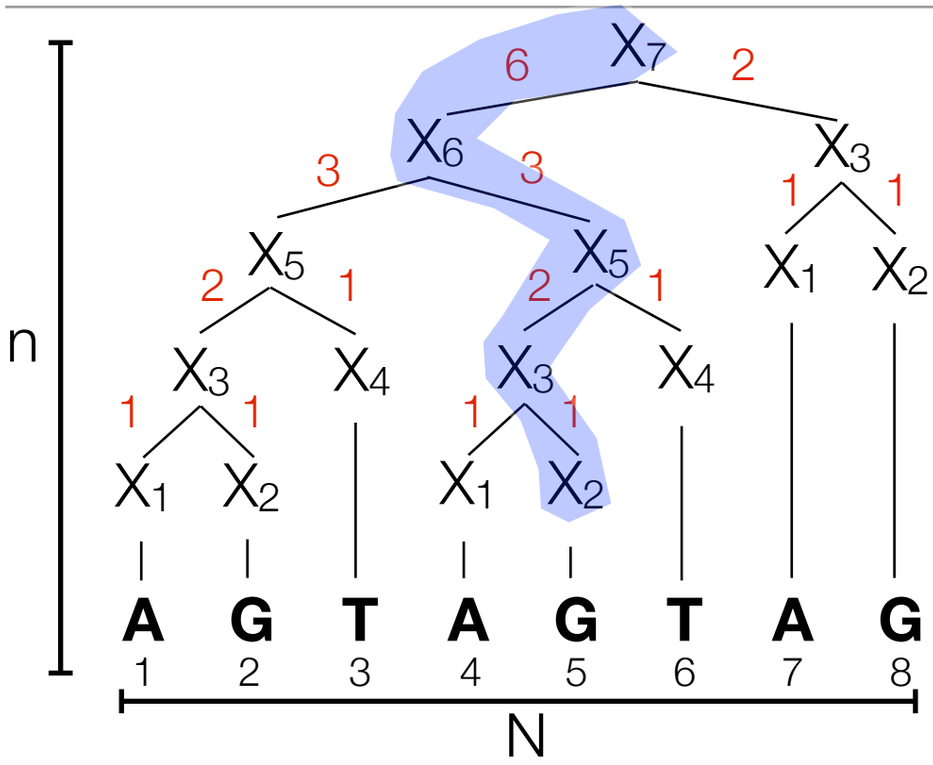
Extension: Compressed Trees



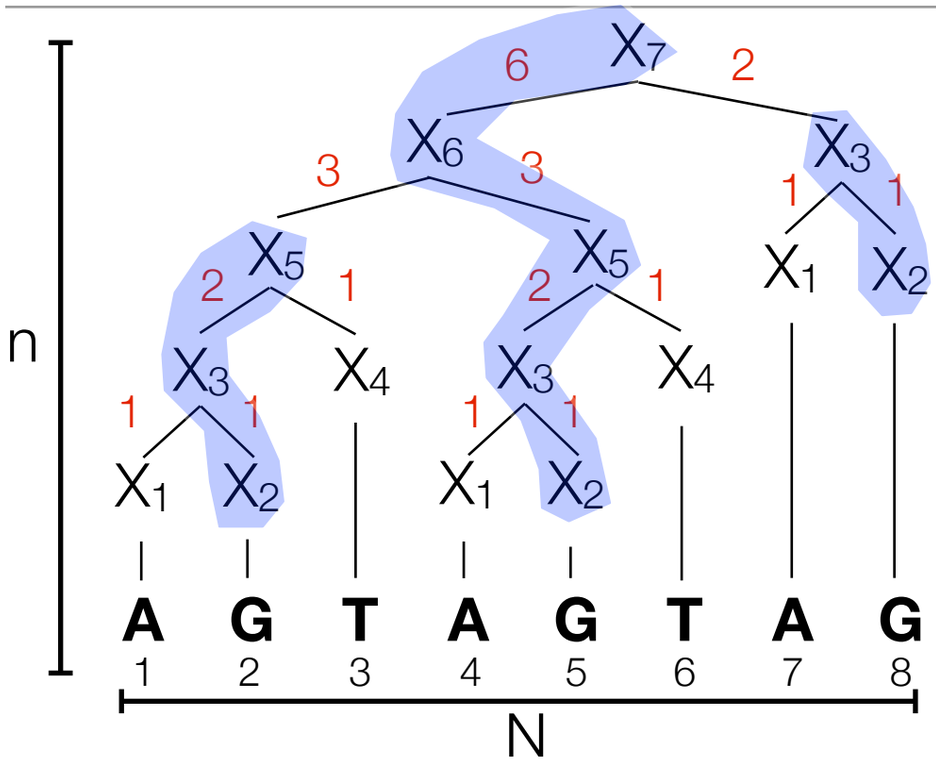


- Linear space in compressed tree.
- Fast navigation operations (select, access, parent, depth, height, subtree_size, first_child, next_sibling, level_ancestor, nca).

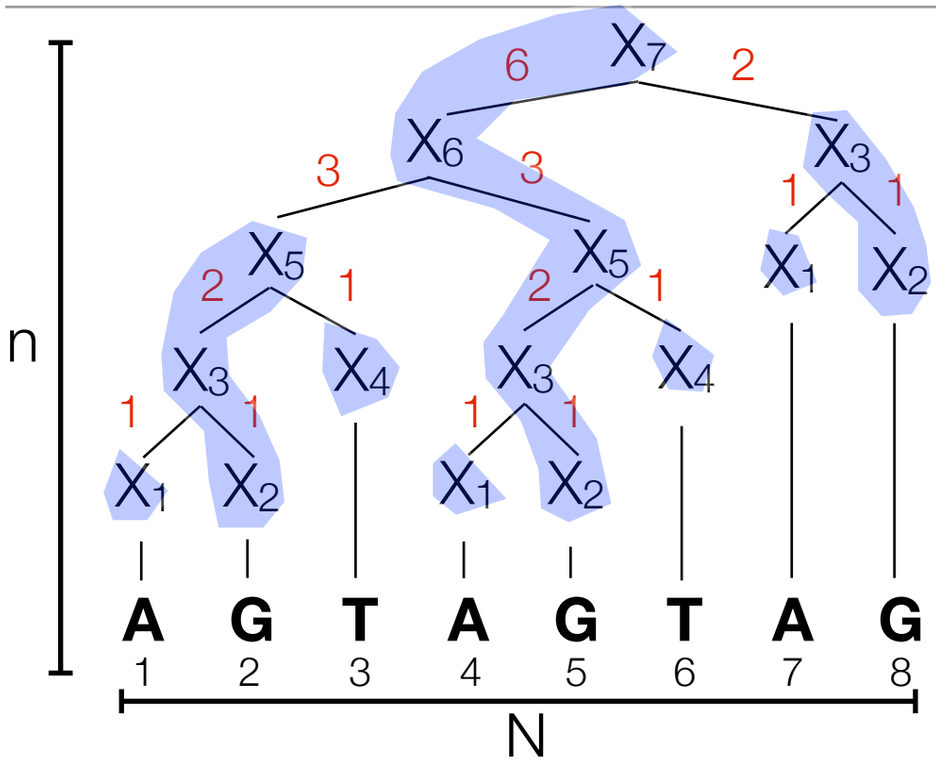
Heavy Path Decomposition



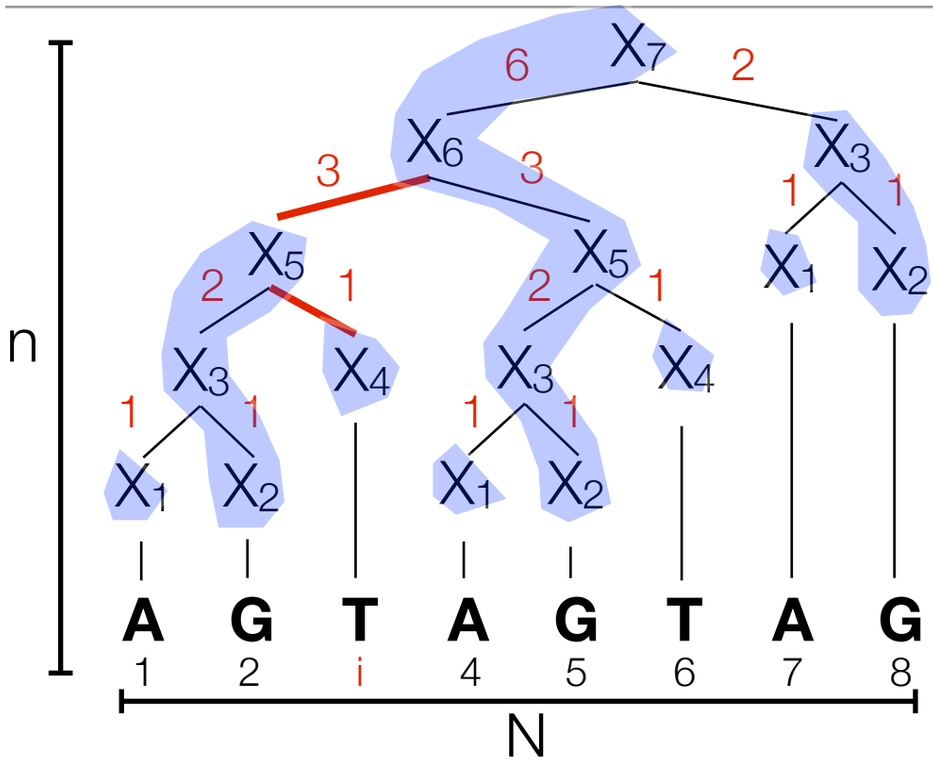
Heavy Path Decomposition



Heavy Path Decomposition



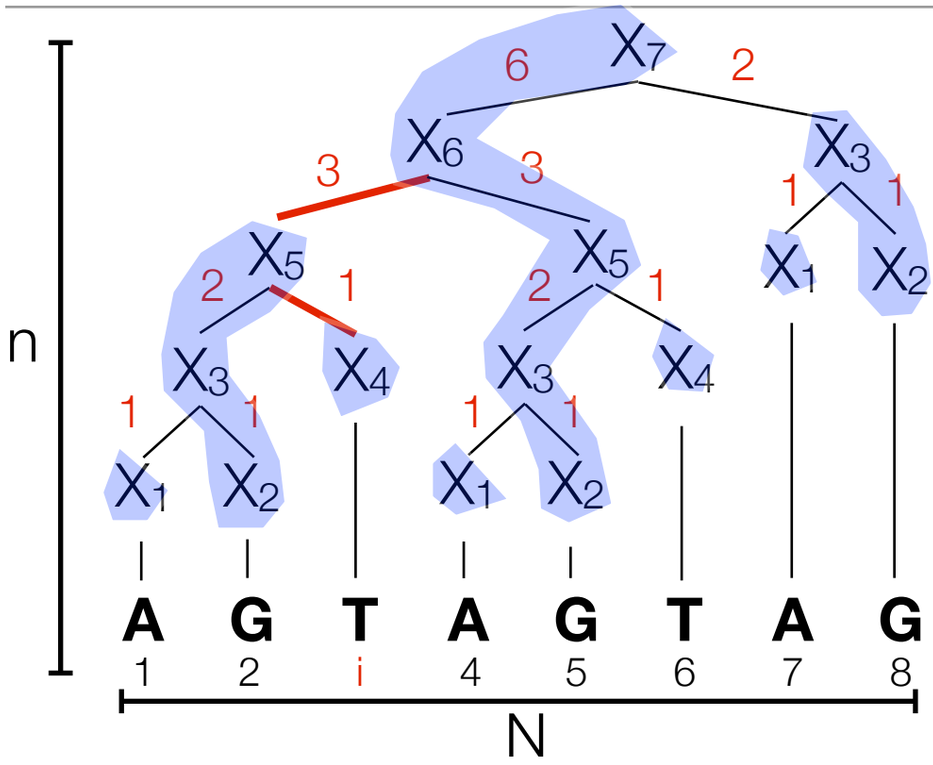
Random Access Query



- The path from root to i goes through O(log N) heavy paths
- Query: Binary search all heavy paths on the way

$$O(\log n) \cdot O(\log N)$$

Random Access Query



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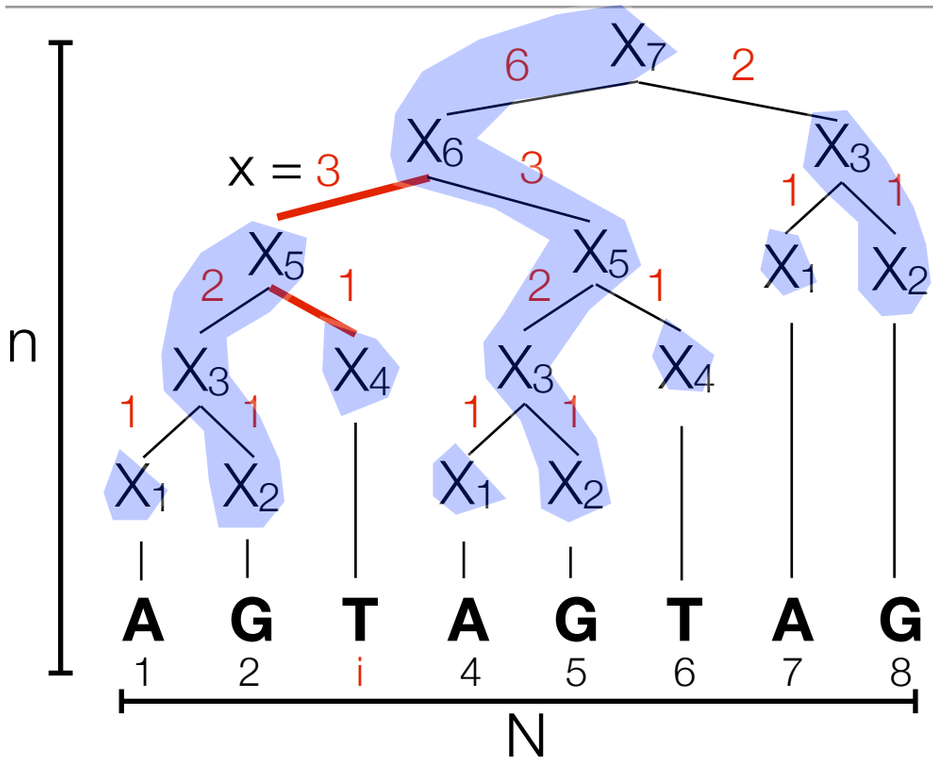
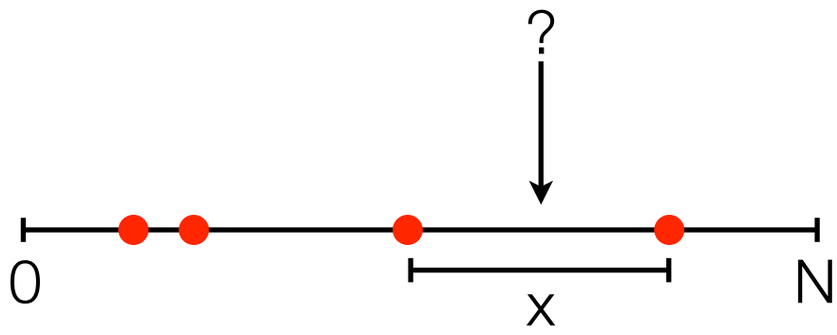
~~O(log n)~~ · O(log N)

Interval Biased Search Tree





- The path from root to i goes through O(log N) heavy paths
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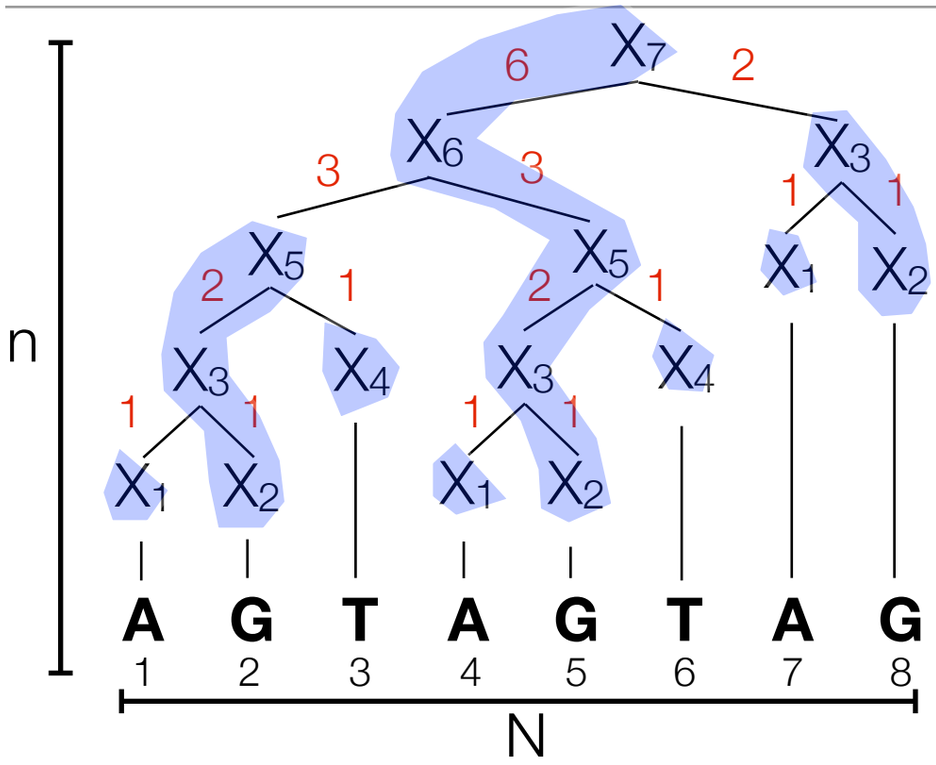
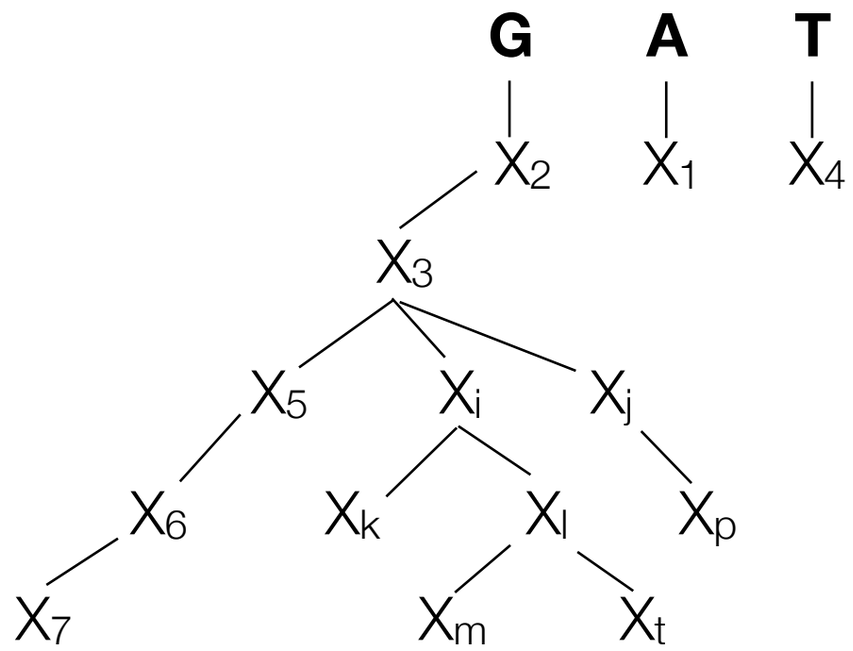
  ~~O(log n)~~ · O(log N)

  O(log N/x) Telescopes to O(log N)

- Space: Each IBSTs uses linear space => total $O(n^2)$ space for all heavy paths.

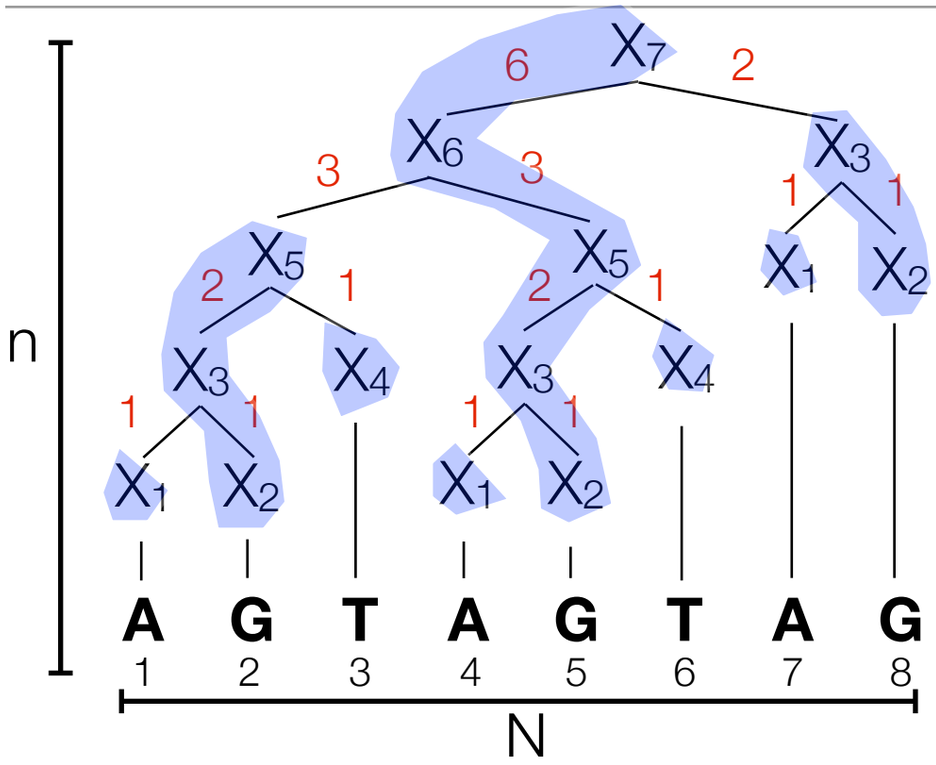
O(n) Representation of Heavy Paths

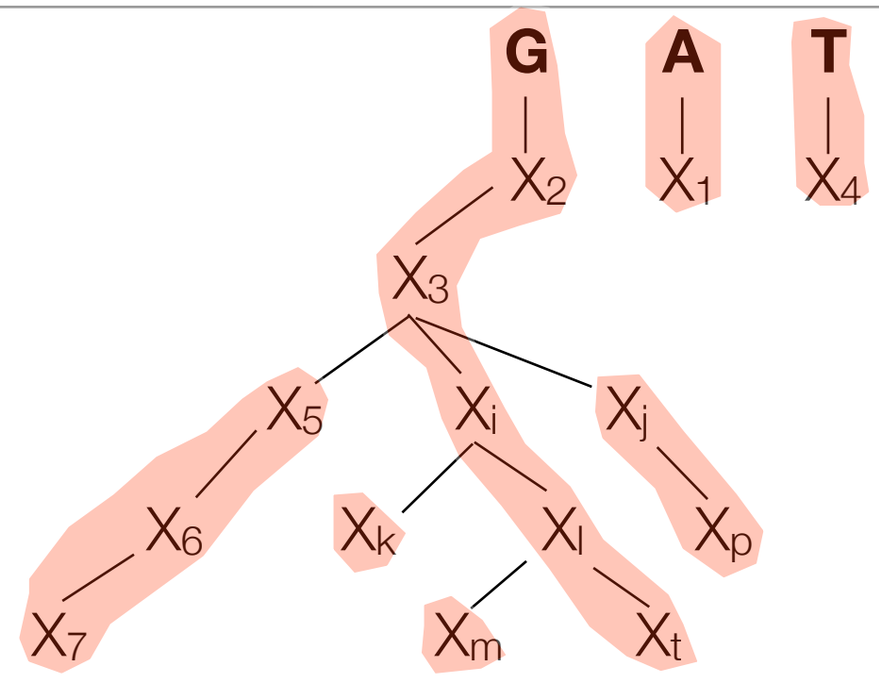




- Search for i on heavy path = lowest ancestor of distance i.

O(n) Representation of Heavy Paths





- Search for i on heavy path = lowest ancestor of distance i.
- A heavy path decomposition of heavy path representation.
- In-path: O(log N/x) time, total O(n) space.

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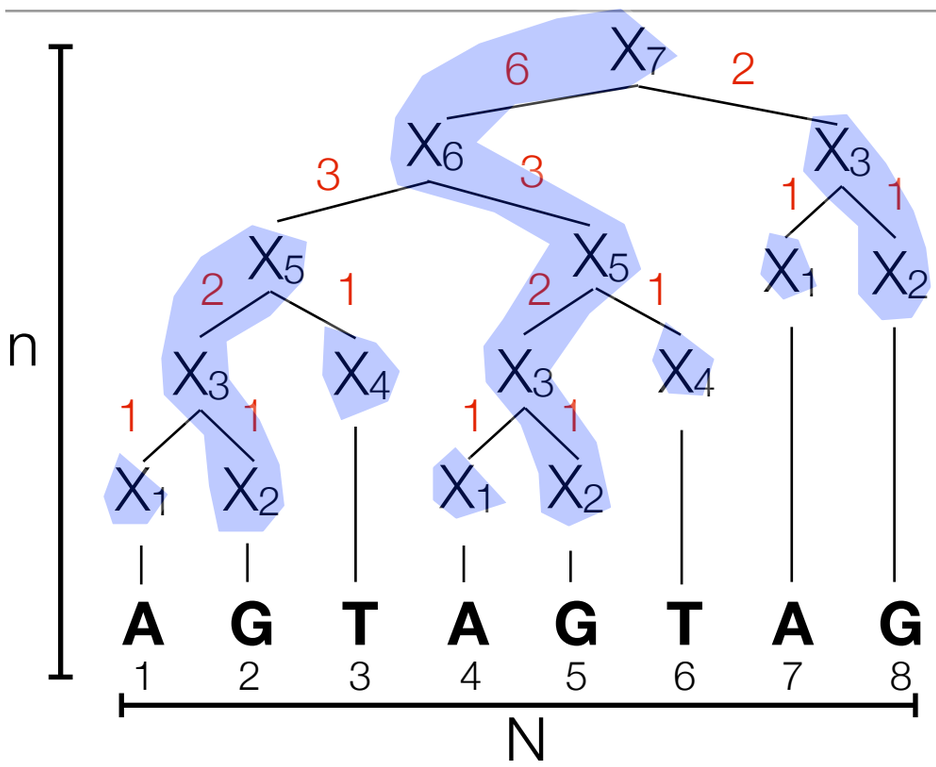

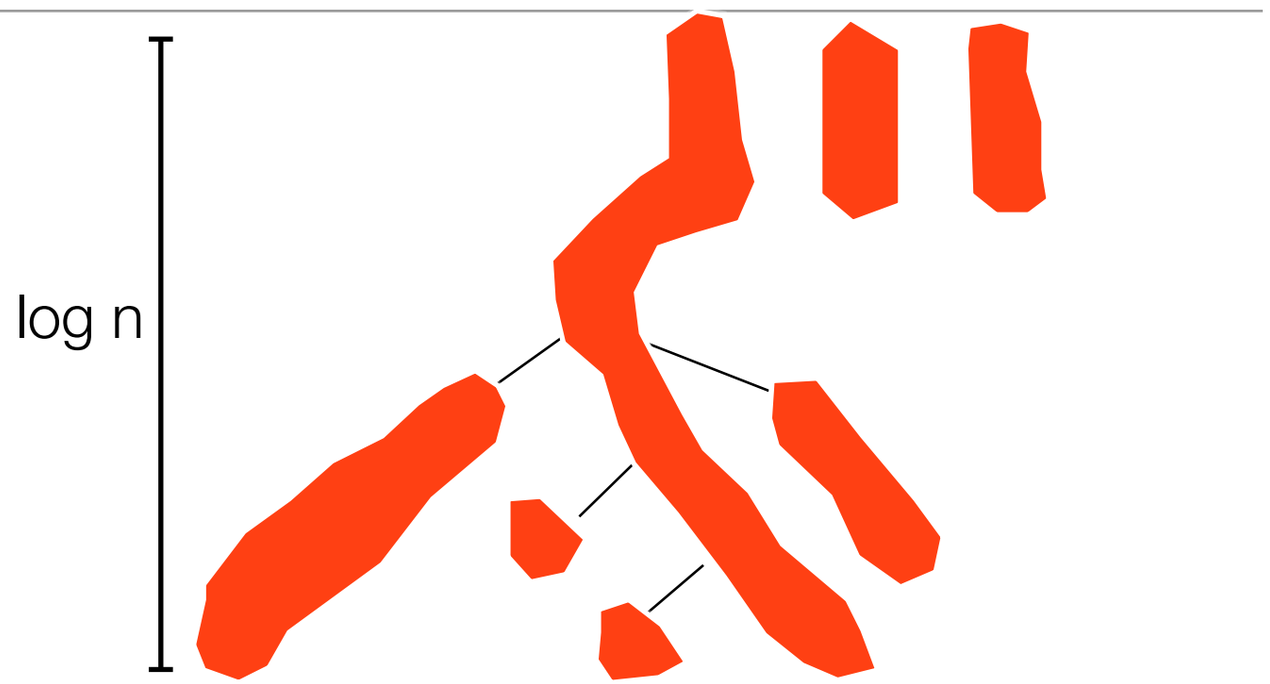

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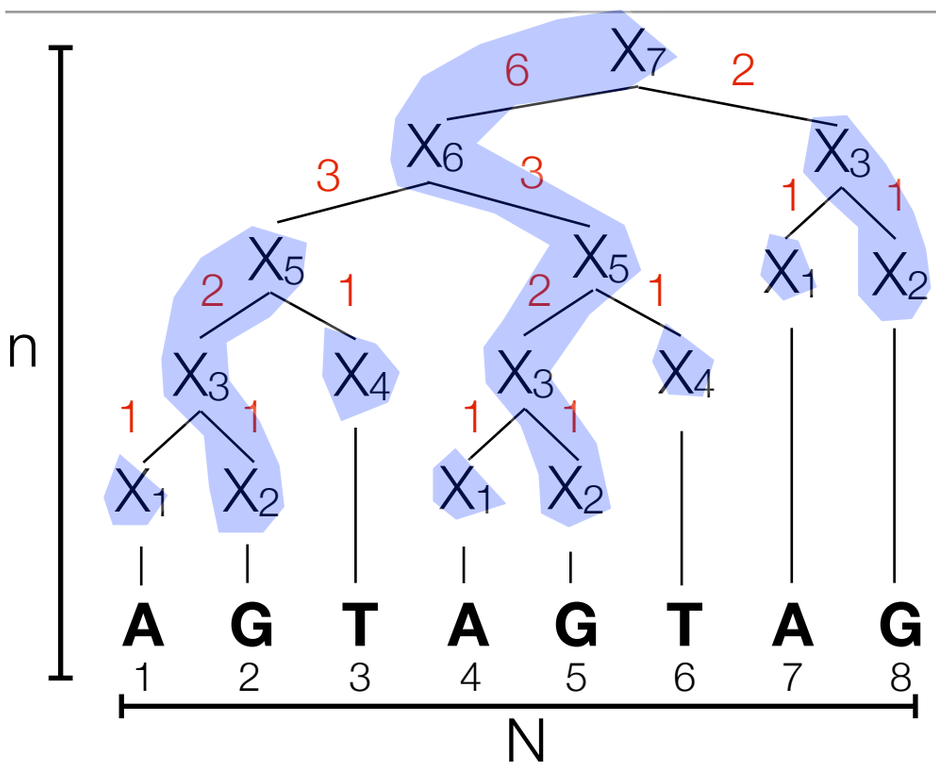

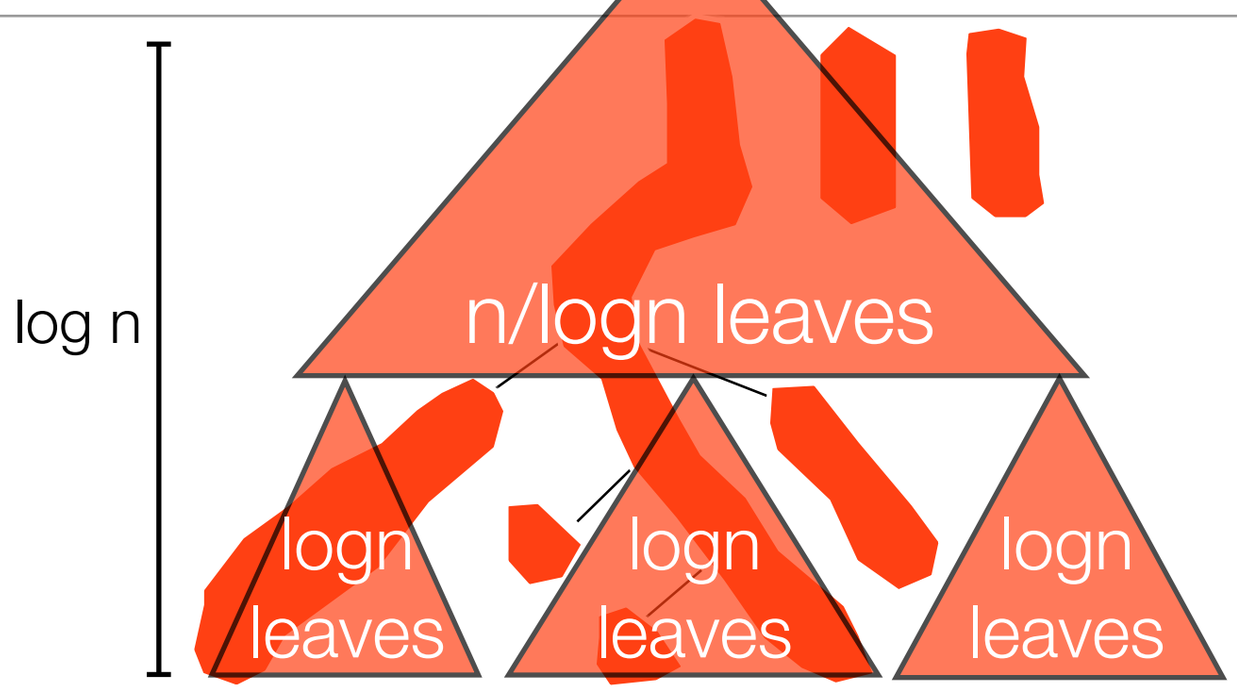

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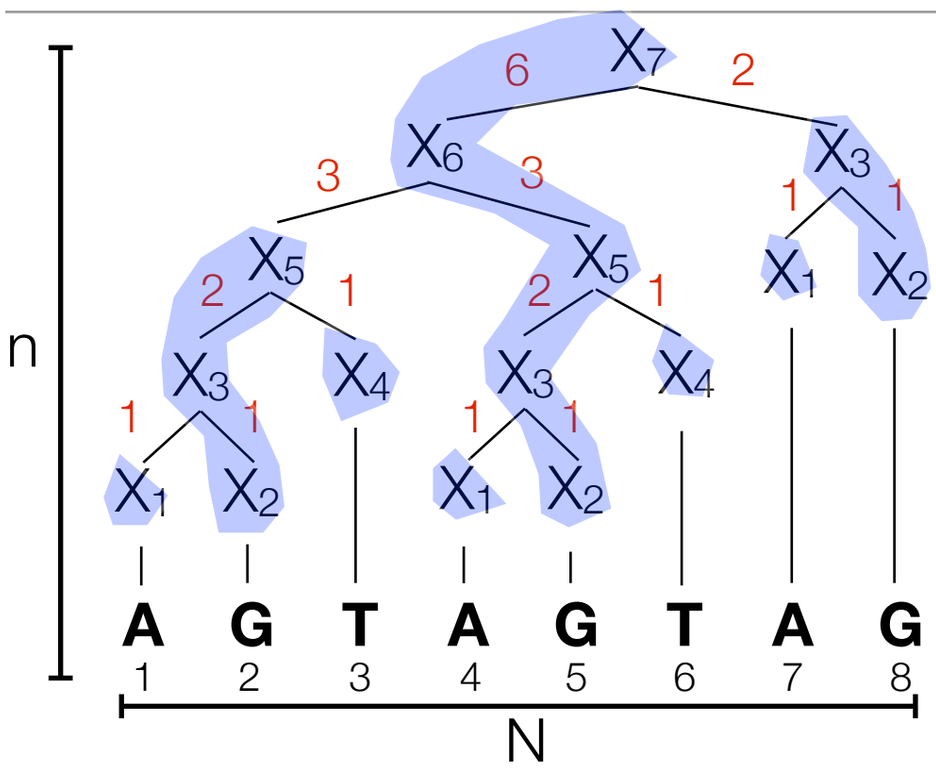

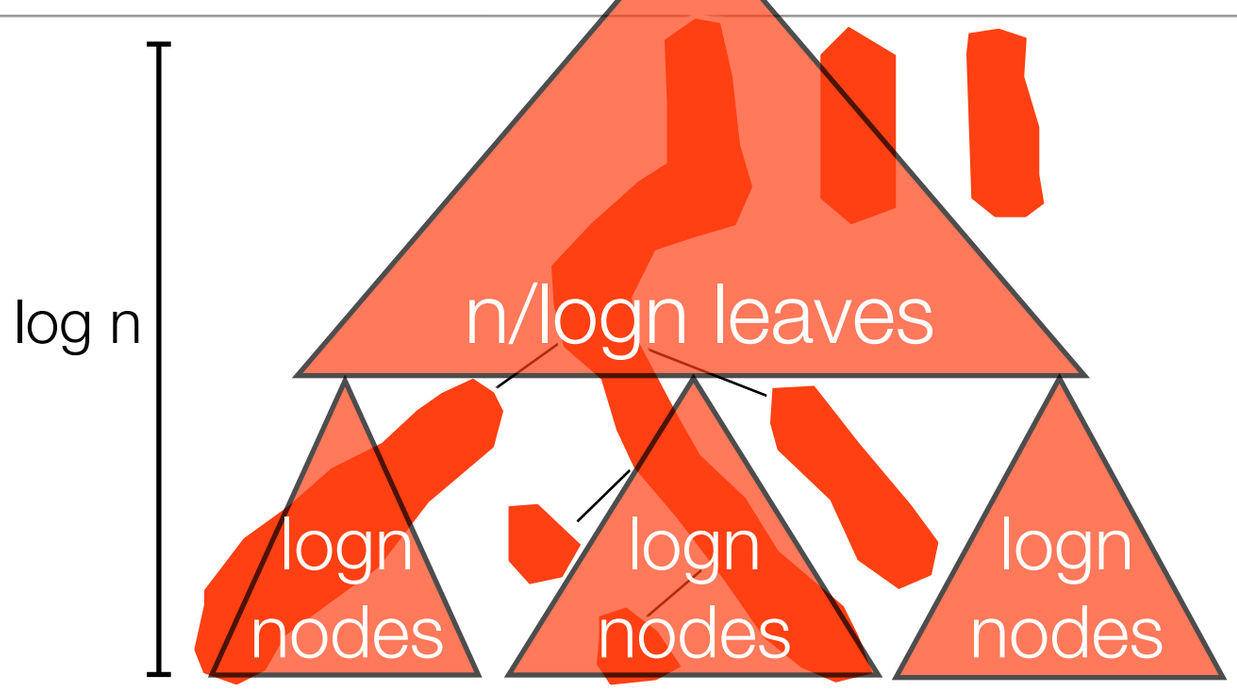

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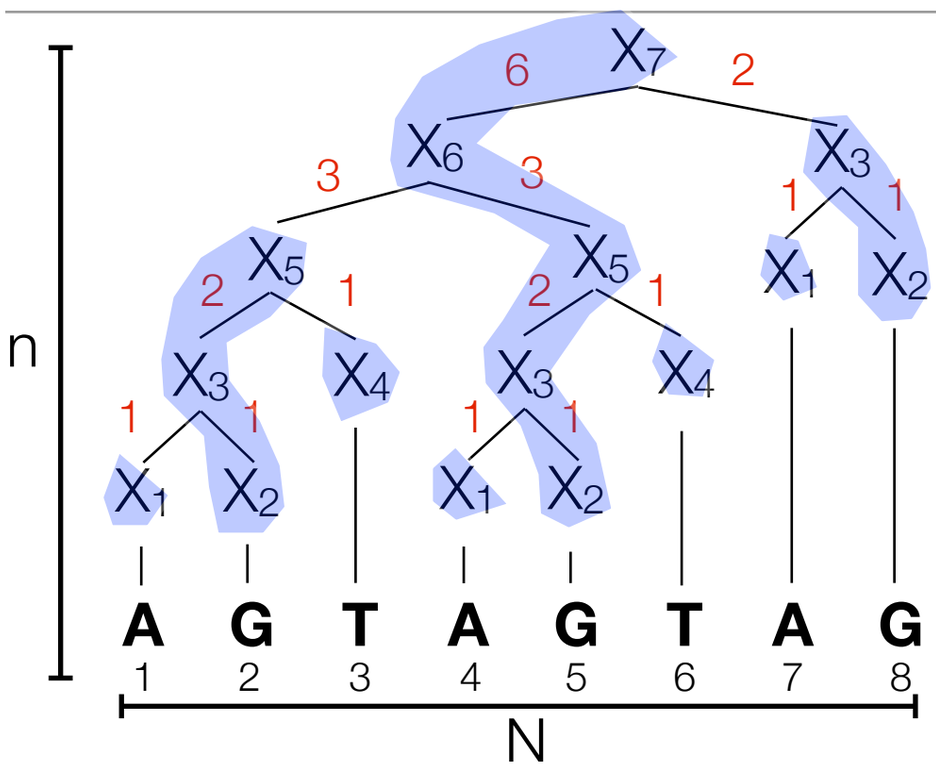

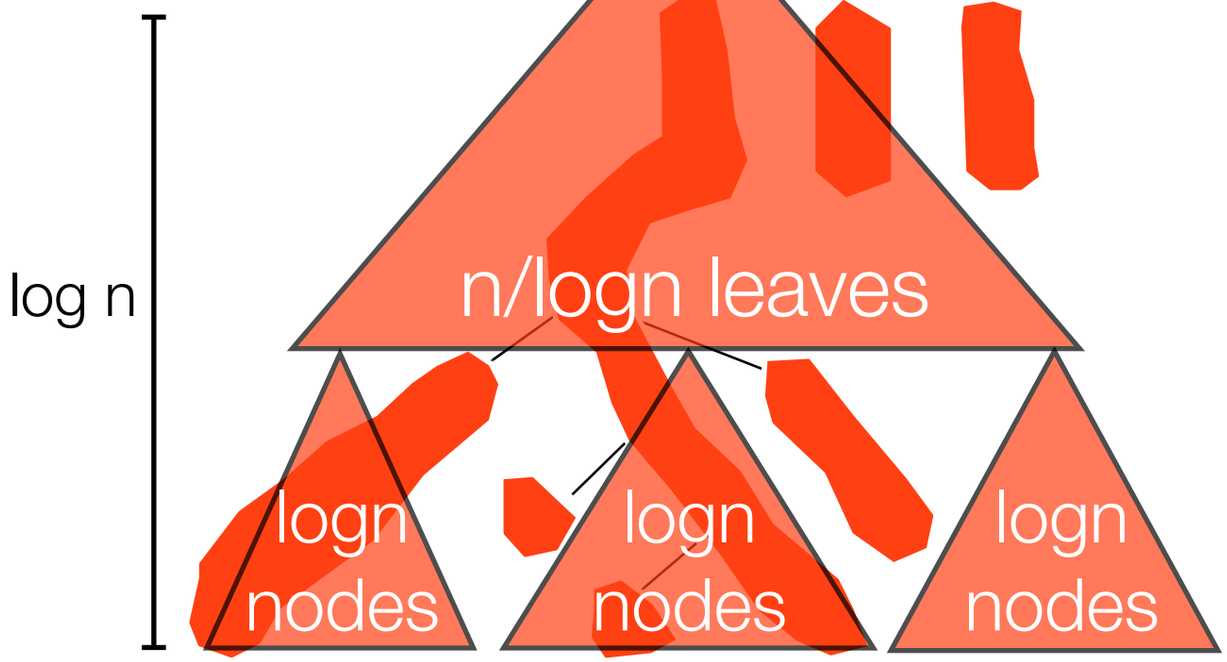

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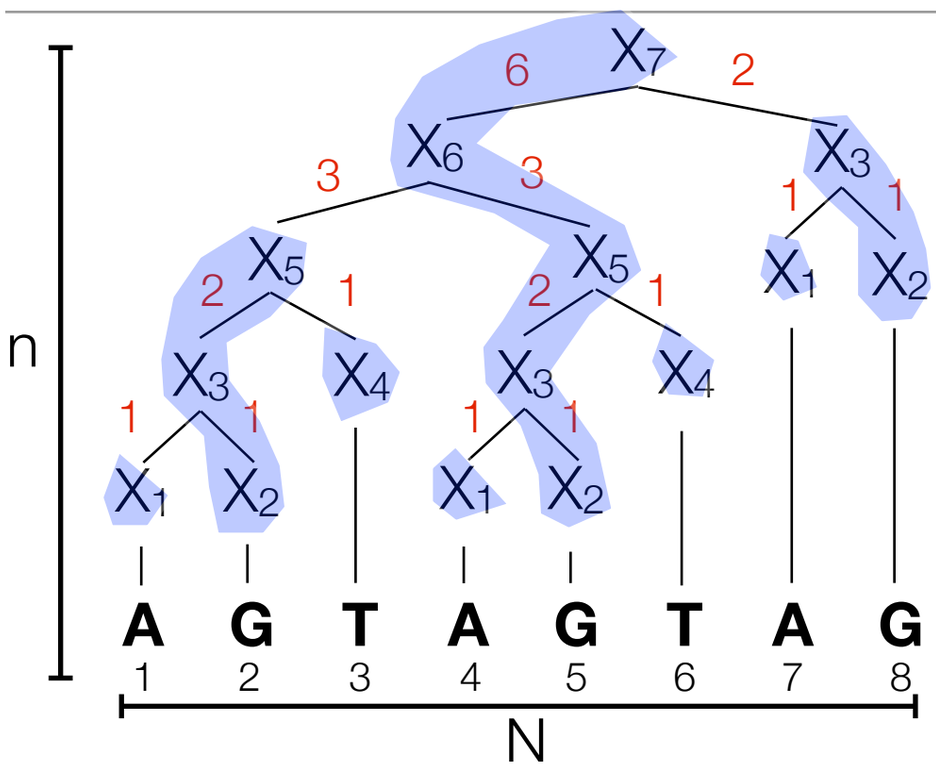
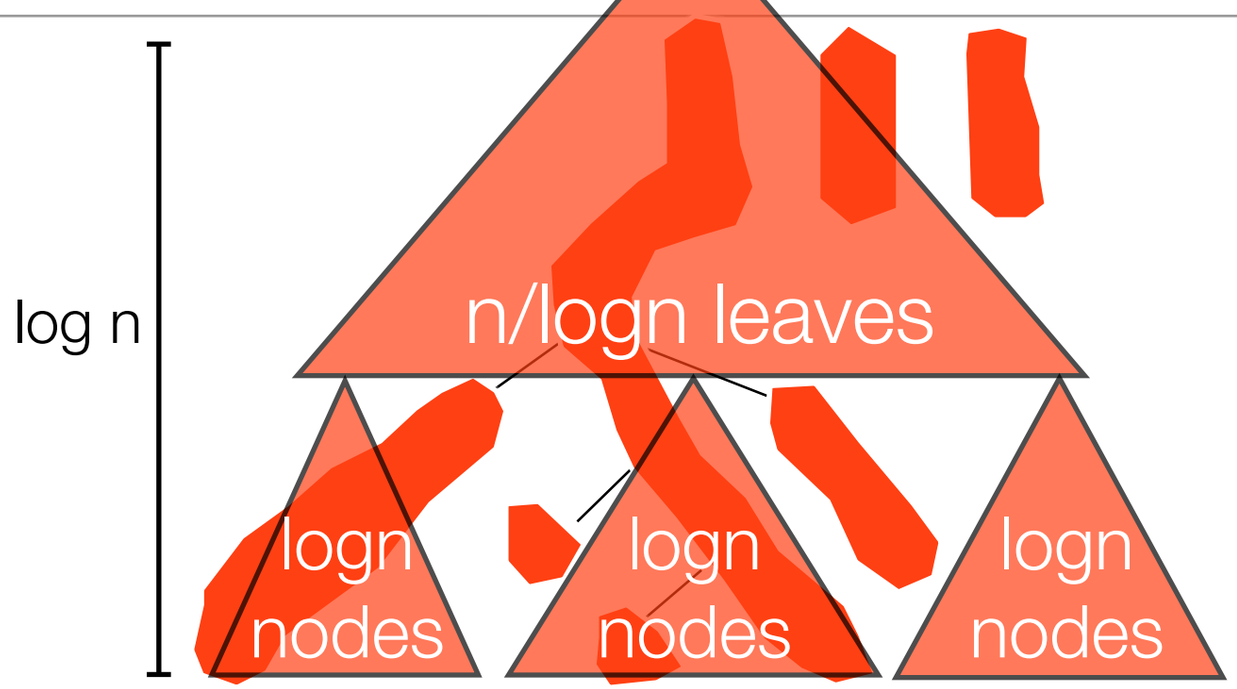
- In-path: O(log N/x) time, total O(n) space.
- Between-paths: O(log N/x) time, total ~~O(n log n)~~ O(nα(n)) space.

O(n) Representation of Heavy Paths





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  ~~O(nα(n))~~

  O(n) with bittricks

Summary

- Random access and substring decompression.
  - O(n) space and O(log N + length of substring) time.
- Black compressed (approximate) string matching.
- Random access in compressed trees.