# Fast Searching in Packed Strings

Philip Bille

# String Matching

- Problem: Given strings P and Q of lengths m and n, resp., report all occurrences of P in Q.

$$Q = \texttt{aababcabbababcabcaabbab}$$

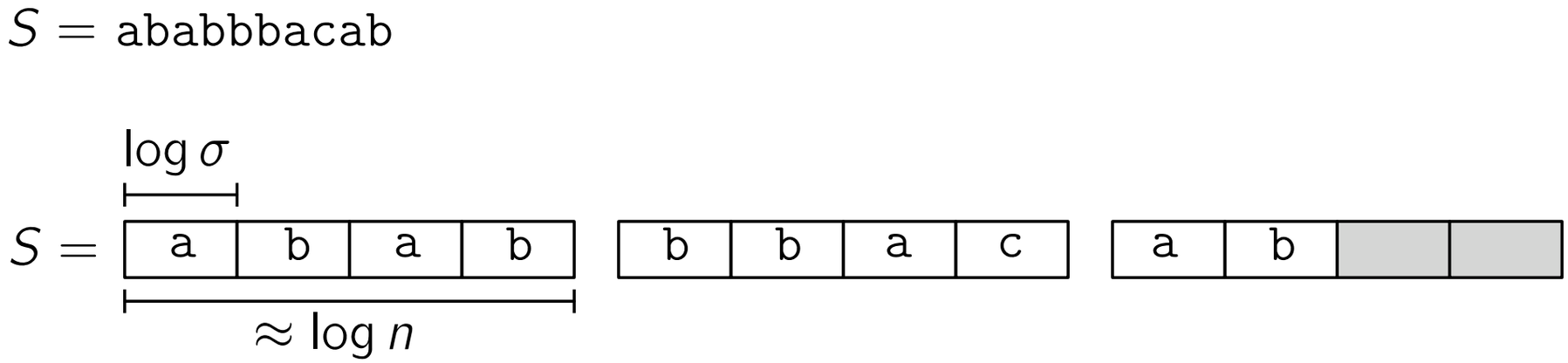
$$P = \texttt{ababca}$$

- KMP-algorithm [KMP1977] uses O(n) time  (assume w.l.o.g. m ≤ n).

- Optimal if strings are stored with one char per memory word.

# Packed Strings

- Real strings are *packed*:



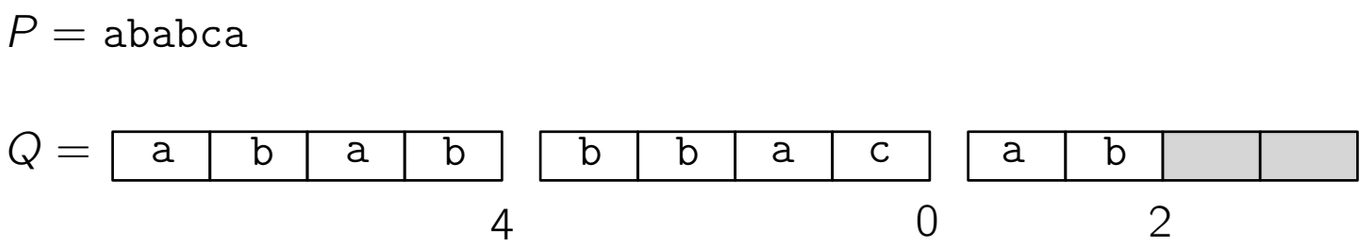

- With word-length log n a memory word holds ≈ log n / log σ characters.

- S uses O(|S| log σ/log n) = O(|S|/ $\log_\sigma$n) words.

# Packed String Matching

- Problem: String matching with P and Q in packed representation.
- Lower bound: $\Omega\left(\frac{n+m}{\log_\sigma n} + \mathrm{occ}\right)$
- What is the best upper bound?
- Can we do better than O(n)?

# A Simple Algorithm: Use a lots of space




- Idea: Traverse Q from left-to-right reading r ≈ #number of characters per word at a time.

- At each step compute the *longest prefix* of P matching the current suffix of Q. (slightly more information needed to also report occurrences).

- To do step in constant time store for each prefix of P and each combination of r characters a pointer to the next prefix. (called a “super-alphabet technique” [Fre02]).

| | | $r = \epsilon \log_\sigma n$ |
|---|---|---|
| Space: | $O(m\sigma^r)$ | $O(mn^\epsilon)$ |
| Time: | $O(n/r + m\sigma^r + \text{occ})$ | $O(n/\log_\sigma n + mn^\epsilon + \text{occ})$ |

# Complexities

| Time | Space | |
|---|---|---|
| $O\left(\frac{n}{r} + m\sigma^r + \mathrm{occ}\right)$ | $O(m\sigma^r)$ | Simple |
| $O\left(\frac{n}{\log_\sigma n} + mn^\varepsilon + \mathrm{occ}\right)$ | $O(mn^\varepsilon)$ | |
| $O\left(\frac{n}{r} + m + \sigma^r + \mathrm{occ}\right)$ | $O(m + \sigma^r)$ | This paper |
| $O\left(\frac{n}{\log_\sigma n} + m + \mathrm{occ}\right)$ | $O(m + n^\varepsilon)$ | |

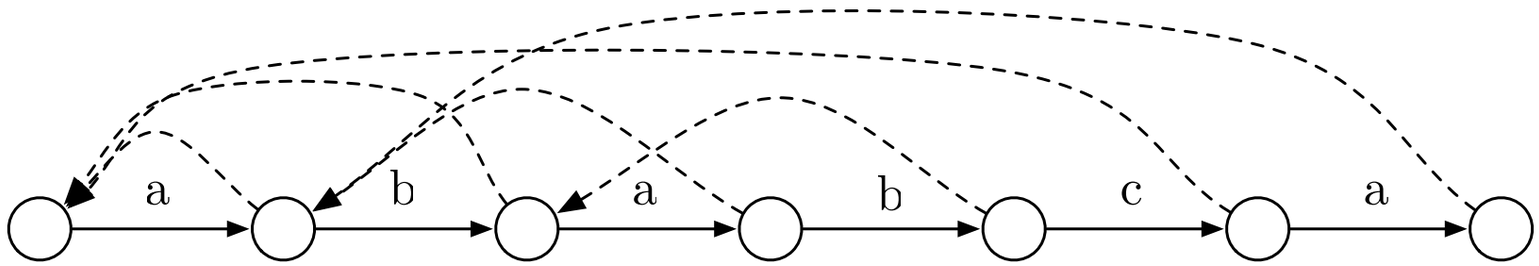
# Algorithm Overview

- Based on the Knuth-Morris-Pratt automaton.

- The “Four-Russian Technique” (divide and tabulate) with new twists.

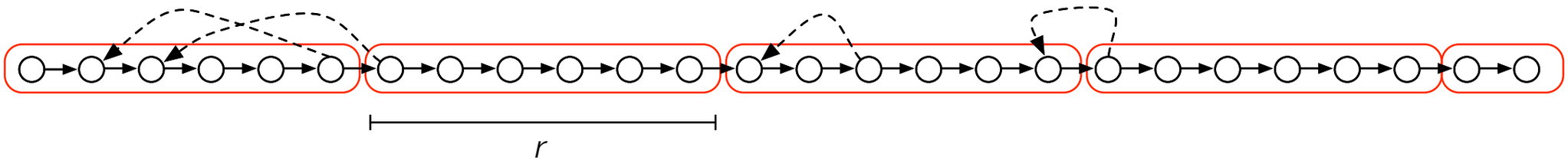
# The Knuth-Morris-Pratt Automaton

$P = \texttt{ababca}$




KMP($P$)

# A First Attempt: The Four-Russian Technique



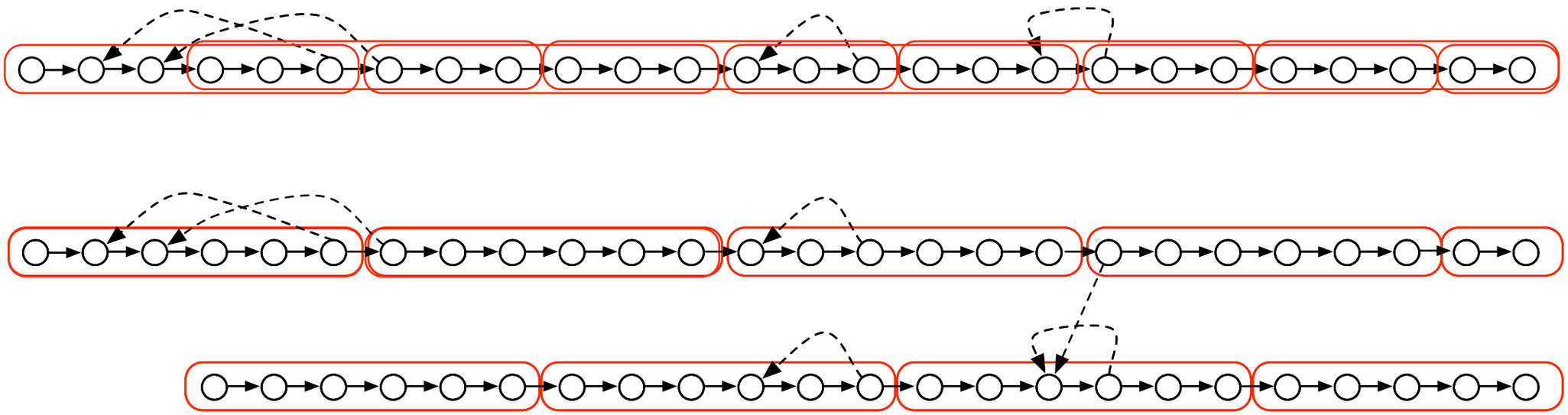

- Tabulate information for each subautomata to allow up to r *internal* transitions in constant time.

- Simulate by doing *external* transitions explicitly and internal transitions using the tabulated information.

- Issue 1: Too many external transitions.

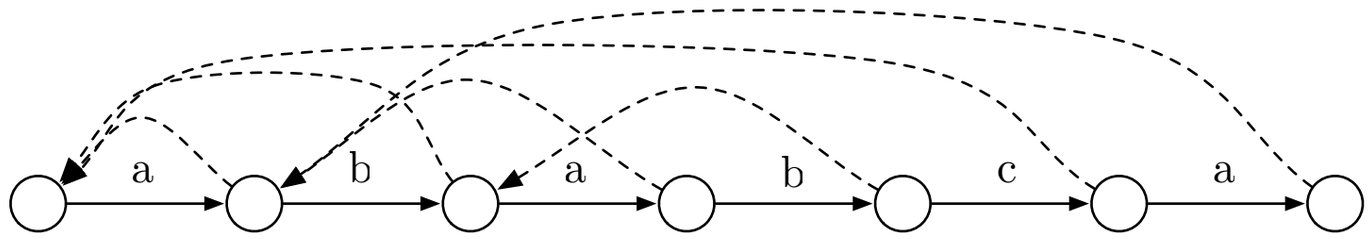
- Issue 2: Representing subautomata compactly.

# Fixing 1: Too Many External Transitions



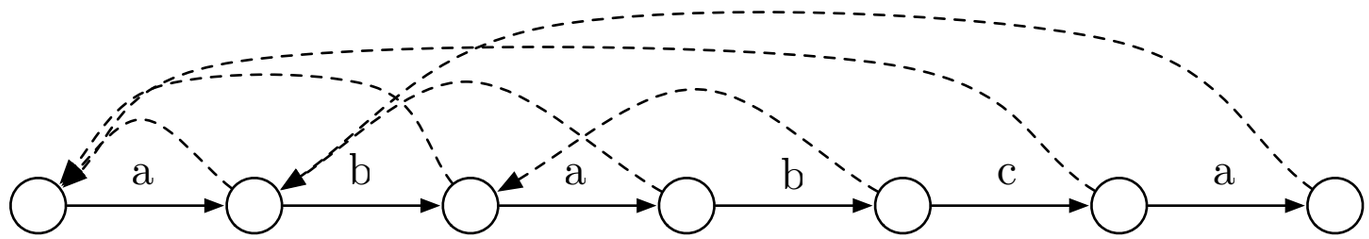
At most O(n/r) external transitions in simulation of Q

# Fixing 2: Representing Subautomata Compactly



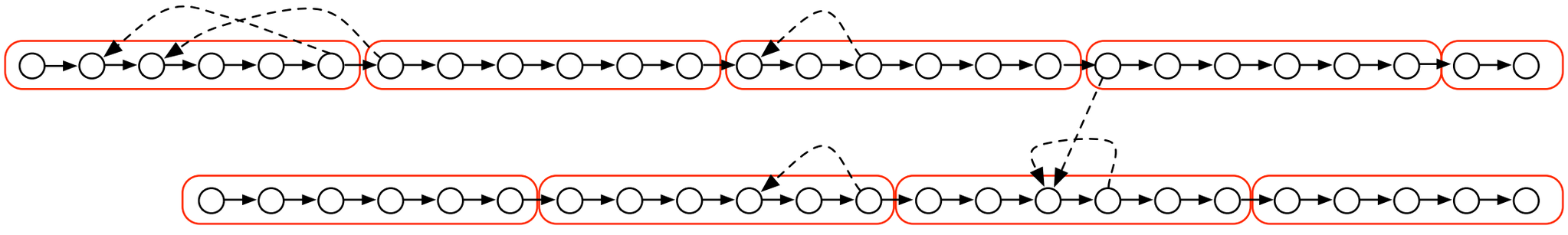

- We want to encode an arbitrary subautomaton of KMP(P) in O(r log σ) bits.
- Non-failure transitions encoded by the sequence of labels in O(r log σ) bits.
- How about the failure transitions in S?

# Fixing 2: Representing Subautomata Compactly




- Storing r explicit pointers uses Ω(r log r) bits.
- Instead we exploit a basic property of KMP-automata: In any subautomaton failure transition endpoints increase by at most 1 between consecutive states.
- => Total increase at most r => Total decrease at most O(r).
- => We can difference encode all failure transitions with O(r) bits.

# Putting the Pieces together



- Construct segment automaton and tabulate transitions for subautomata using the compact encoding.

- Simulate the segment automaton. Each external transitions is done explicitly. Internal transitions are done using the tabulation.

- Complexity:

| | | $r = \epsilon \log_\sigma n$ |
|---|---|---|
| Space: | $O(m + \sigma^r)$ | $O(m + n^\epsilon)$ |
| Time: | $O(n/r + m + \sigma^r + \mathrm{occ})$ | $O(n/\log_\sigma n + m + \mathrm{occ})$ |

# Directions

- Packed string matching:
    - Practical?
    - Long word lengths?
        - Multi-string matching?
- Packed problems appear everywhere.
    - Longer word lengths => more packing.
    - Most packed problems are not well-solved.