Regular Expression Matching with Multi-Strings and Intervals

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Outline

- Definition
- Applications
- Previous work
- Two new problems: Multi-strings and character class intervals
- Algorithms
    - Thompson's algorithm with multi-strings.
    - Decomposition-based algorithms with multi-strings.
    - Character class intervals extensions.

Regular Expressions

- A character α is a regular expression.
- If S and T are regular expressions, then so is
    - The *union* S | T
    - The *concatenation* ST (S·T)
    - The *kleene star* S*

Languages

- The *language* $L(R)$ of a regular expression R is:
- $L(\alpha) = \{\alpha\}$
- $L(S|T) = L(S) \cup L(T)$
- $L(ST) = L(S)L(T)$
- $L(S^*) = \{\varepsilon\} \cup L(S) \cup L(S)^2 \cup L(S)^3 \cup \ldots$

Example

- R = a(a*)(b|c)
- L(R) = {ab, ac, aab, aac, aaab, aaac, ...}

Regular Expression Matching

- Given regular expression R and string Q the regular expression matching problem is to decide if $Q \in L(R)$.

Applications

- Primitive in large scale data processing:
  - Internet Traffic Analysis
  - Protein searching
  - XML queries
- Standard utilities and tools
  - Grep and Sed
  - Perl

Previous Work (Worst-Case Efficient Algorithms)

- Let |R| = m and |Q| = n.
- Standard textbook algorithm [Thompson 1968] simulates a non-determinstic automaton (NFA) in O(nm) time.
- NFA-decomposition algorithms [Myers 1992], [B 2006], [B ,Farach-Colton 2005], [B, T 2009]:
  - Decompose NFA into tree of small NFAs and combine with tabulation and/or word-level parallelism to speedup Thompson's algorithm.
  - We will need O(n (m log w/ w + log m)) time algorithm [B 2006] for our results. Fastest known algorithm for large w.

Problem 1: Multi-Strings

- Many regular expressions consist k << m strings.
- Example: Gnutella download stream detection:
- `(Server:|User-Agent:)( |\t)*(LimeWire|BearShare|Gnucleus|Morpheus|XoloX|gtk-gnutella|Mutella|MyNapster|Qtella|AquaLime|NapShare|Comback|PHEX|SwapNut|FreeWire|Openext|Toadnode)`
- k = 21 vs. m = 174.
- Can we exploit k << m in algorithms for regular expression matching?

Problem 2: Character Class Intervals

- For a subset of characters C a *character class interval* C{x,y} represents a string of character from C of length at least x and at most y.
- Example: `[afg]{13,42}`
- Special case of *gaps* (Σ{x,y}) is important in protein searching.
- We can always convert a character class interval operator to standard operators but this increases the length of regular expression by y.
- Can we efficiently implement character class interval operators in regular expression matching?

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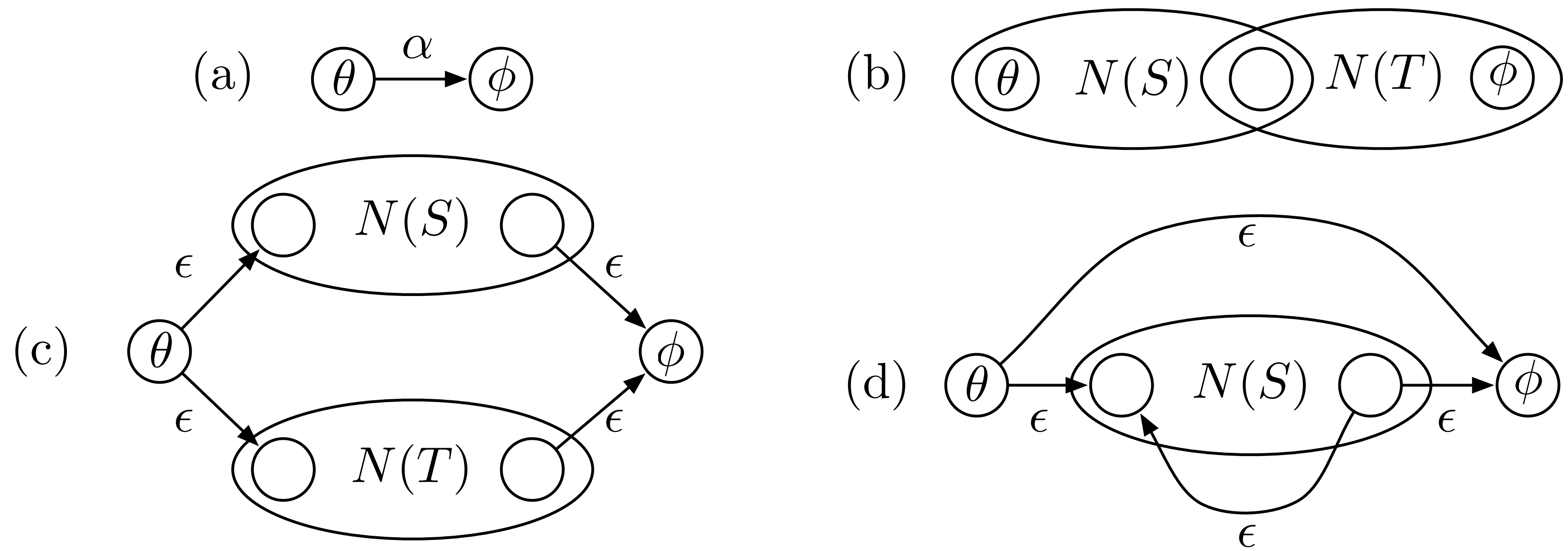
- **Algorithms**
    - Thompson’s algorithm with multi-strings.
    - Decomposition-based algorithms with multi-strings.
    - Character class intervals extensions.

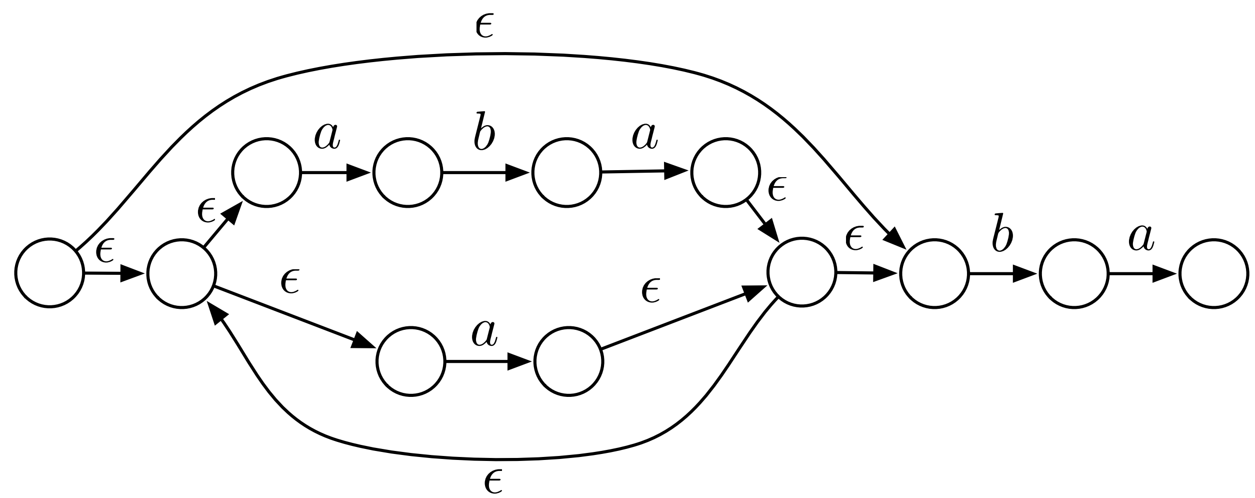
Thompson's Algorithm



- Recursively construct non-deterministic finite automaton (NFA) from R.

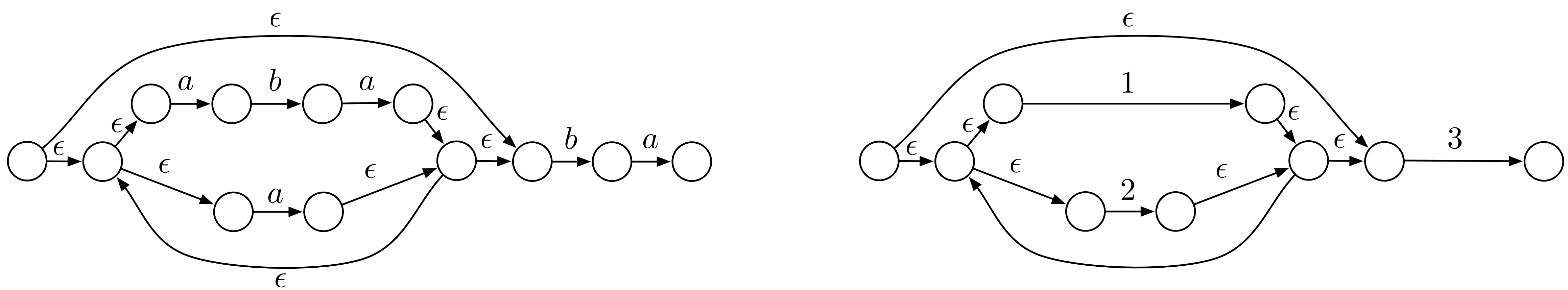
Thompson's Algorithm

R = (aba|a)*·ba



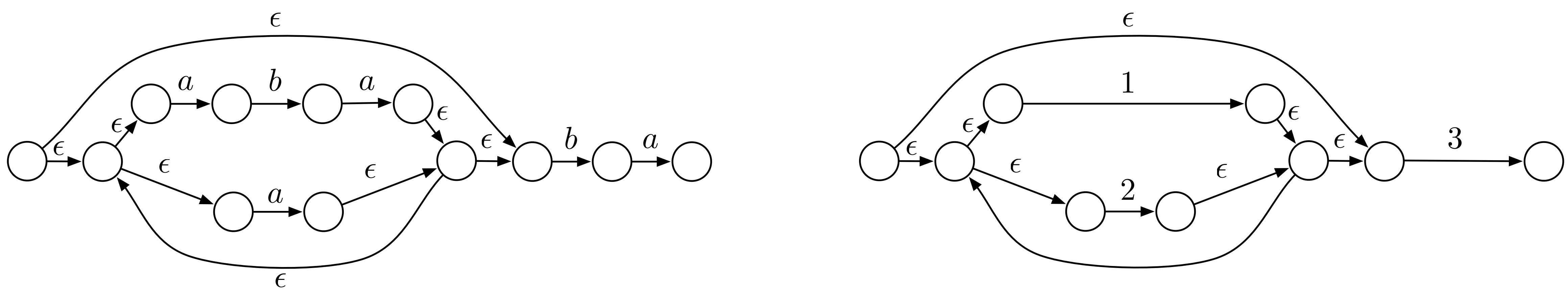
- *Thompson NFA* (TNFA) N(R) has O(|R|) = O(m) states and transitions.
- N(R) *accepts* L(R). Any path from start to accept state corresponds to a string in L(R) and vice versa.
- Traverse TNFA on Q one character at a time.
- O(m) per character => O(|Q|m) = O(nm) time algorithm.
- Can we get O(nk)?

Thompson's Algorithm with Multi-Strings



- Construct *pruned* TNFA: Replace strings L = {L_1, ..., L_k} with single transitions => number of states and transitions is O(k).
- Maintain FIFO bit queue for L_i of length $|L_i|$.
- Preprocess L for fast multi-string matching (Aho-Corasick automaton).

Thompson's Algorithm with Multi-Strings



- Interleaved traversal of TNFA and multi-string matching on one character from Q at a time:
  - Startpoint of string transition active => Enqueue 1 else 0.
  - Front of queue 1 and match of string => Make endpoint active.
- O(k) states and transition, k queues, multi-string matching is fast => O(k) time per character => Total time O(nk + m log k) and space O(m).

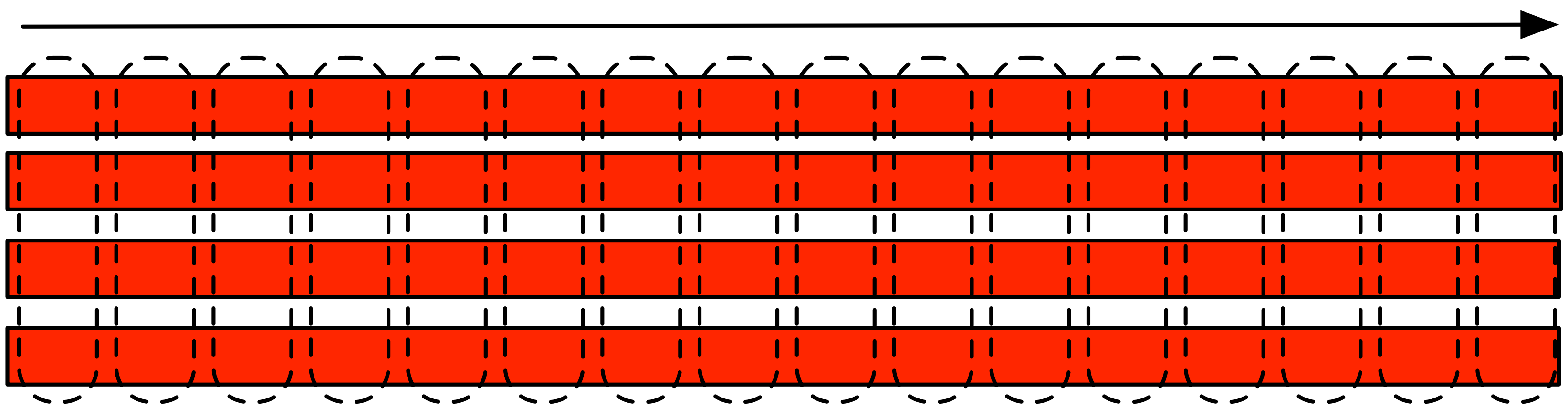
Decomposition Algorithms

- We use NFA-decomposition algorithm based on word-level parallelism [B 2006]:
- Simplifying assumption: $m \geq w$.
- Decompose TNFA into tree of $O(m/w)$ micro TNFAs, each with at most w states.
- Encode each micro TNFA state-set in $O(w)$ bits.
- Micro TNFA traversal on a single character in $O(\log w)$ time using word-level parallelism.
- => $O(m/w \cdot \log w)$ on a single character for entire TNFA
- => $O(nm\log w/ w)$ algorithm for regular expression matching.
- Fastest known for large w.

Decomposition Algorithms with Multi-Strings

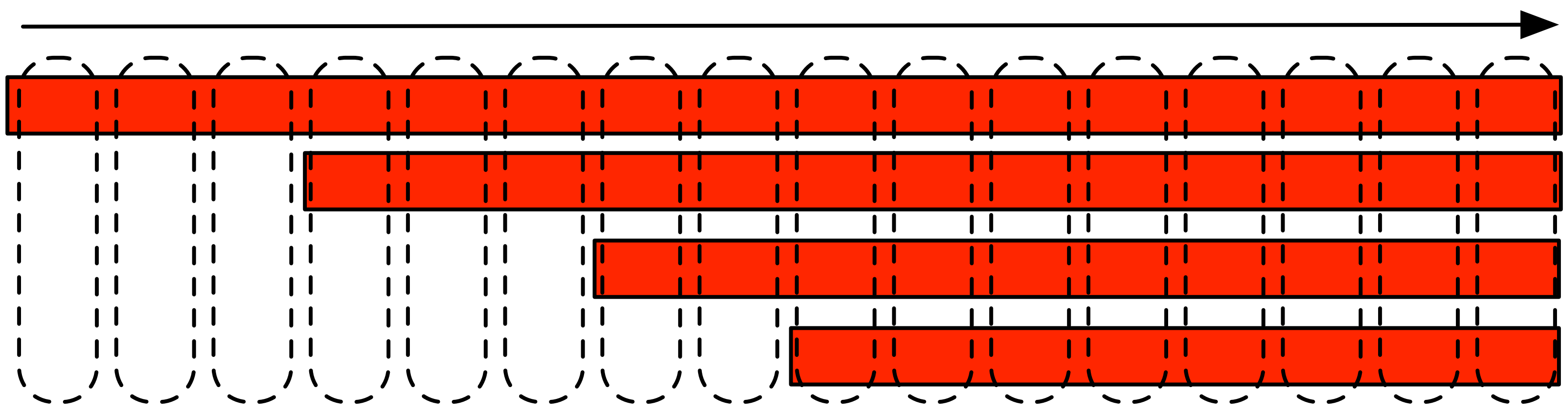
- Goal: Replace m with k. Process a character in O(k log w/w) time.
- Apply decomposition on pruned TNFA: Tree of O(k/w) micro TNFAs with at most w states and w strings.
- Reuse ε-transition traversal => O(log w) per micro TNFA
- Reuse multi-string matching algorithm.
- **The missing piece**: How can we maintain w bit queues in O(log w) time per operation?

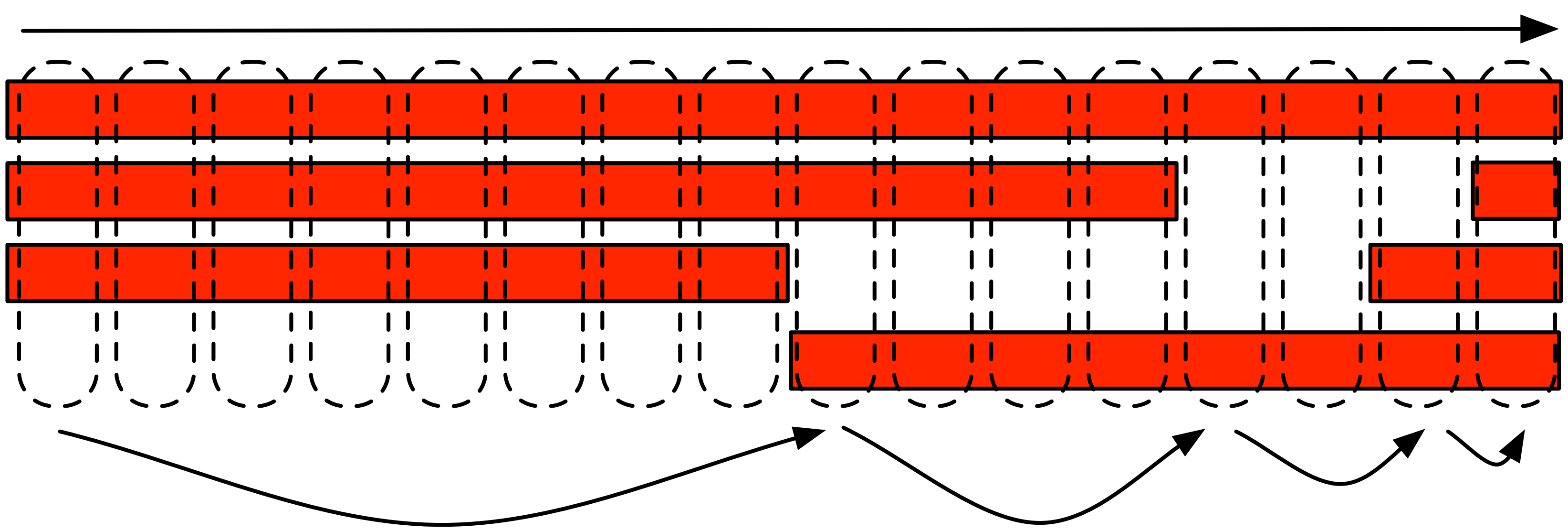
Case 1: Short Bit Queues (length ≤ 2w)



- First, suppose all queues have the same length!
- Represent queues “vertically”.
- In each step insert input bits in back of queue and output the front of the queue.
- Implicitly move all bits forward by updating the pointer to the start of the queues.
- => O(1) time per step.

Case 1: Different lengths?

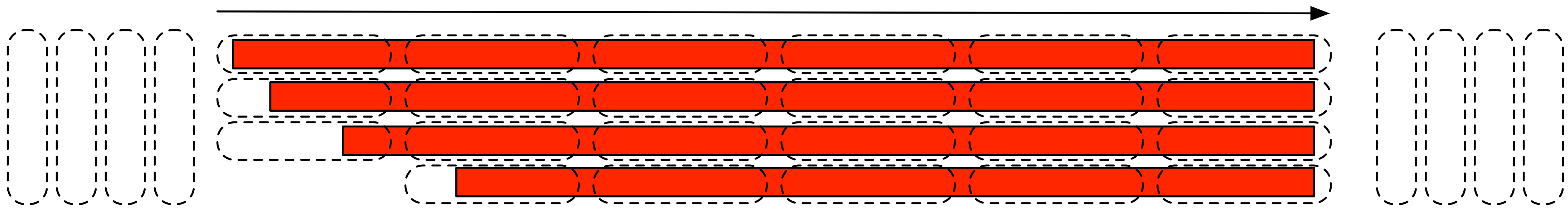




Case 1: Short Bit Queues (length $\leq 2w$)

- With bit mask and standard bitwise operation we can implement each jump point in $O(1)$ time.
- => $O(\log w)$ time per step.

Case 2: Long Bit Queues (length > 2w)



- Horizontal representation with vertical front and back buffers of length w.
- Enqueue and dequeue from buffers in O(1) time.
- Every w steps (full buffers):
    - Transpose the back buffer and insert into horizontal representation.
    - Transpose the front w entries of the horizontal representation and insert into the front buffer.
- Transpose takes time O(w log w) [T 1997] => Amortized O(log w) time per step.

Algorithm Summary

- $O(\log w)$ per character per micro TNFA => $O(k \log w / w)$ per character.
- => total time $O(n (k \log w/w + \log k) + m \log k)$ and space $O(m)$.

Character Class Intervals

- New technique to maintain w counters in parallel with reset and decrement operations.
- Combine with bit queues to support character class intervals.