Finding Patterns in Trees and Strings

Philip Bille

Agenda

- Background for PhD
- Tree Matching
    - Tree Inclusion Problem
- String Matching
    - Regular Expression Matching Problem
- Core Techniques and Future Research
- Short Break
- Question Session

Background For PhD

- Worked on data structures.
    - Labeling Schemes for Small Distances in Trees.
      Stephen Alstrup, Philip Bille, and Theis Rauhe.
      SIAM J. Disc. Math., 2005 and SODA 2003.
- PhD funded by EU-project “Deep Structure, Singularities, and Computer Vision” working on tree matching problems.

- A Survey on Tree Edit Distance and Related Problems. Philip Bille. Theoret. Comp. Sci., 2005.

- The Tree Inclusion Problem: In Optimal Space and Faster. Philip and Inge Li Gørtz. ICALP 2005.

- Matching Subsequences in Trees. Philip Bille and Inge Li Gørtz. CIAC 2006.

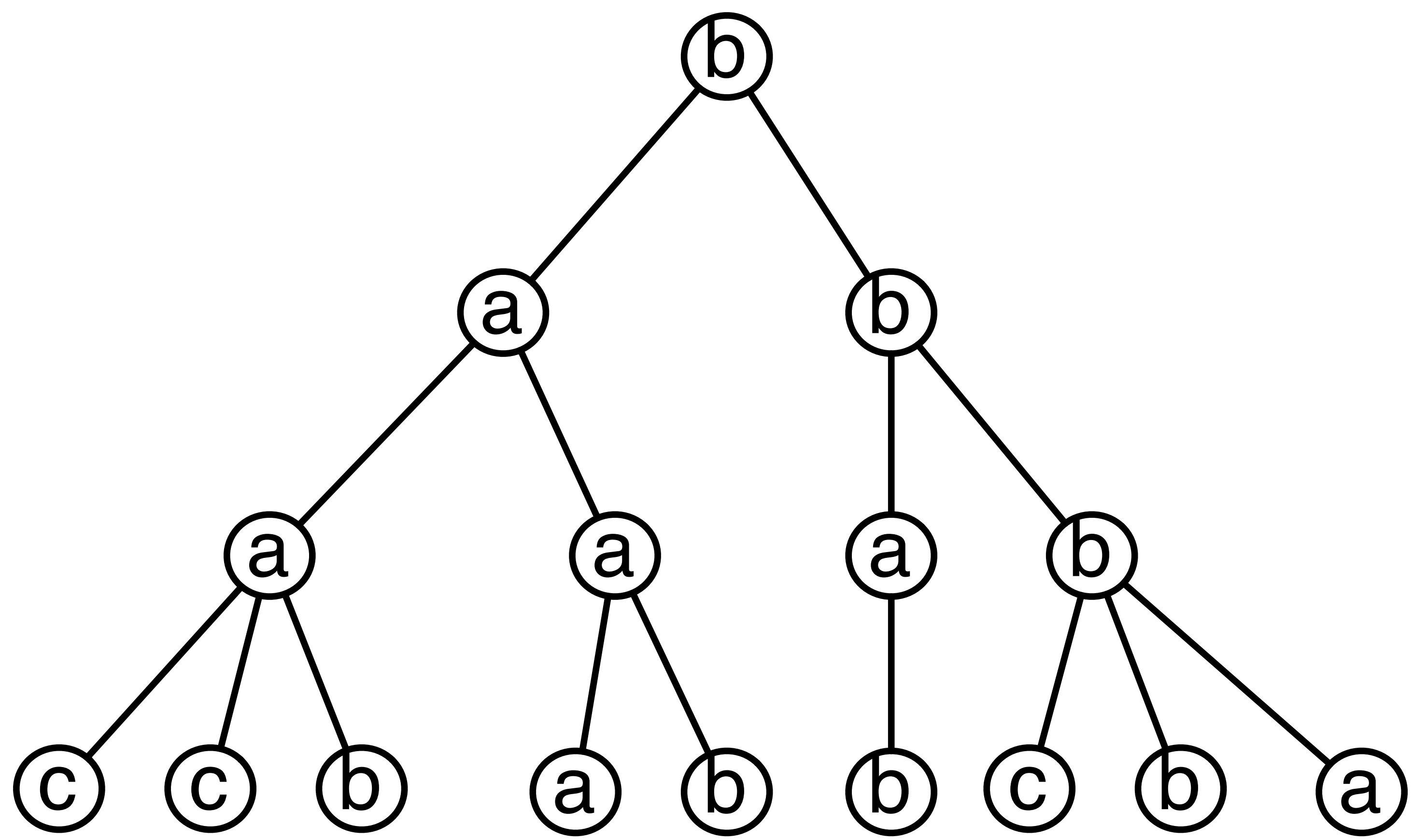
- From a 2D Shape to a String Structure using the Symmetry Set. Arjan Kuijper, Ole Fogh Olsen, Peter Giblin, Philip Bille, and Mads Nielsen. ECCV 2004.

- Matching 2D Shapes using their Symmetry Sets. Arjan Kuijper, Ole Fogh Olsen, Peter Giblin, and Philip Bille. ICPR 2006.

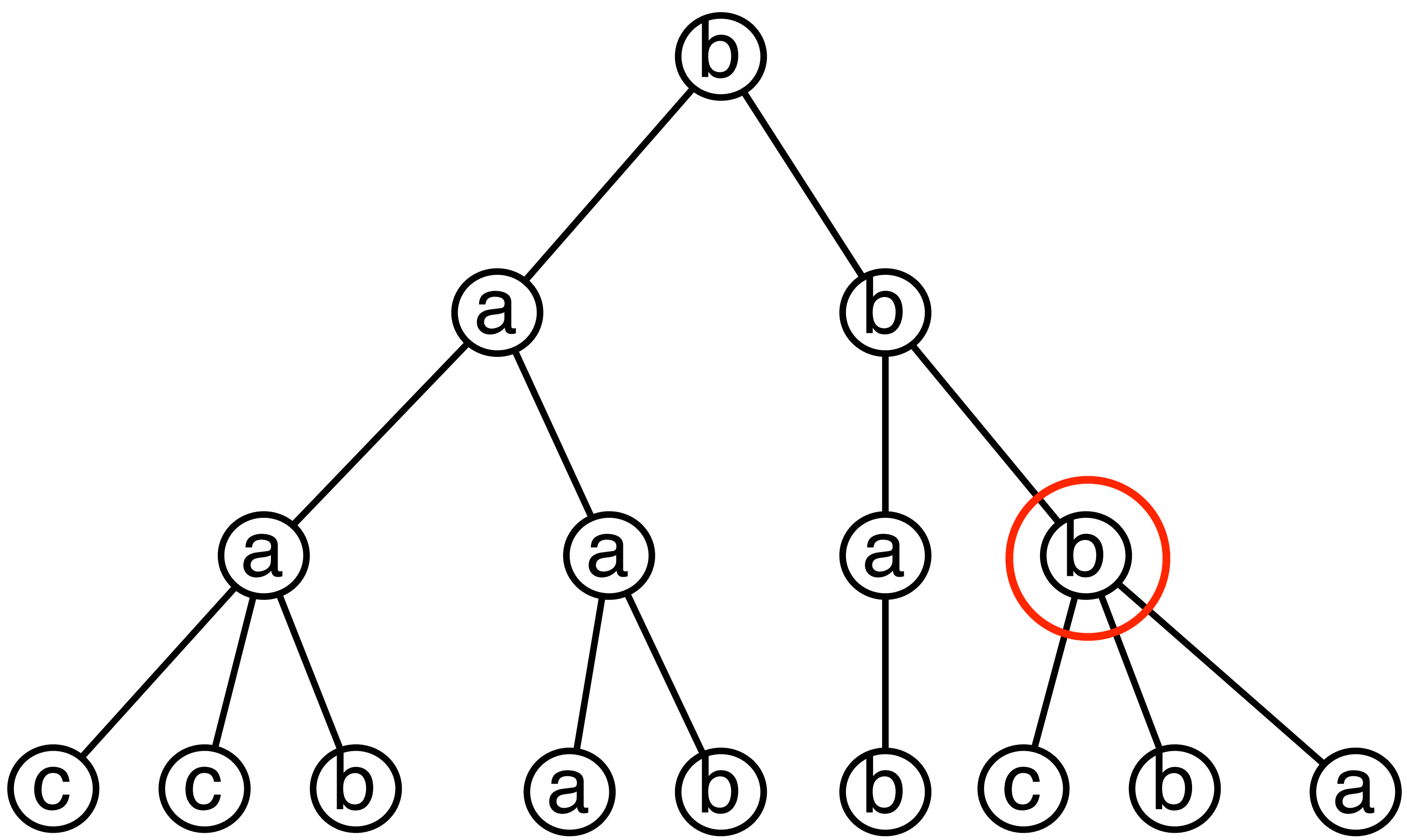
Basic Setup

- Trees are rooted, labeled, and ordered.
    - Rooted: A specific node is designated to be the root.
    - Labeled: Each node is assigned a label from an alphabet Σ.
    - Ordered: There is a given left-to-right ordering among siblings.
- We compare trees by deleting nodes.

Deleting a Node



Deleting a Node



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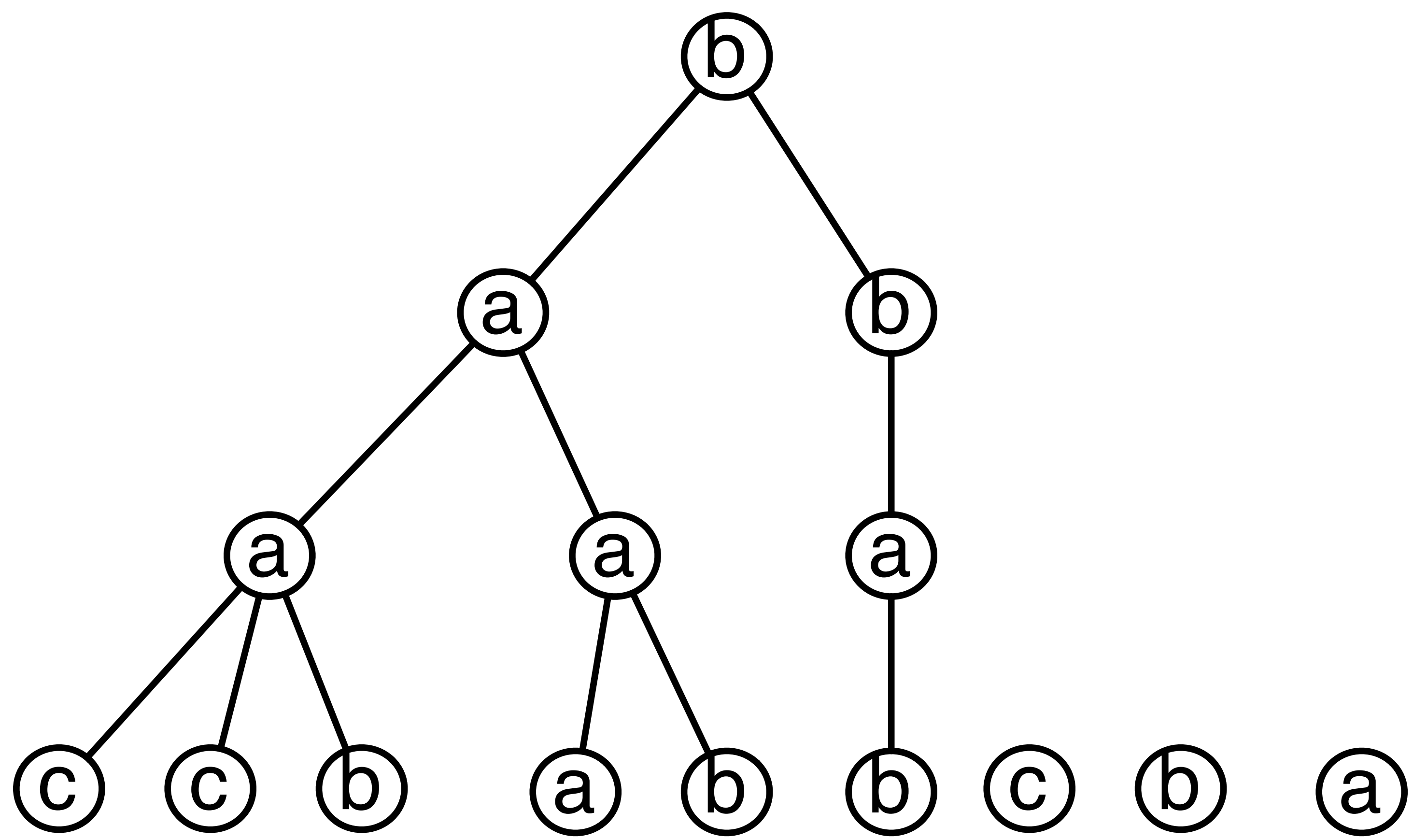

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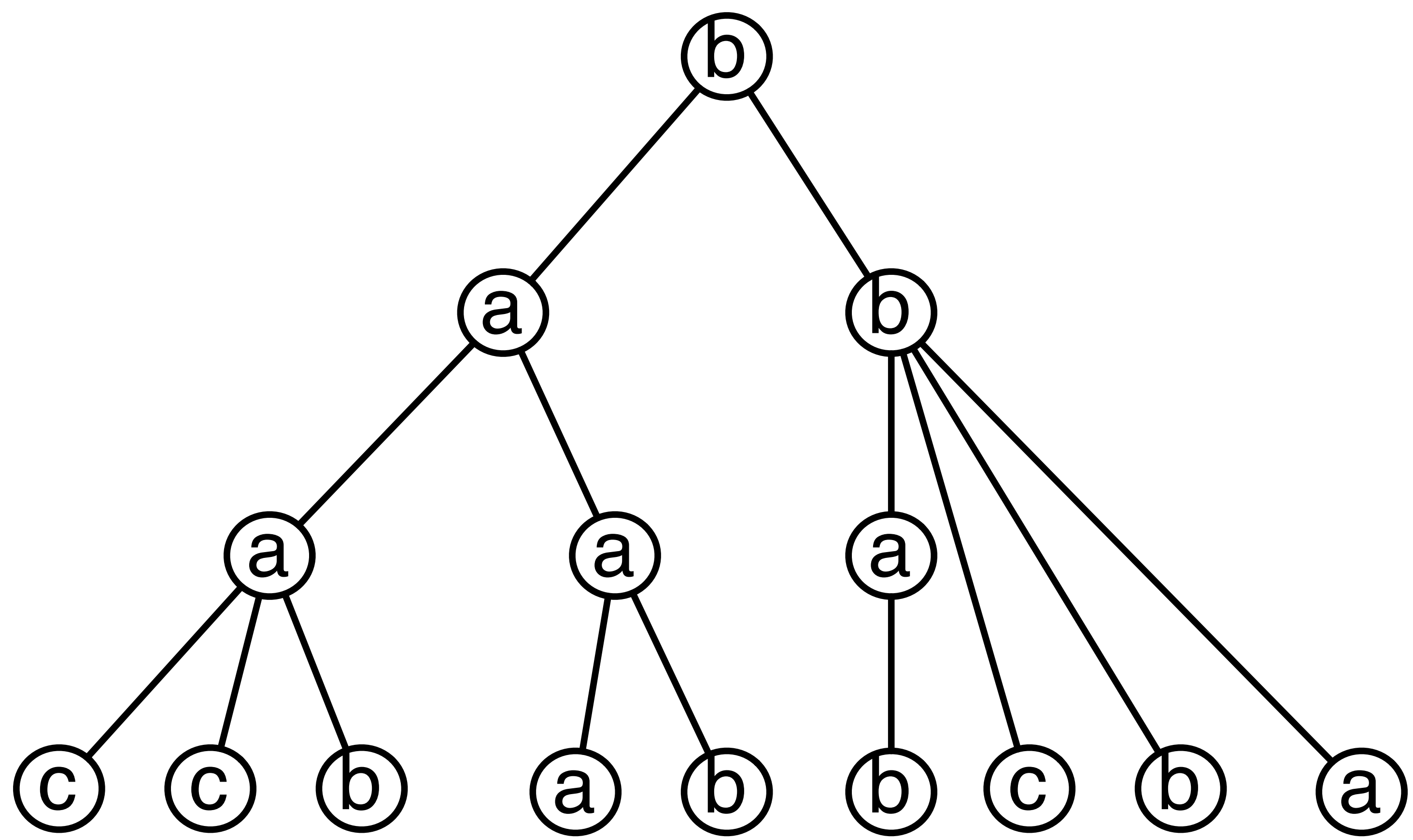


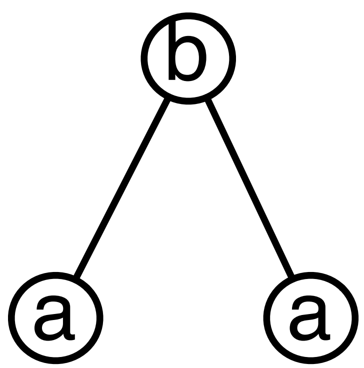
Tree Inclusion

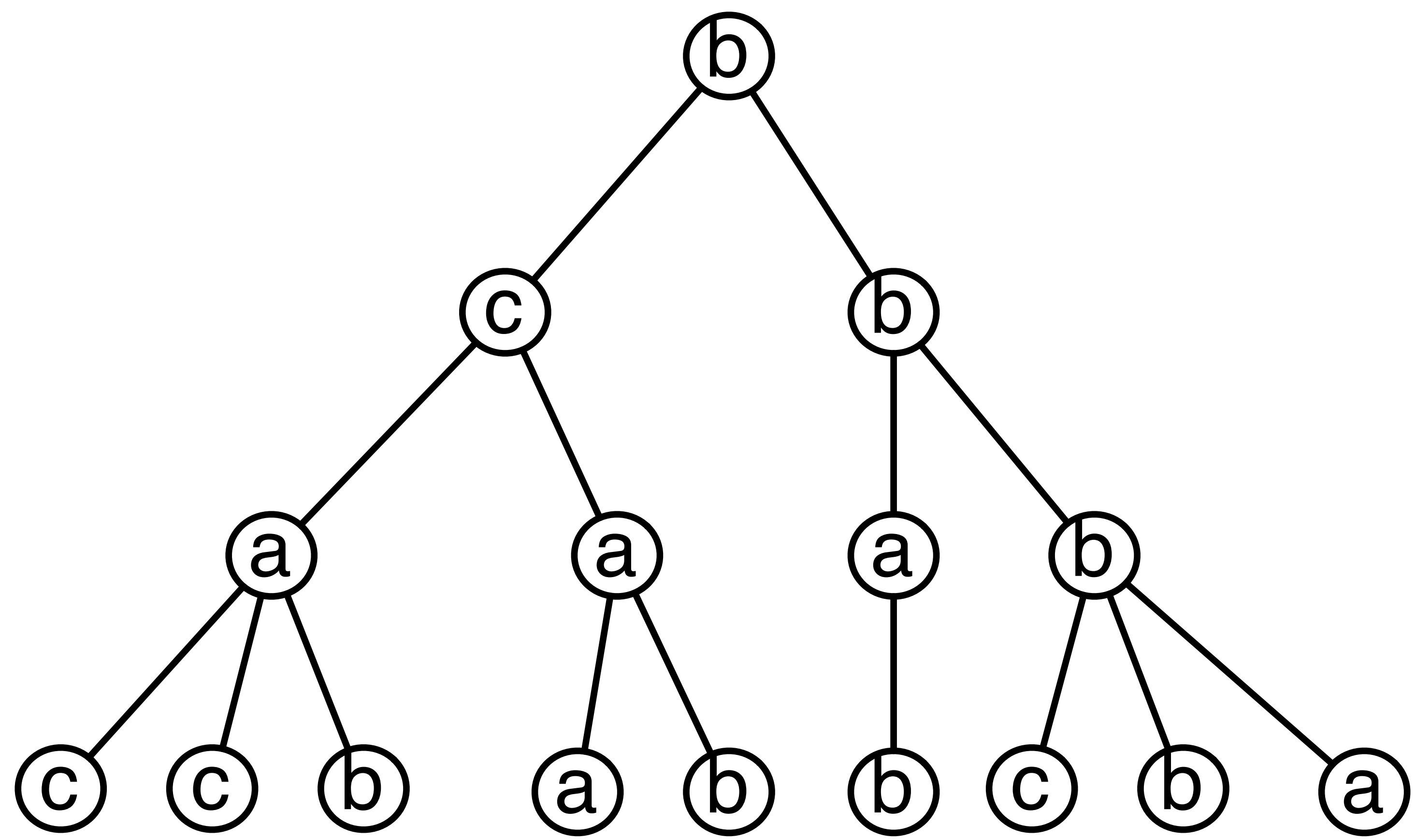
- P is *included* in T if P can be obtained from T by deleting nodes in T.

- P is *minimally included* in T if P is not included in any proper subtree of T.

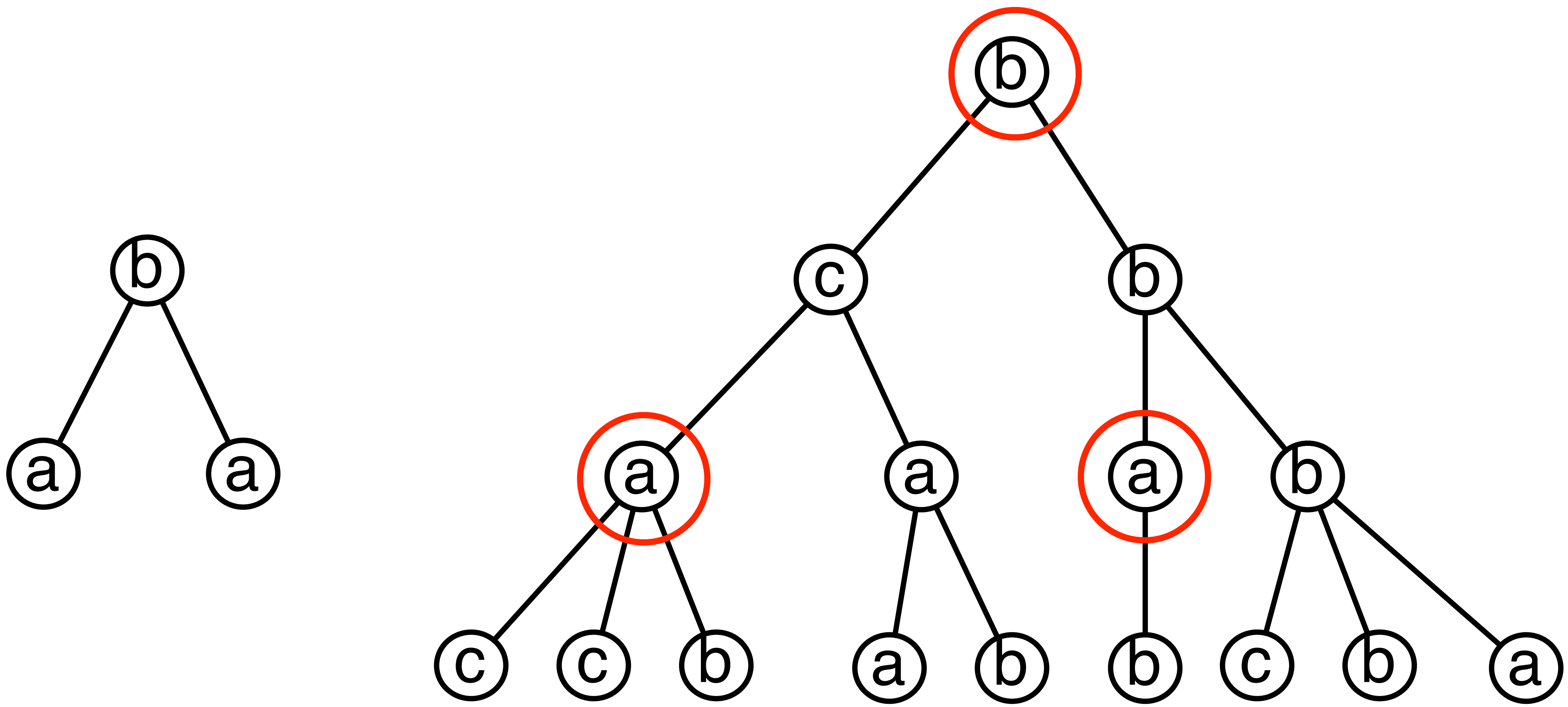
- The *tree inclusion problem* is to decide if P is included in T, and if so, compute all subtrees of T which minimally include P.

Example

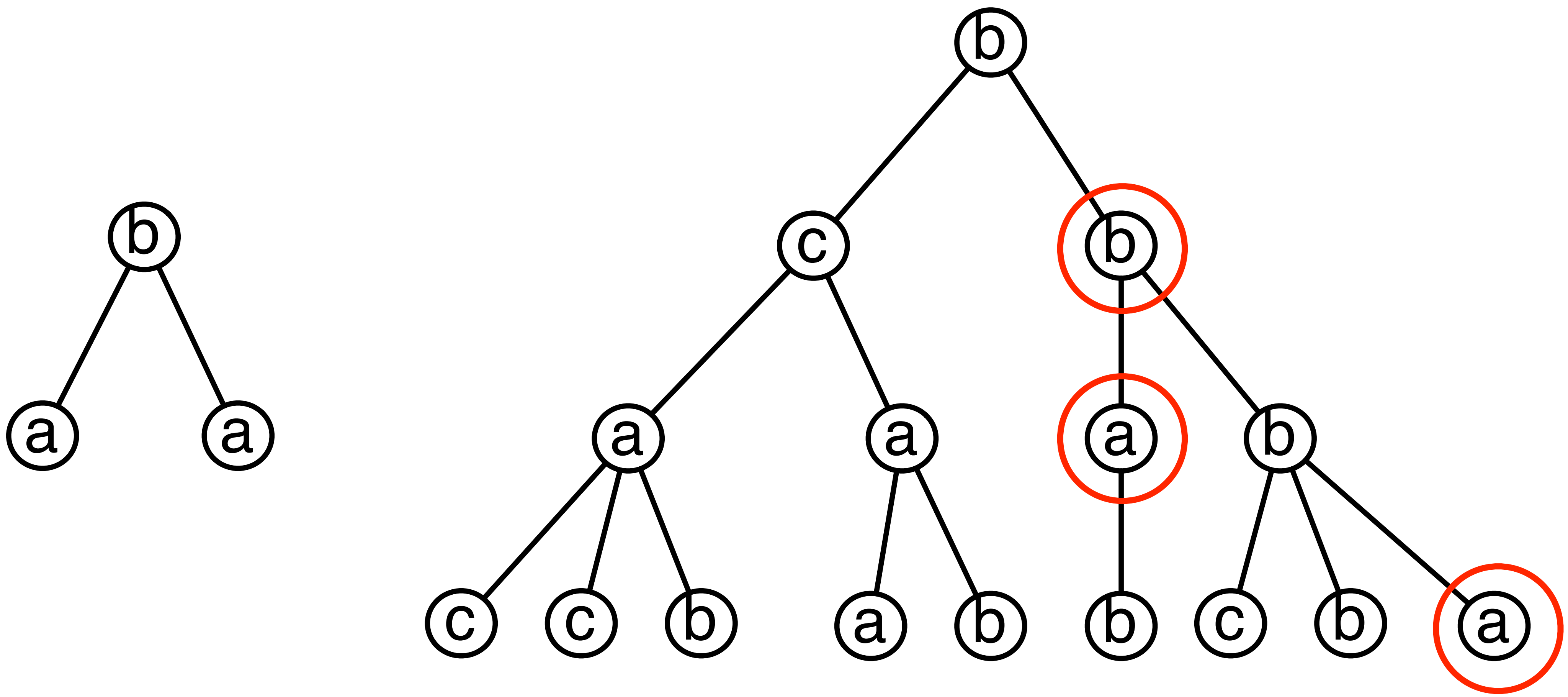




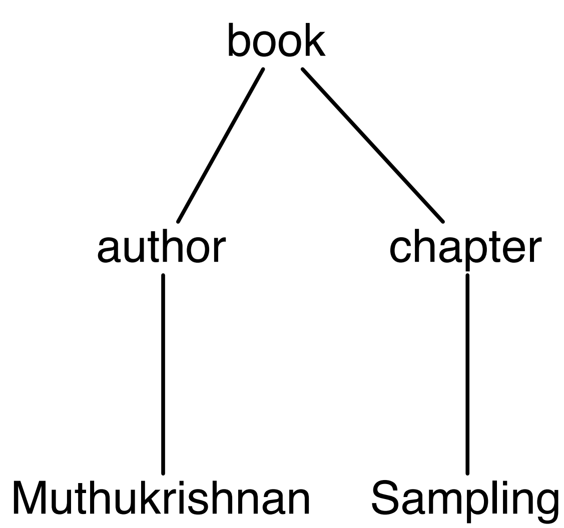
Example

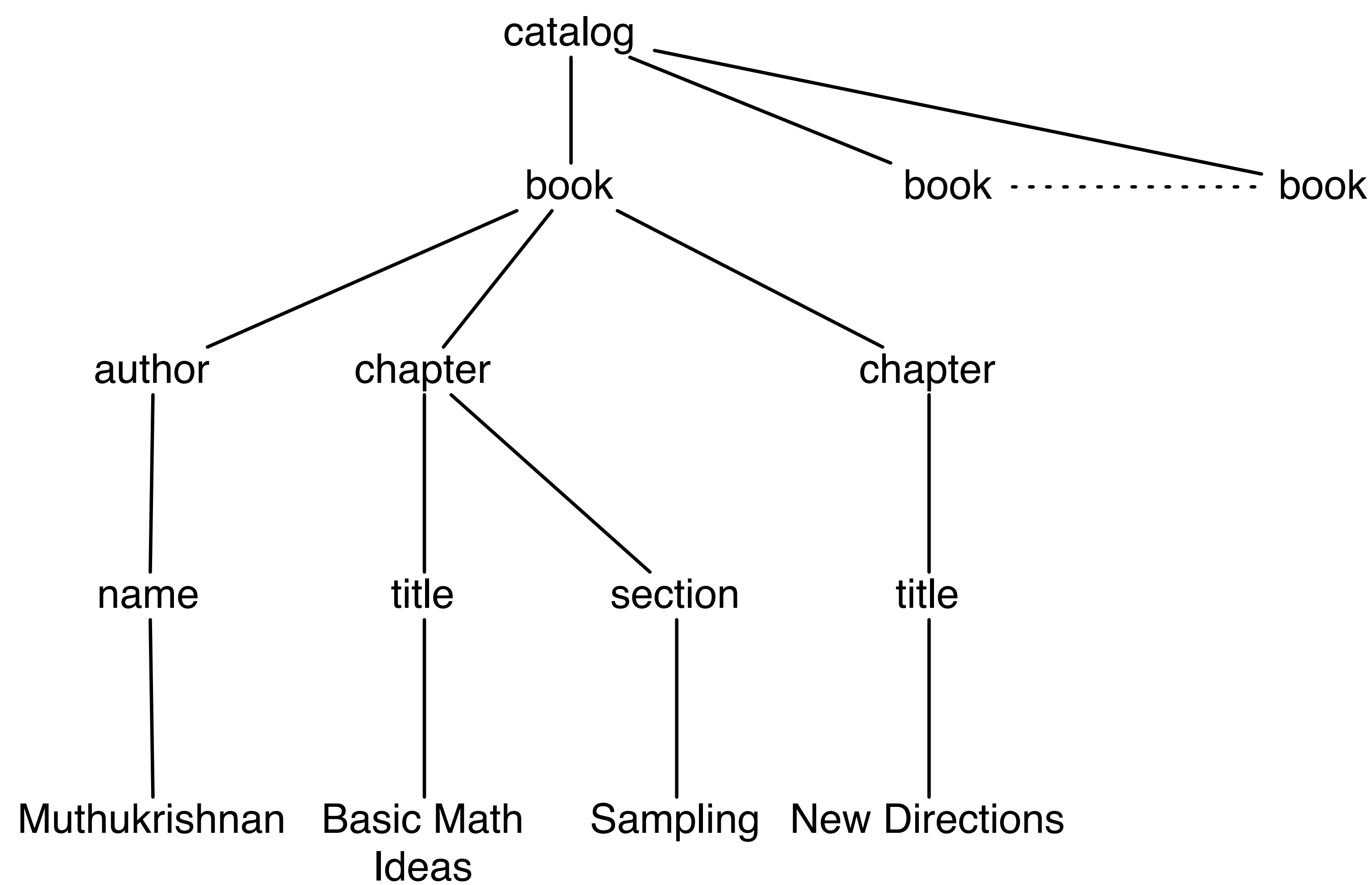


Example



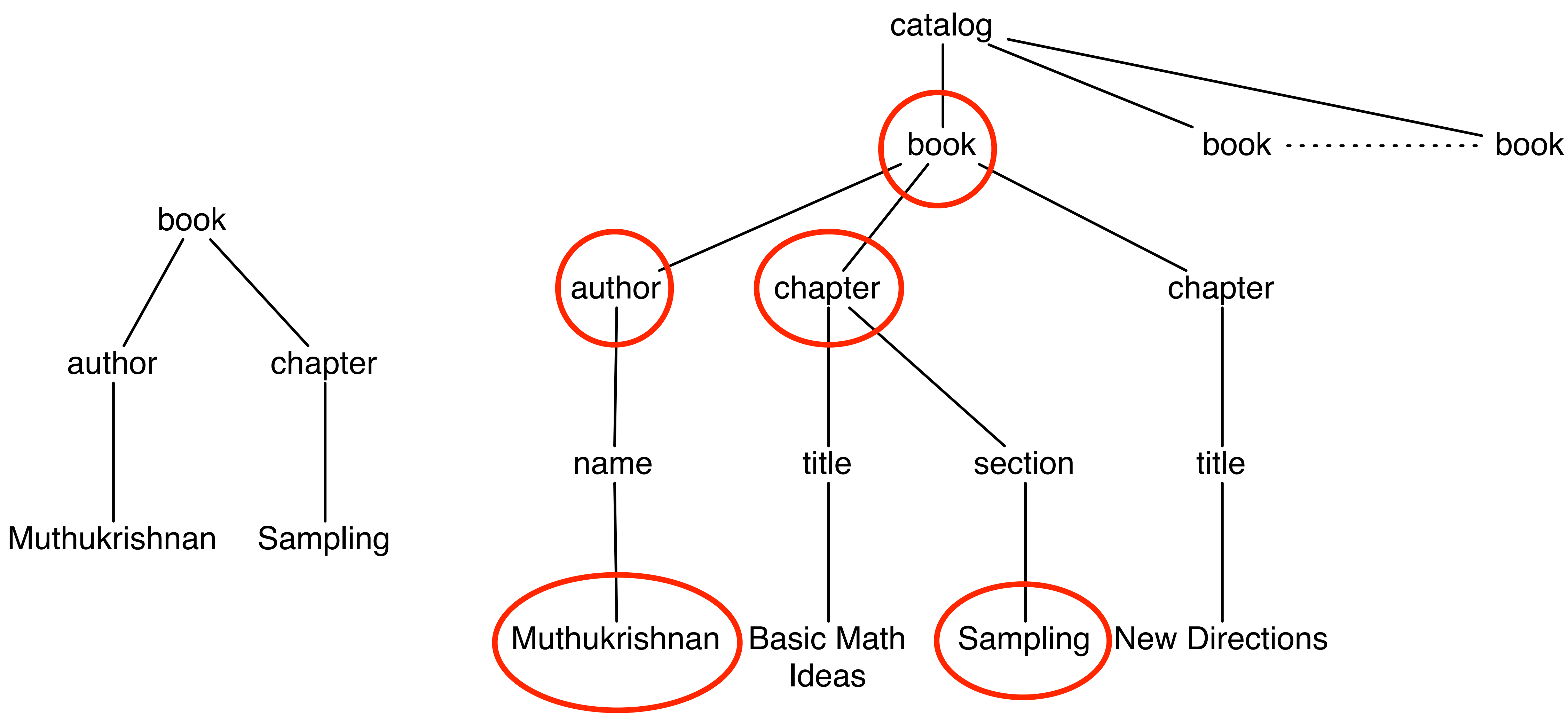
Application: Querying XML Data Bases





Query: “Find all books written by Muthukrishnan with a chapter that has something to do with sampling”.

Application: Querying XML Data Bases



Query: "Find all books written by Muthukrishnan with a chapter that has something to do with sampling".

Results

Time	Space	Ref
$O(n_P n_T)$	$O(n_P n_T)$	[KM92]
$O(l_P n_T)$	$O(l_P \min(d_T, l_T))$	[Che98]
$O(l_P n_T)$	$O(n_P + n_T)$	[Here]
$O(n_P l_T \log\log n_T + n_T)$		
$O\left(\frac{n_P n_T}{\log n_T} + n_T \log n_T\right)$		

Practical Implications

- Significant space reduction:
  - Feasible to query large XML databases.
  - Faster query time since more computation can be kept in main memory.

Algorithm Overview

- Reduce tree inclusion to *tree embedding.*
- Compute tree embeddings using a simple general framework.
- Implement the framework in 3 different ways to get the results.

Tree Inclusion and Embeddings

- An injective function f from the nodes of P to the nodes of T is an *embedding* if for all nodes v and w:

  1. $\mathrm{label}(v) = \mathrm{label}(f(v))$,

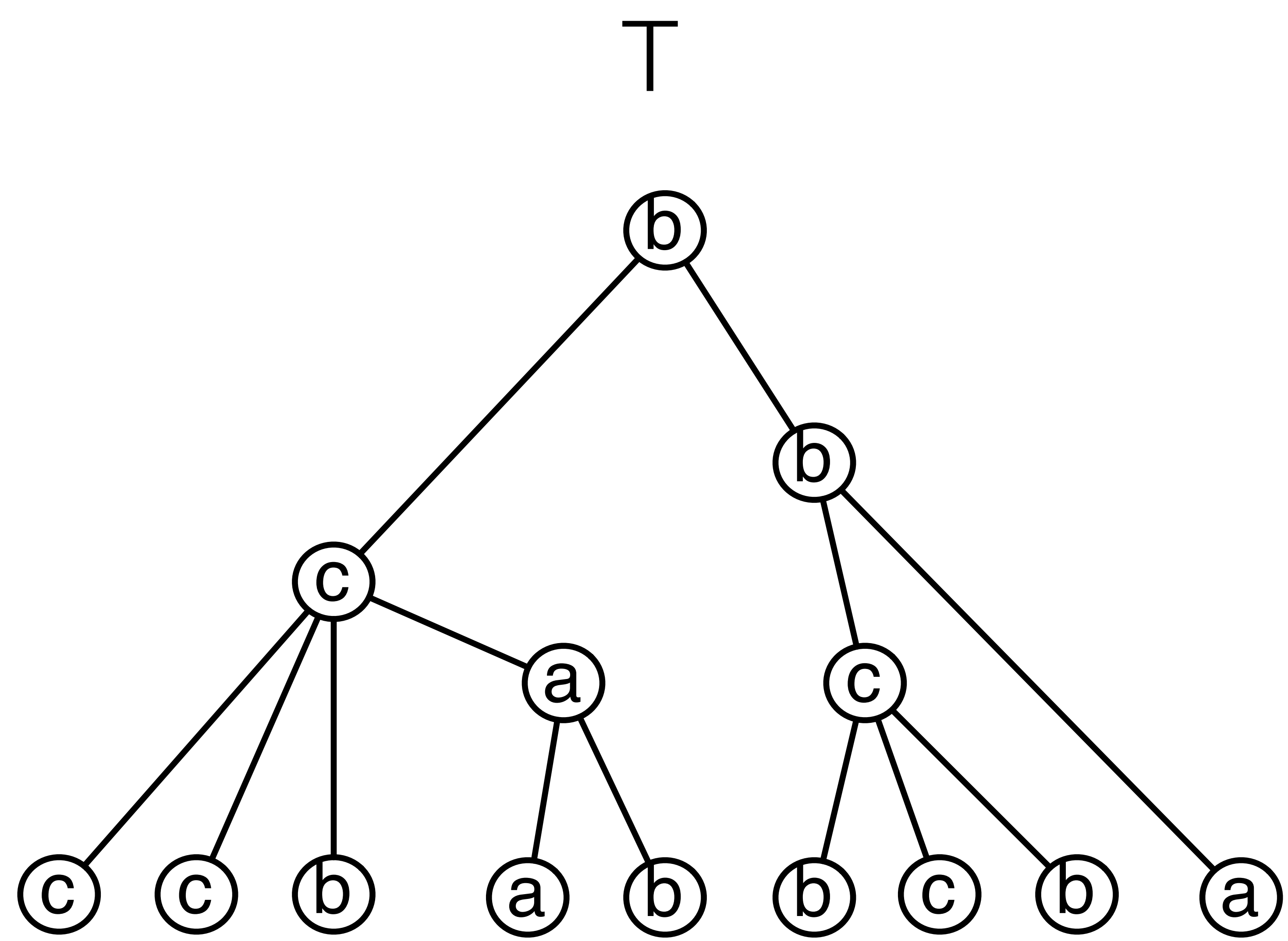
  2. v is a proper ancestor of w if and only if $f(v)$ is a proper ancestor of $f(w)$,

  3. v is to the left of w if and only if $f(v)$ is to the left of $f(w)$.

- P is included in T if and only if there is an embedding from P to T.

- P is minimally included in T if and only if there is an embedding from P to T and P cannot be embedded in a proper subtree of T.

Computing Embeddings: P is a Path



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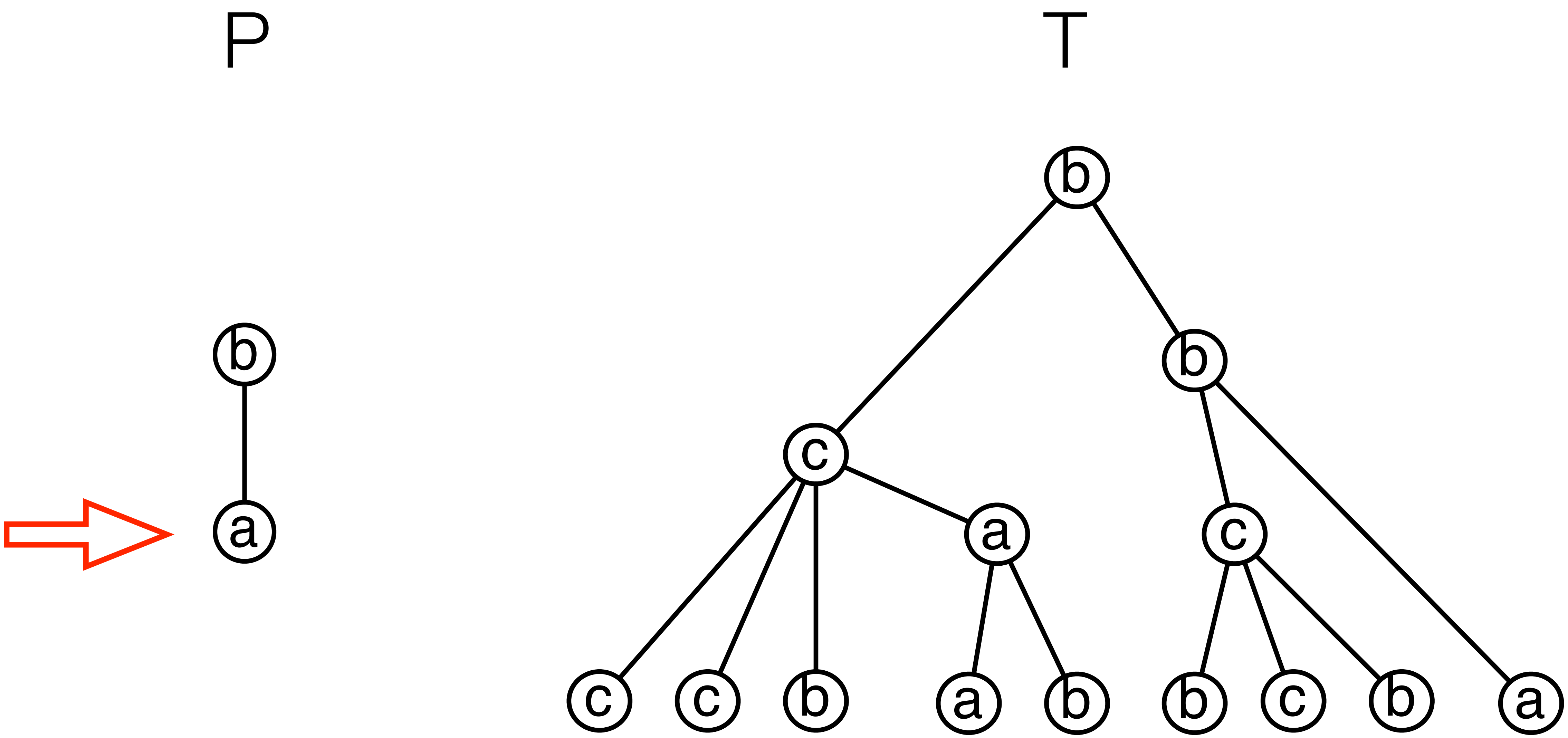

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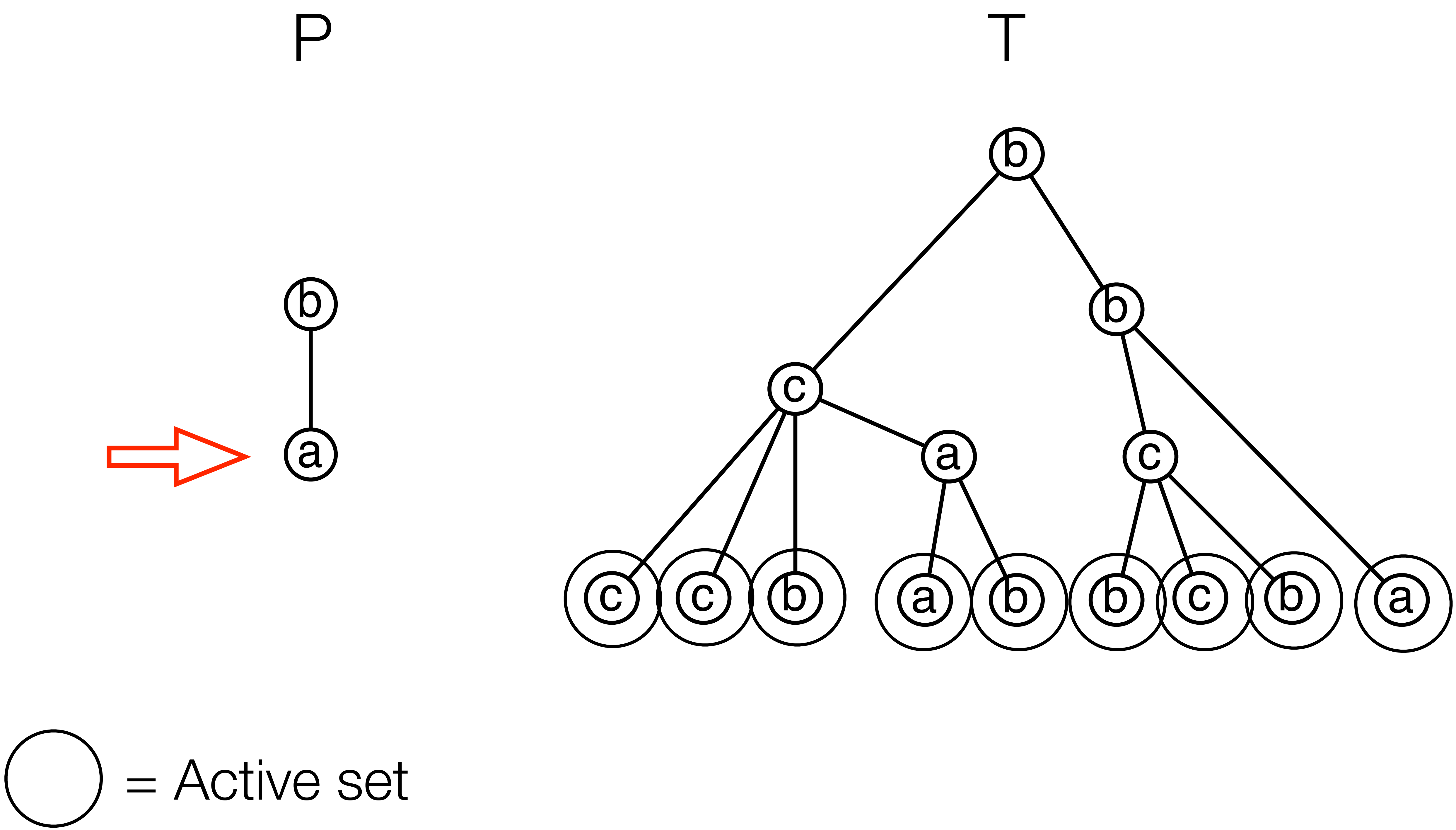

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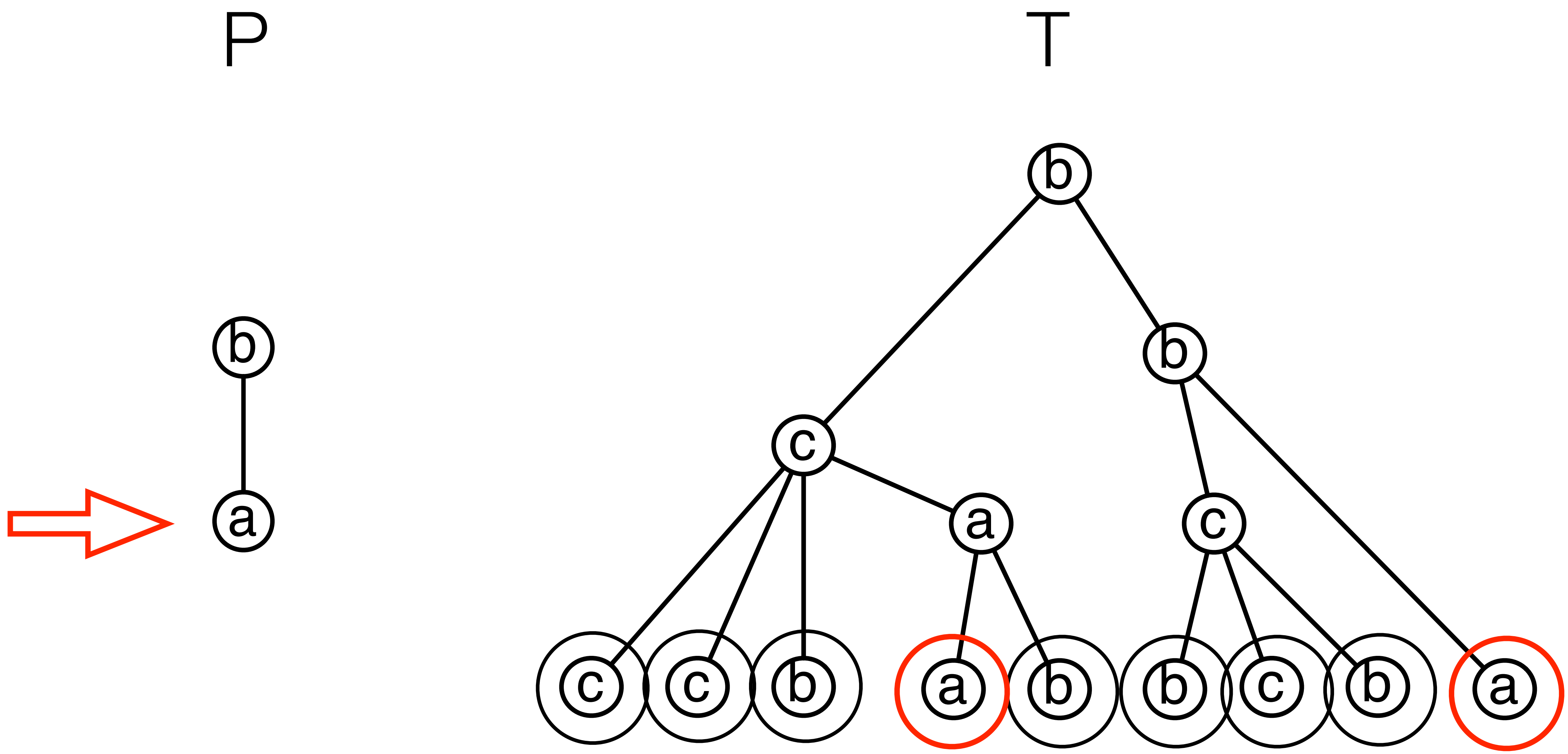

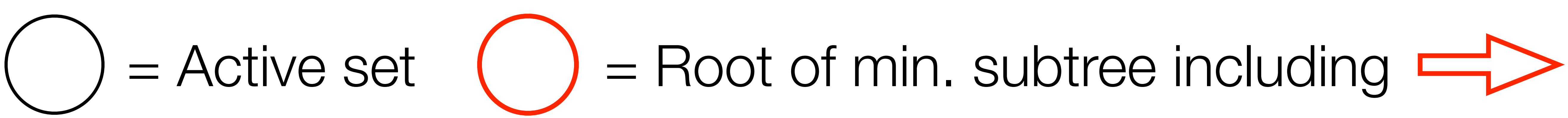

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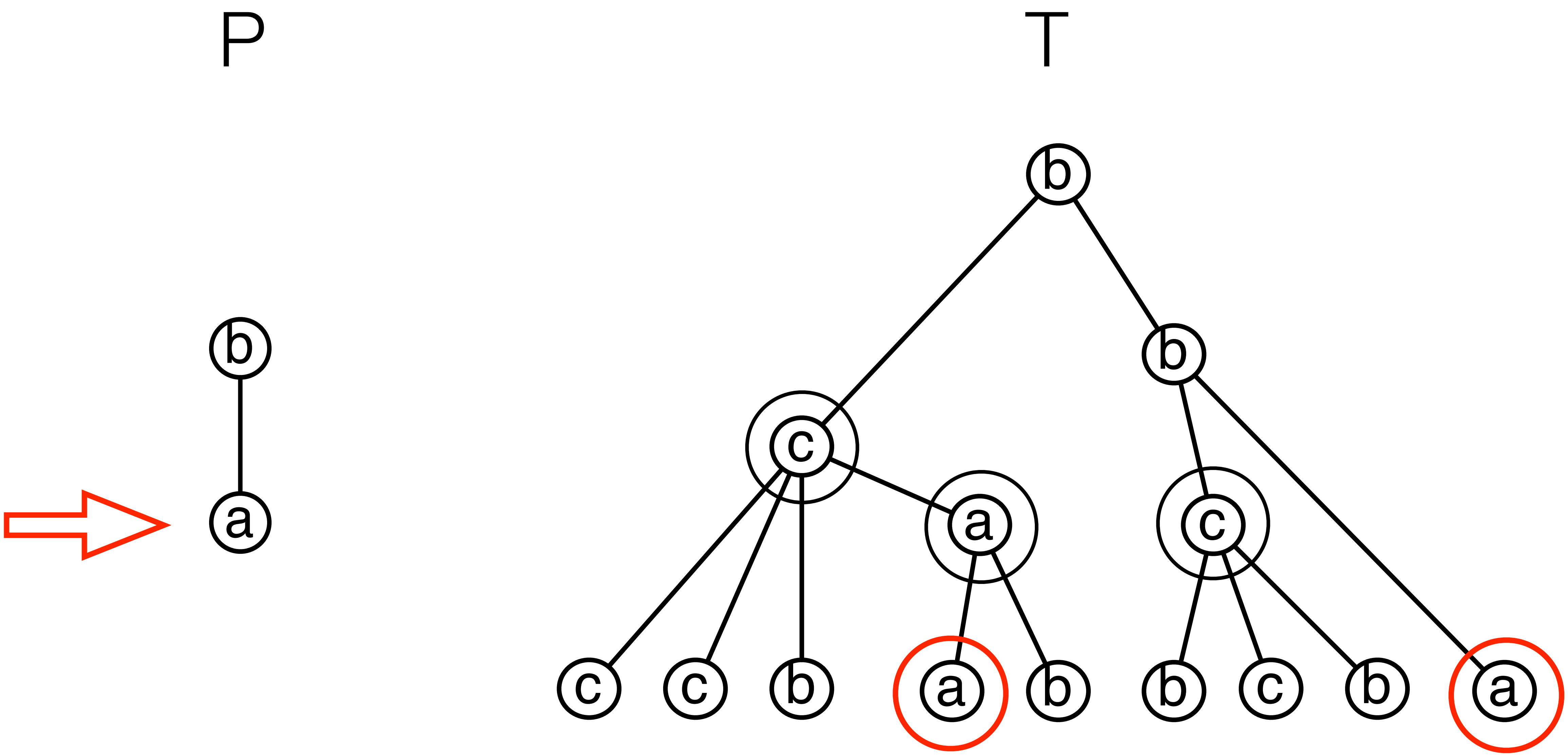
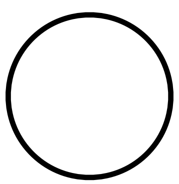
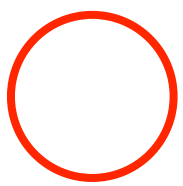
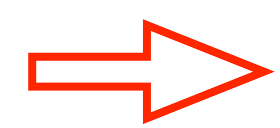



= Active set


= Root of min. subtree including



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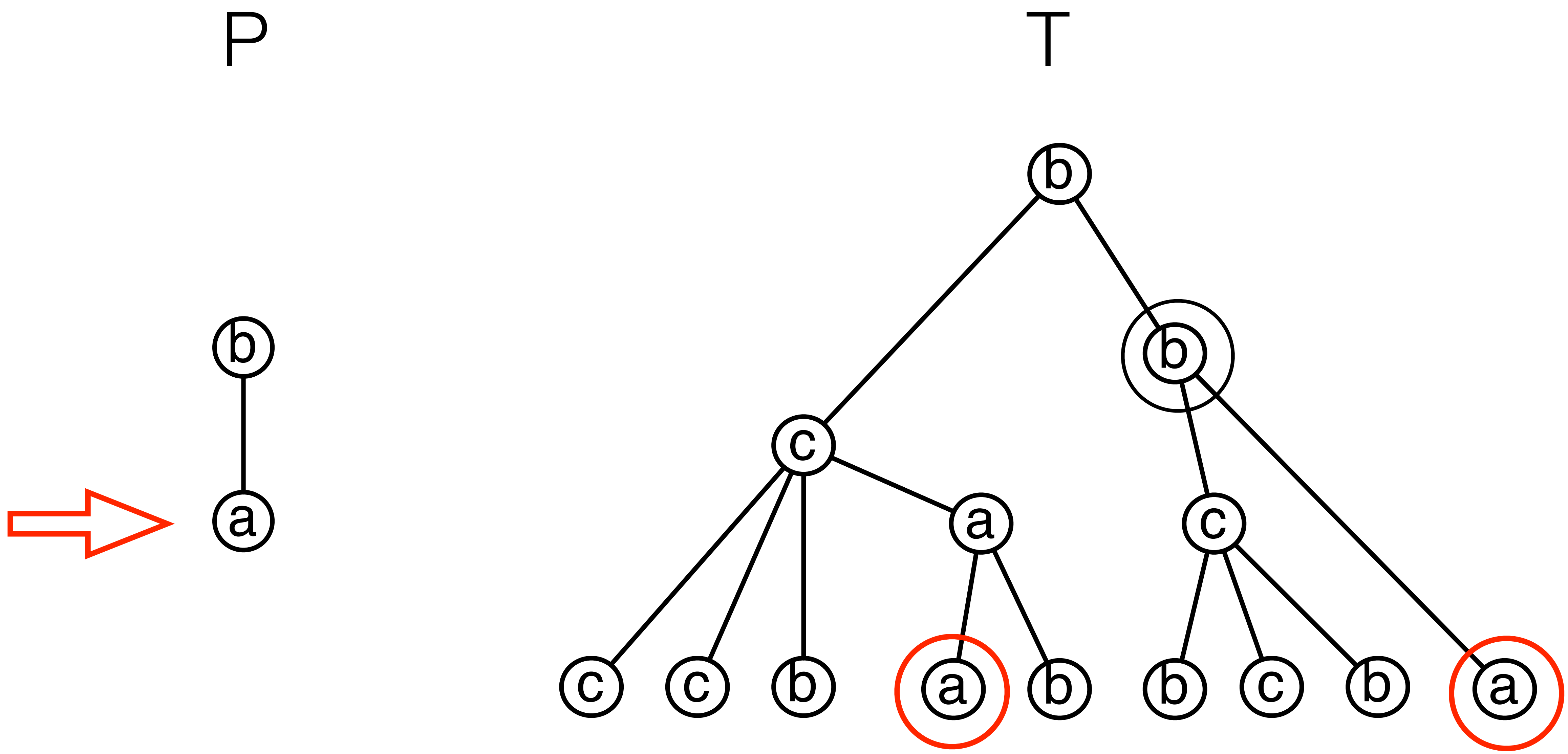
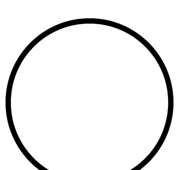
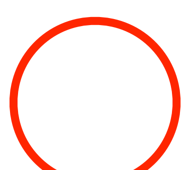



= Active set

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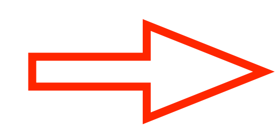

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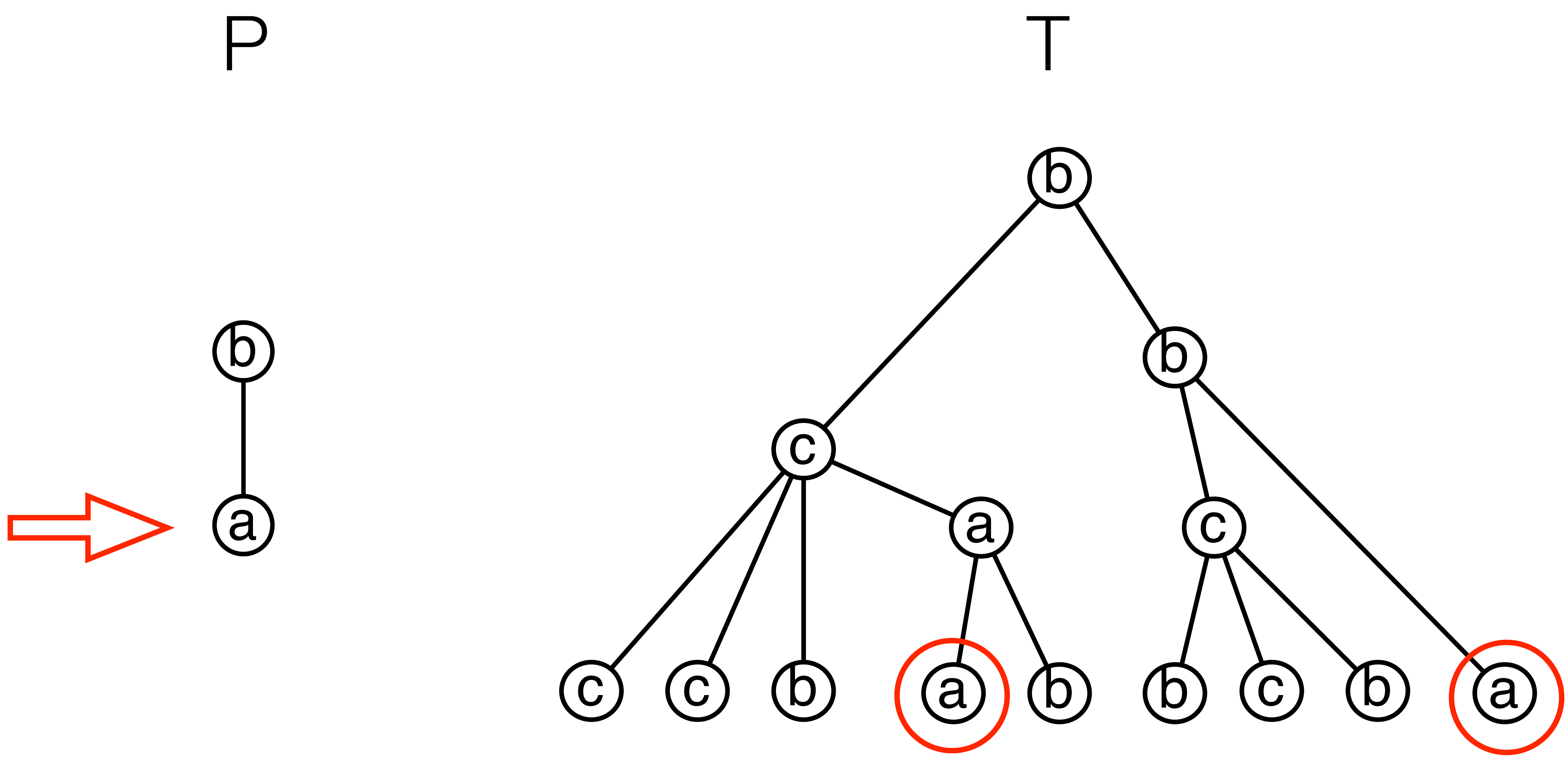
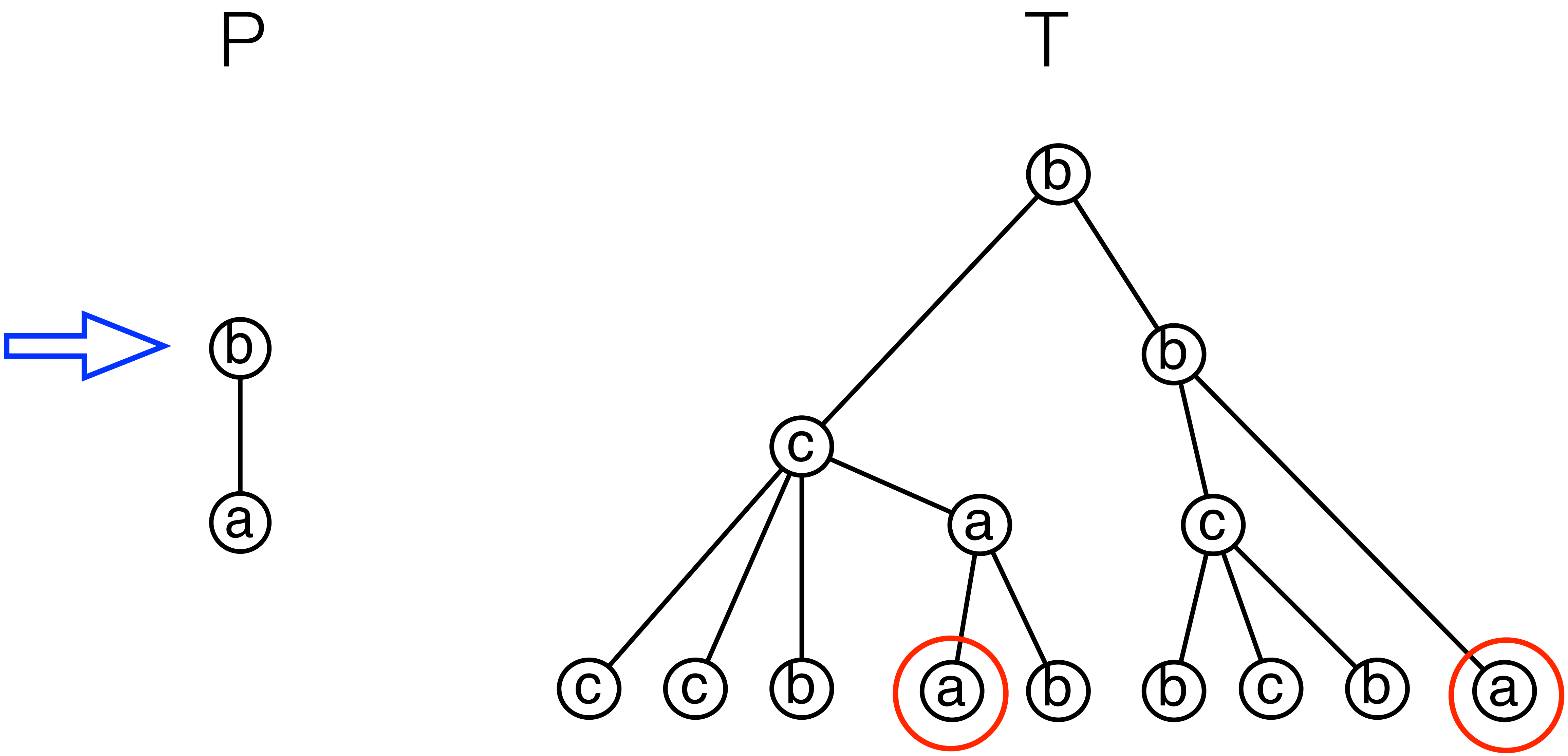


= Active set = Root of min. subtree including

Computing Embeddings: P is a Path



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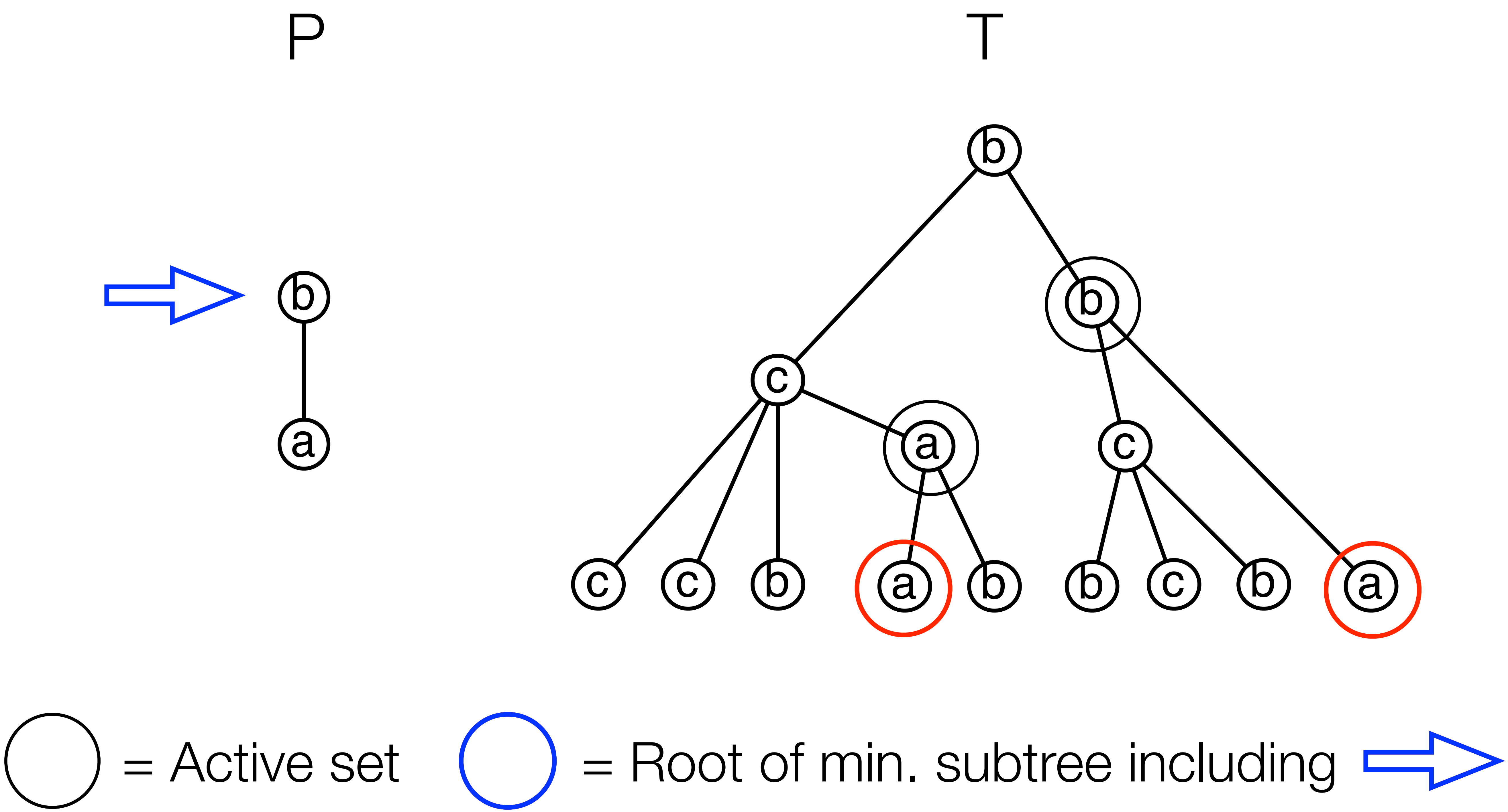

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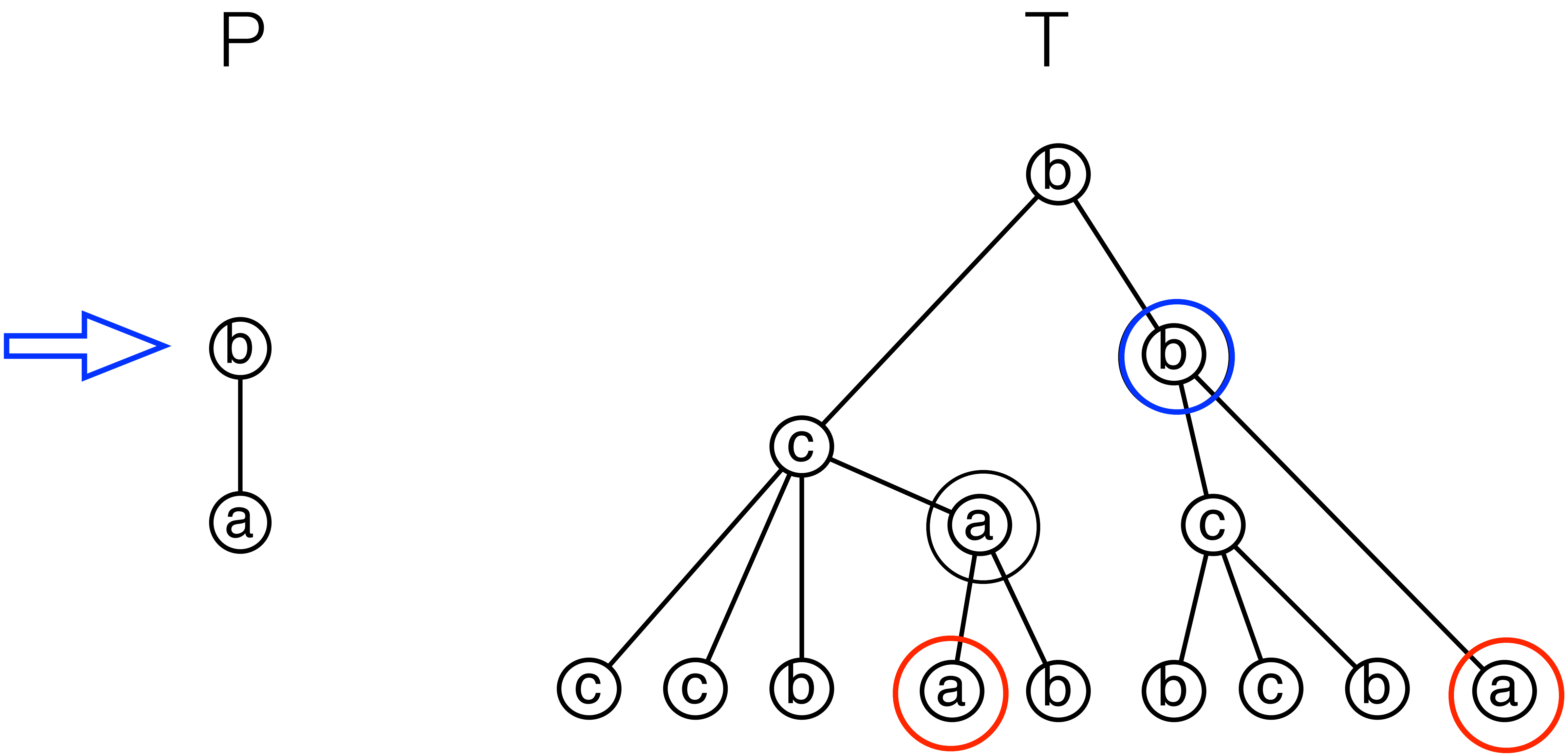
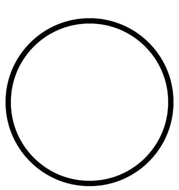
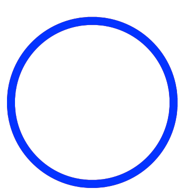
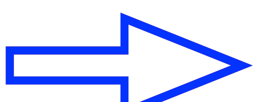



= Active set


= Root of min. subtree including



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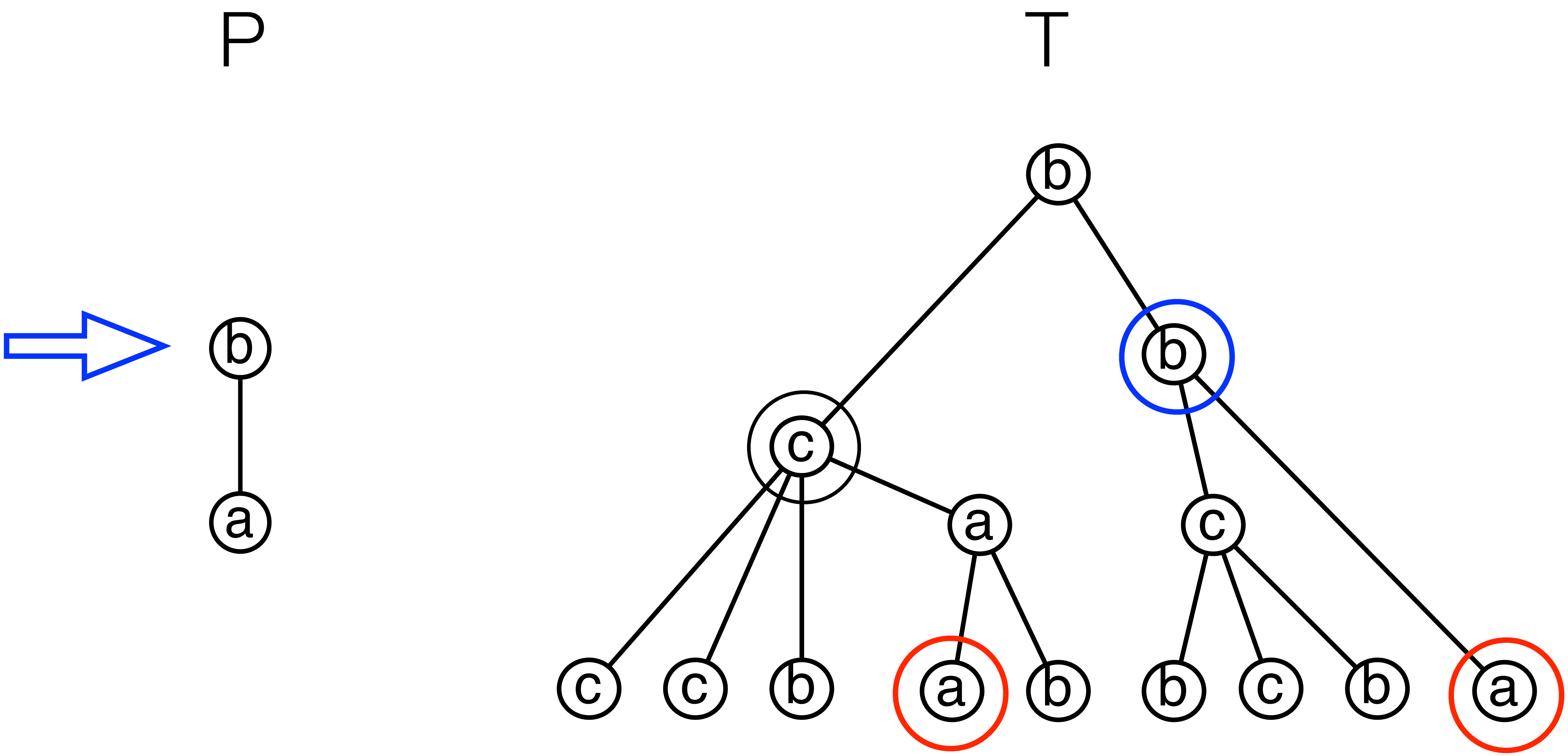

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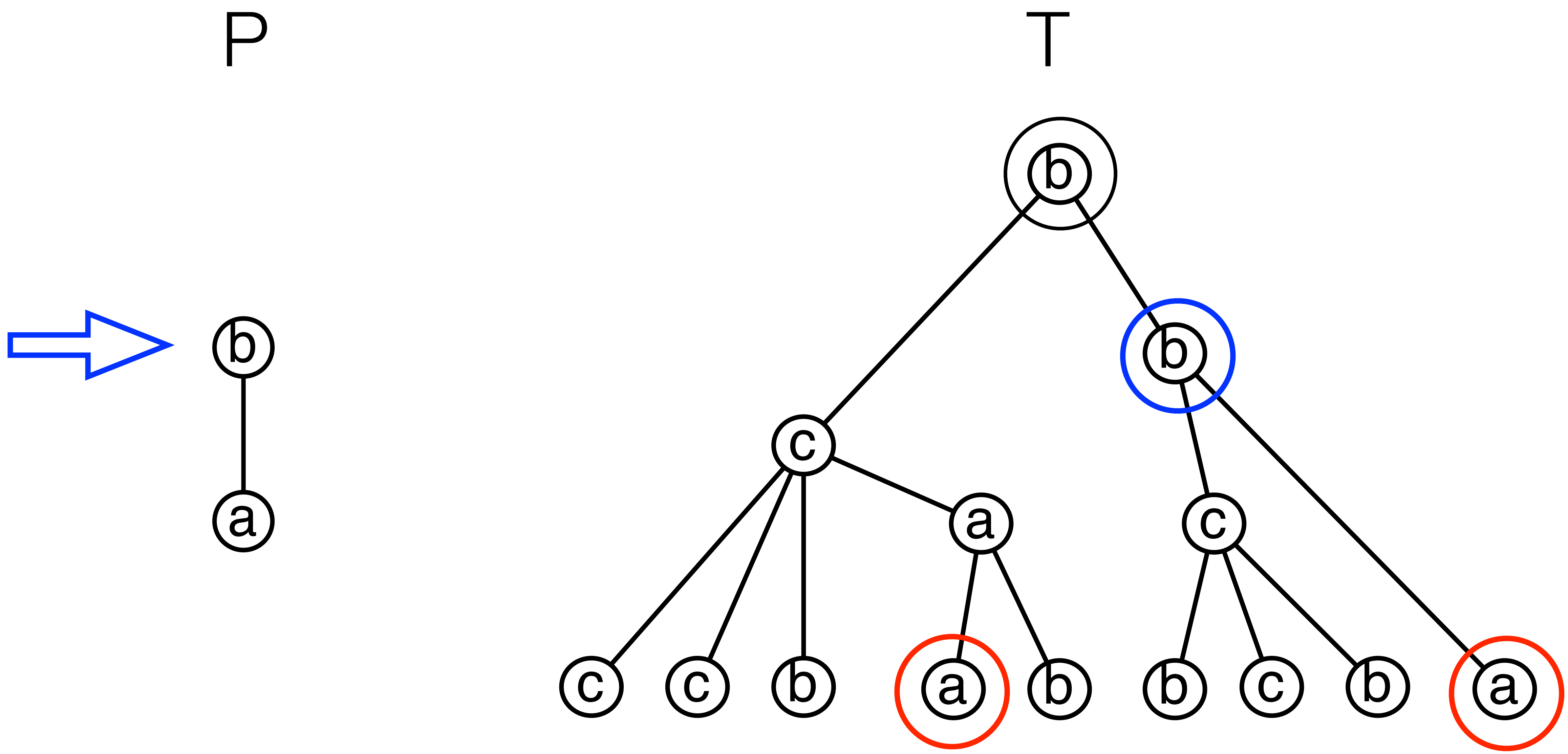

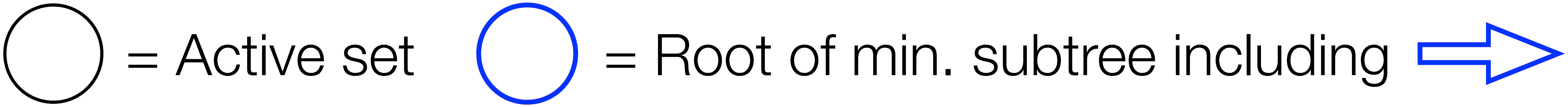
= Active set
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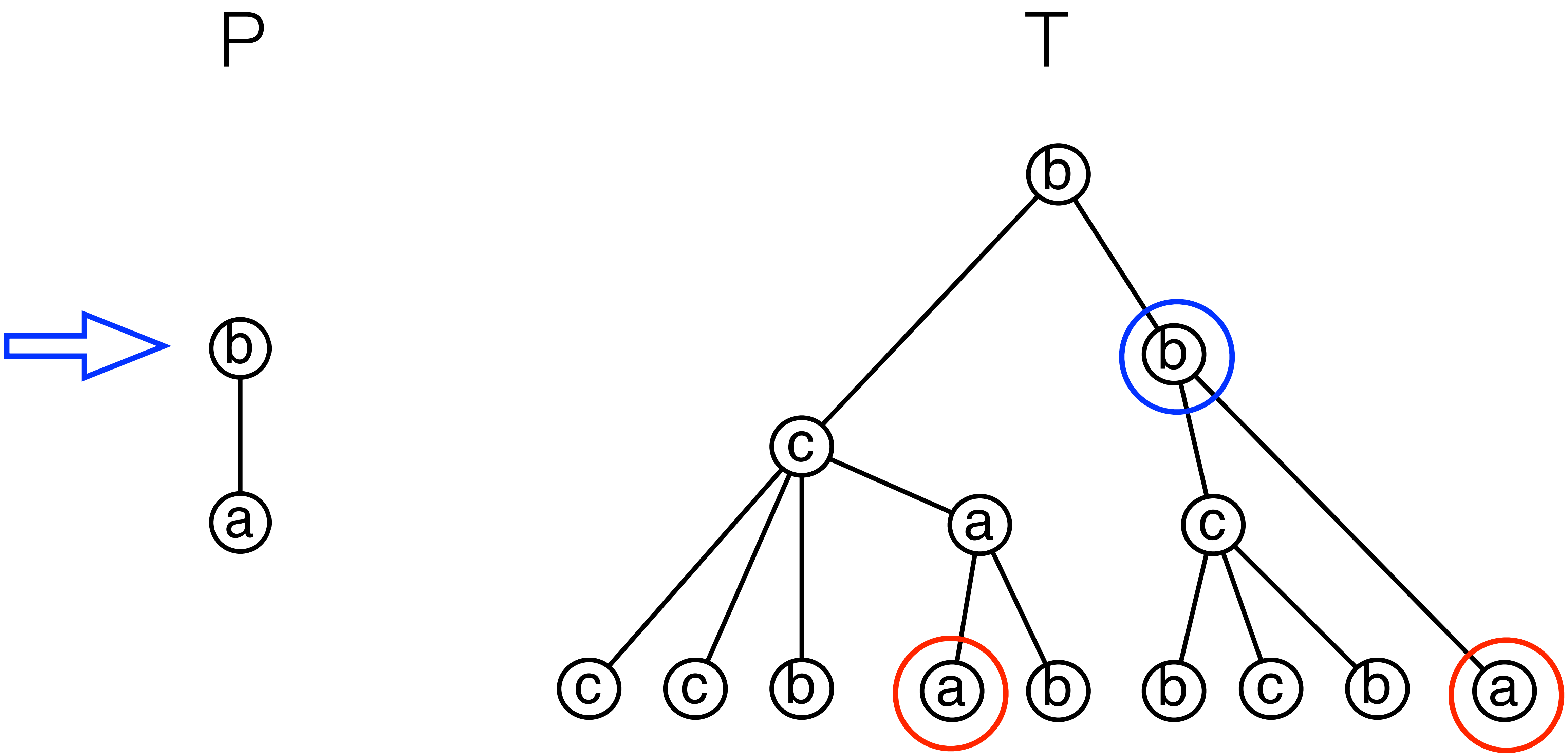
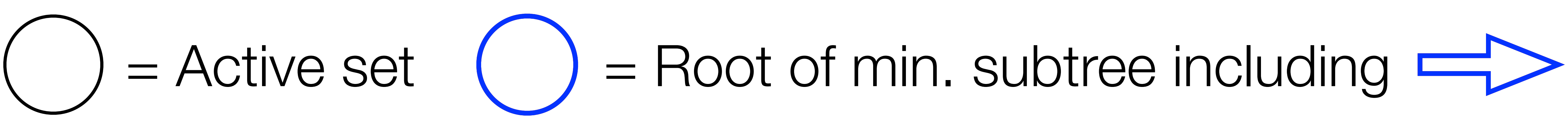
Computing Embeddings: P is a Path

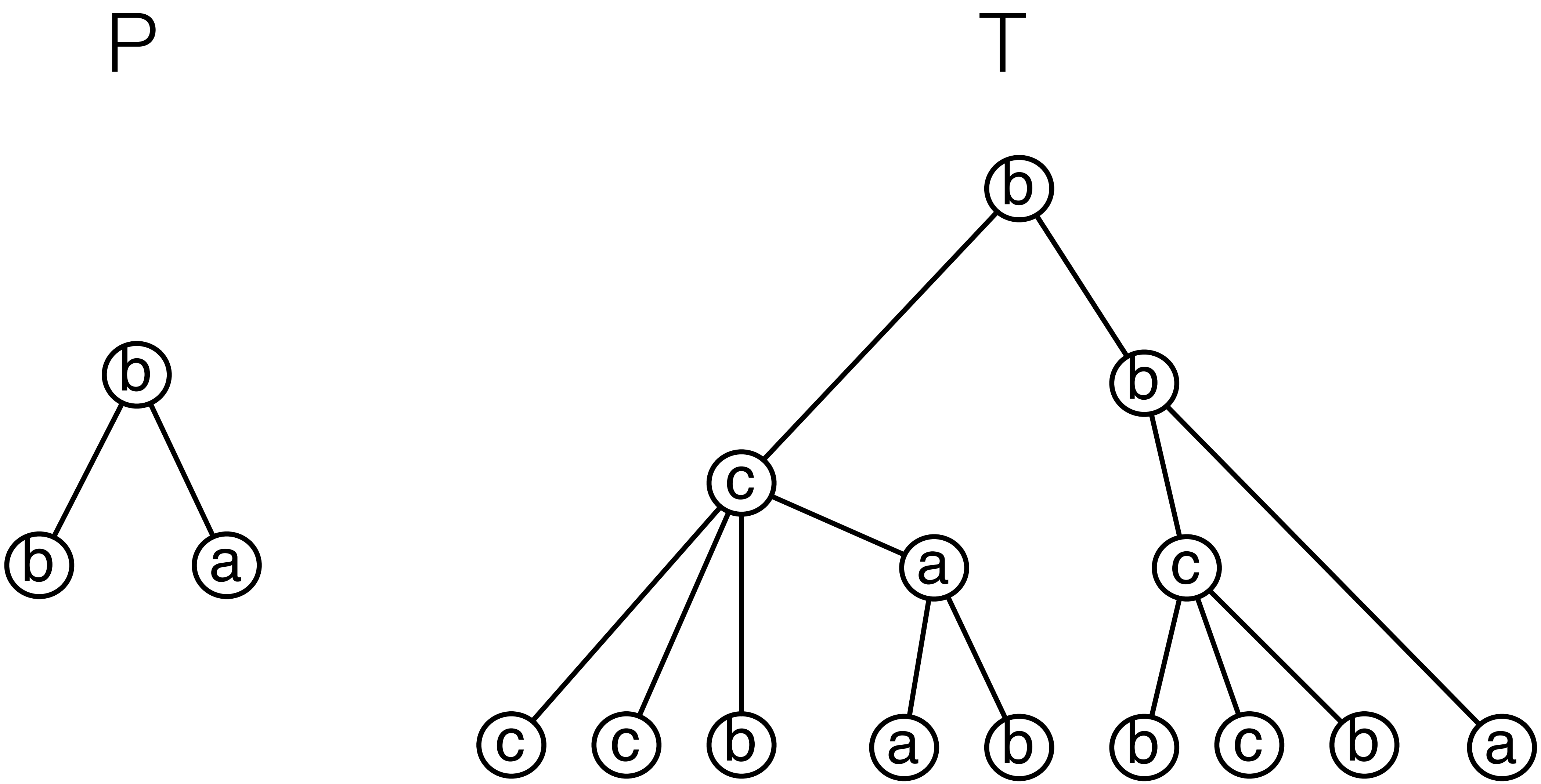




Time Complexity

- At each step of the algorithm the active set “moves up”.
- Each parent pointer in T is traversed a constant number of times.
- Using a simple data structure and exploiting the ordering of the nodes we get a total running time of $O(n_T)$.

Computing Embeddings: P is not a Path



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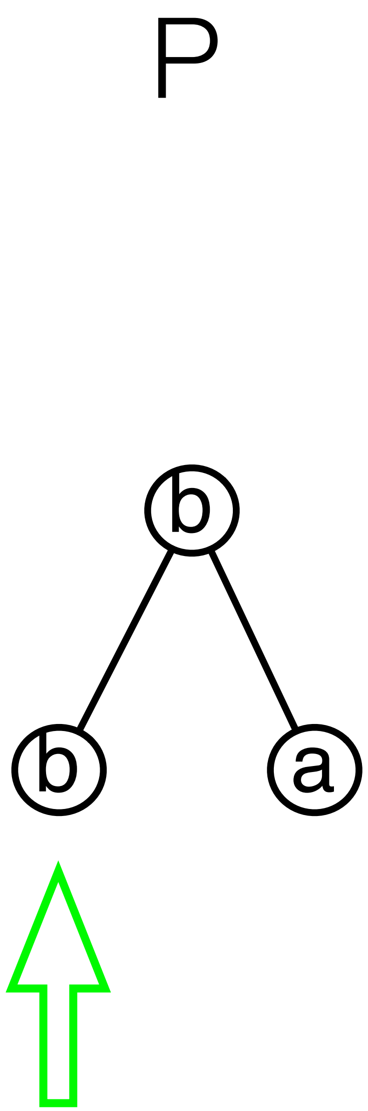

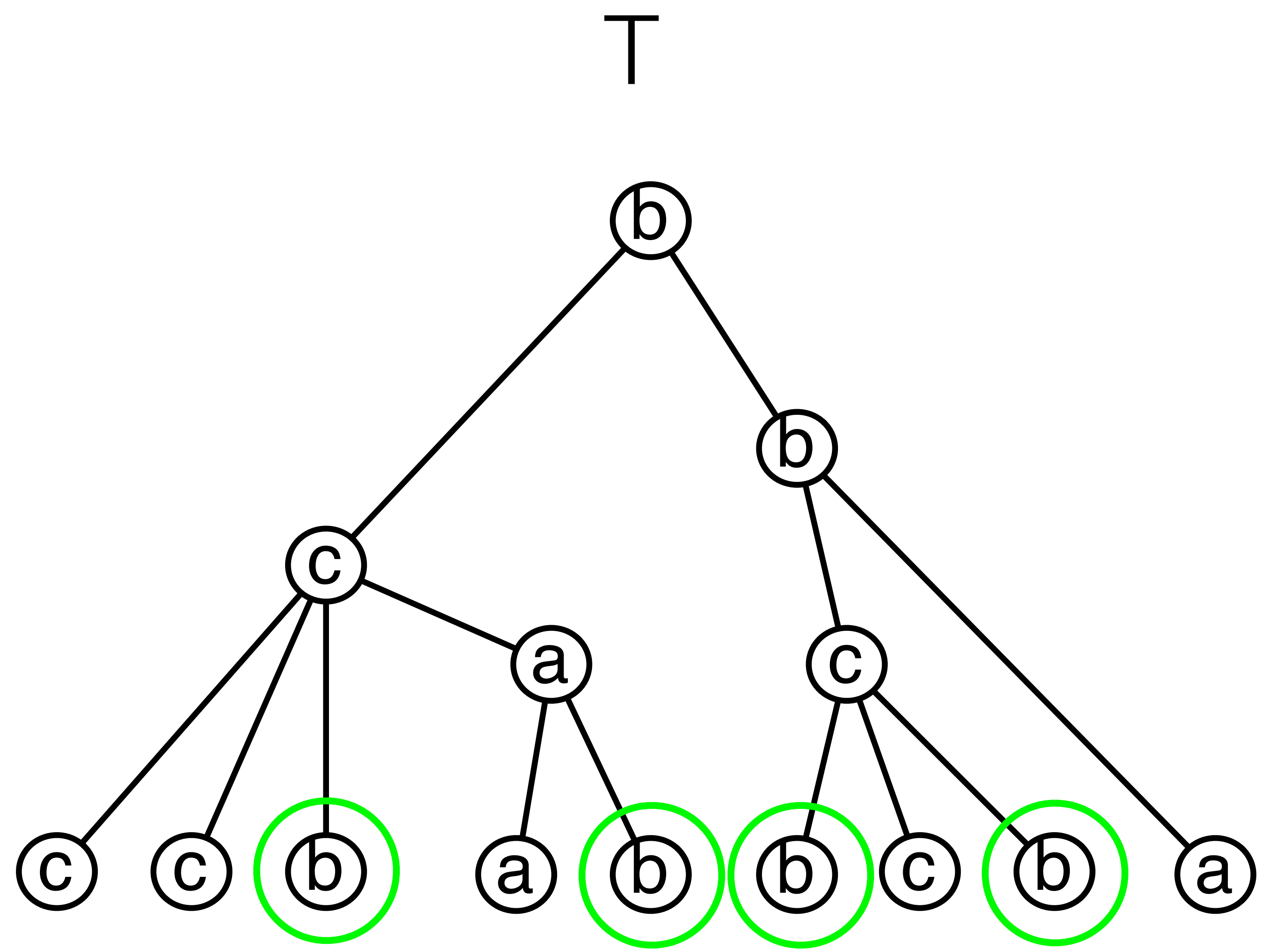

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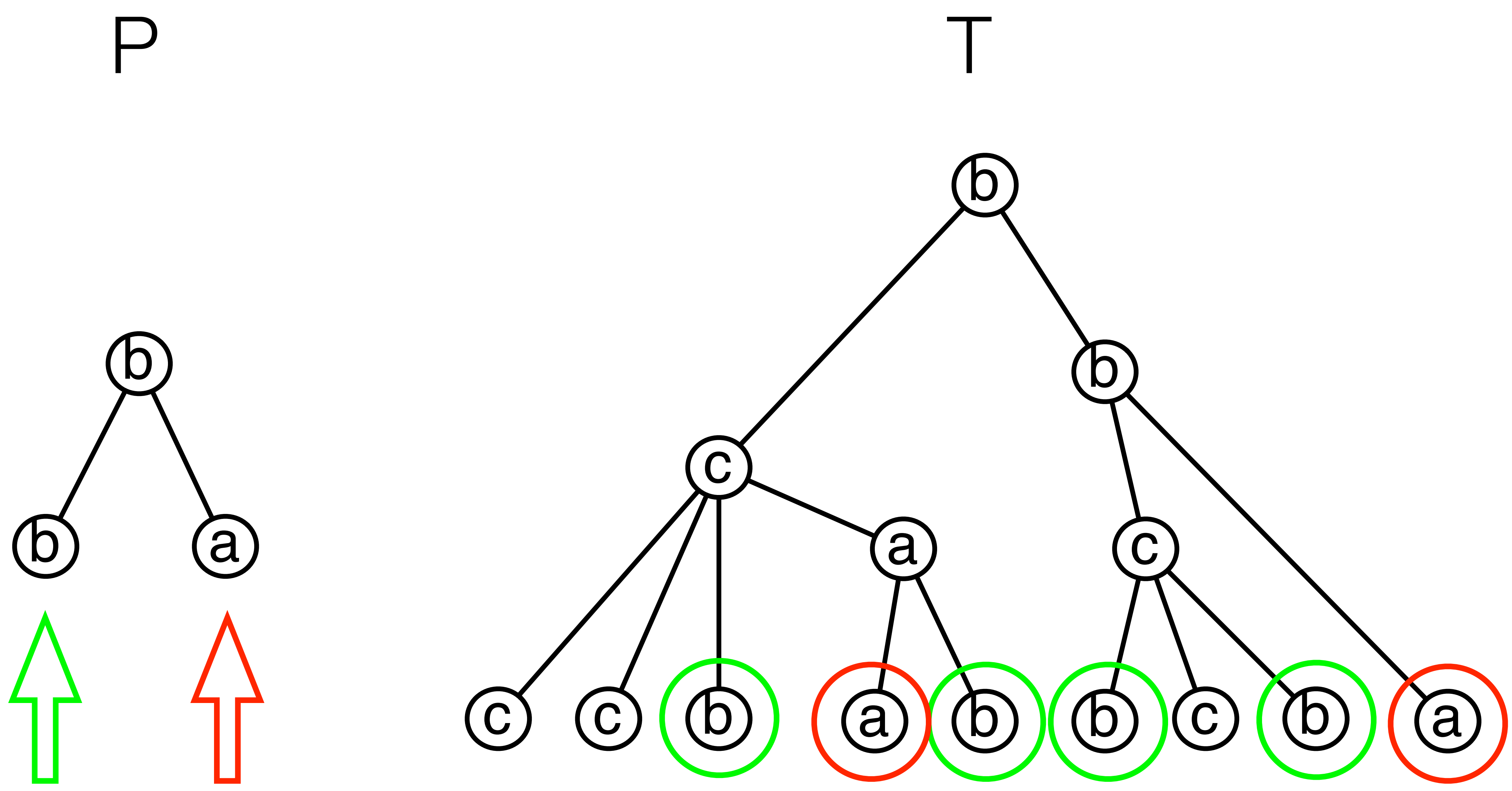
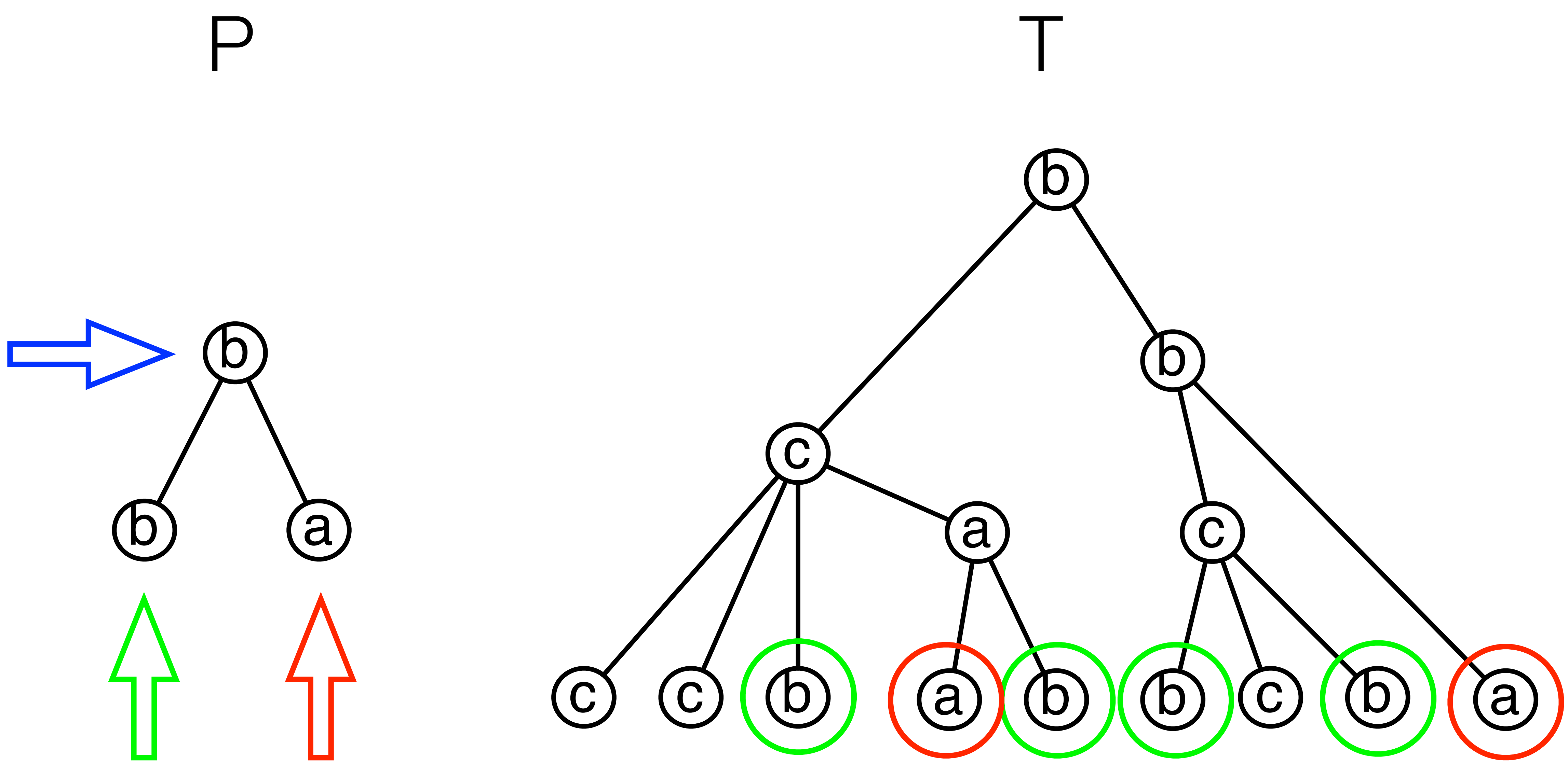


Computing Embeddings: P is not a Path



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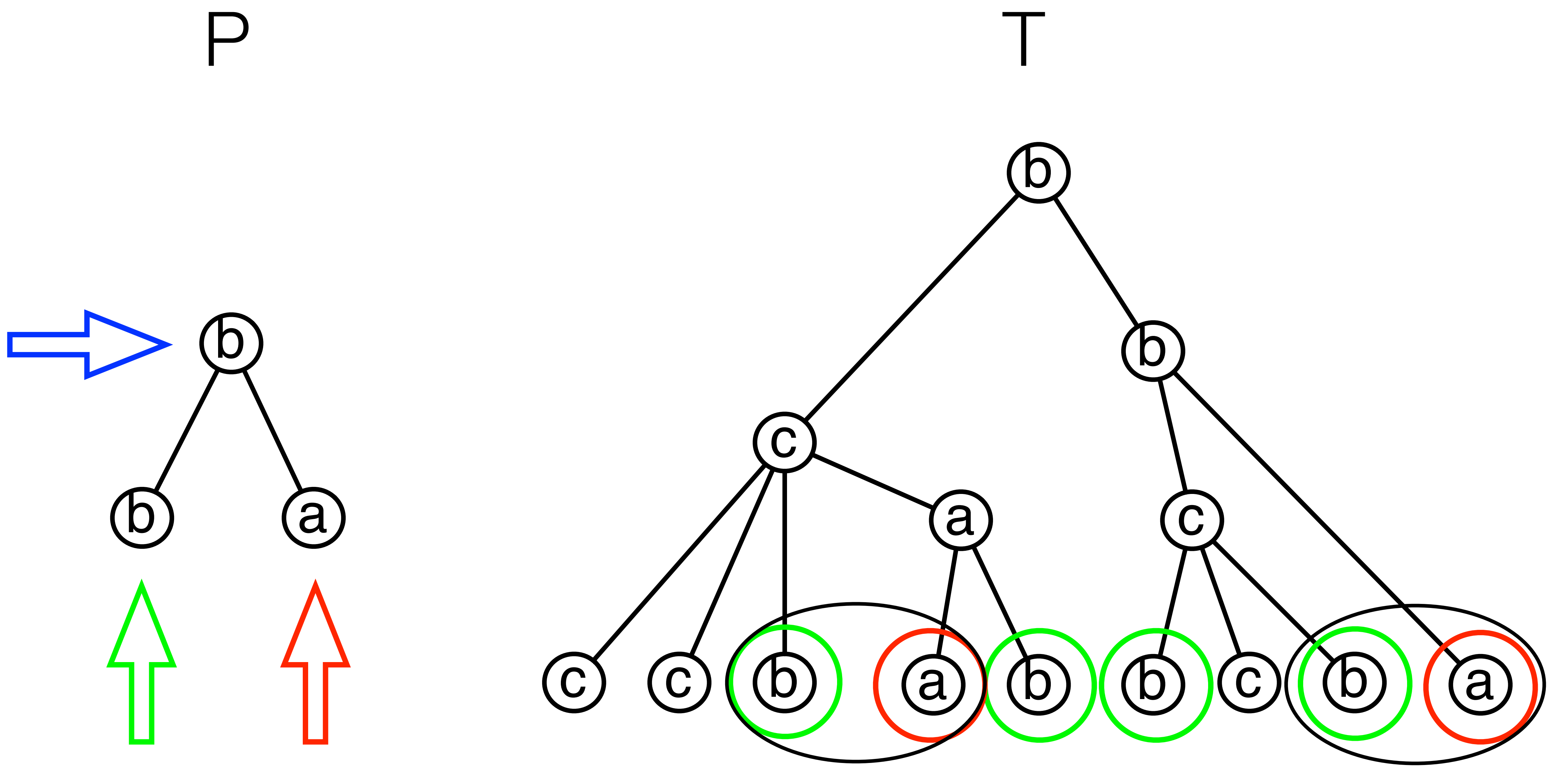

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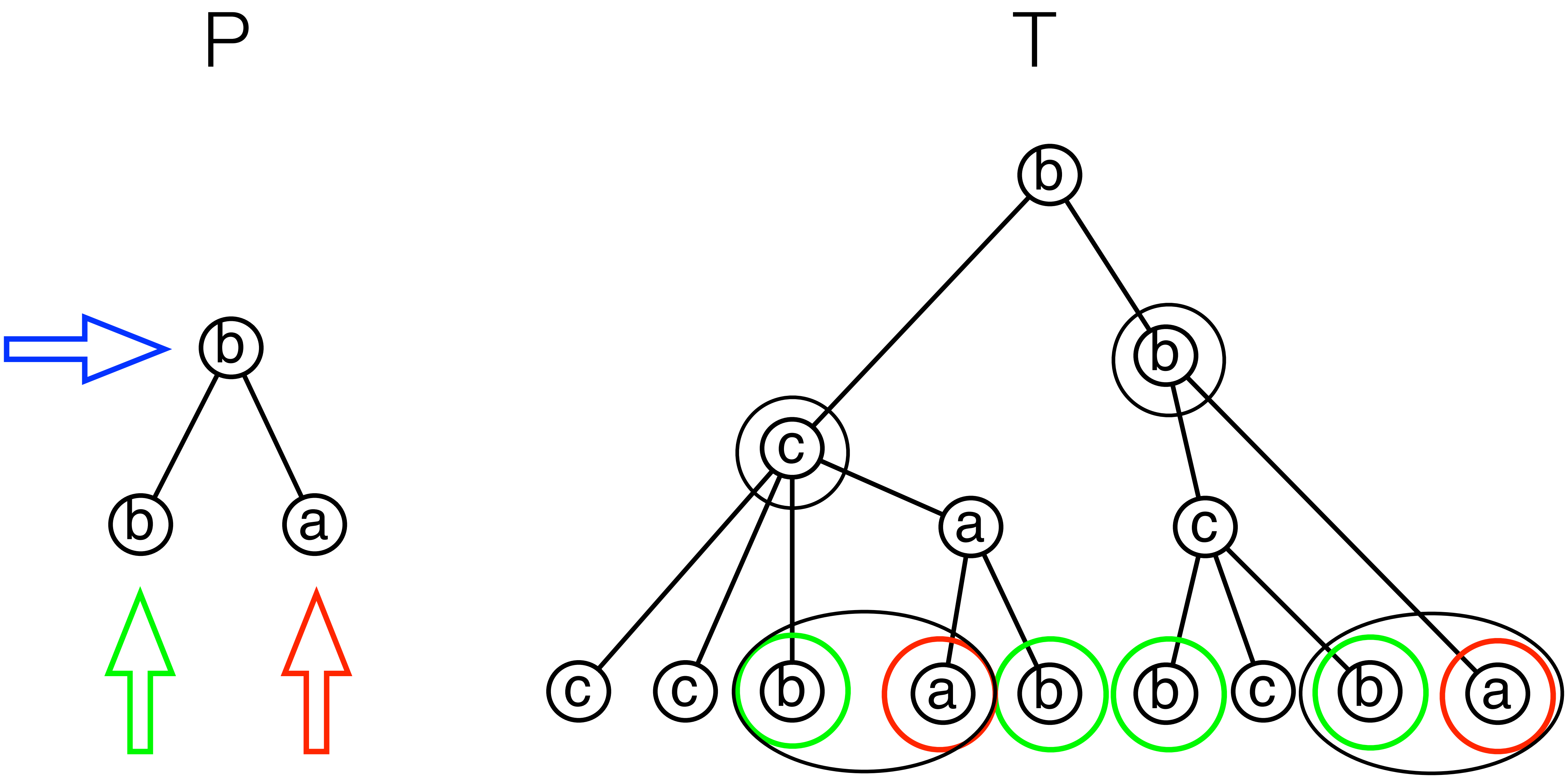

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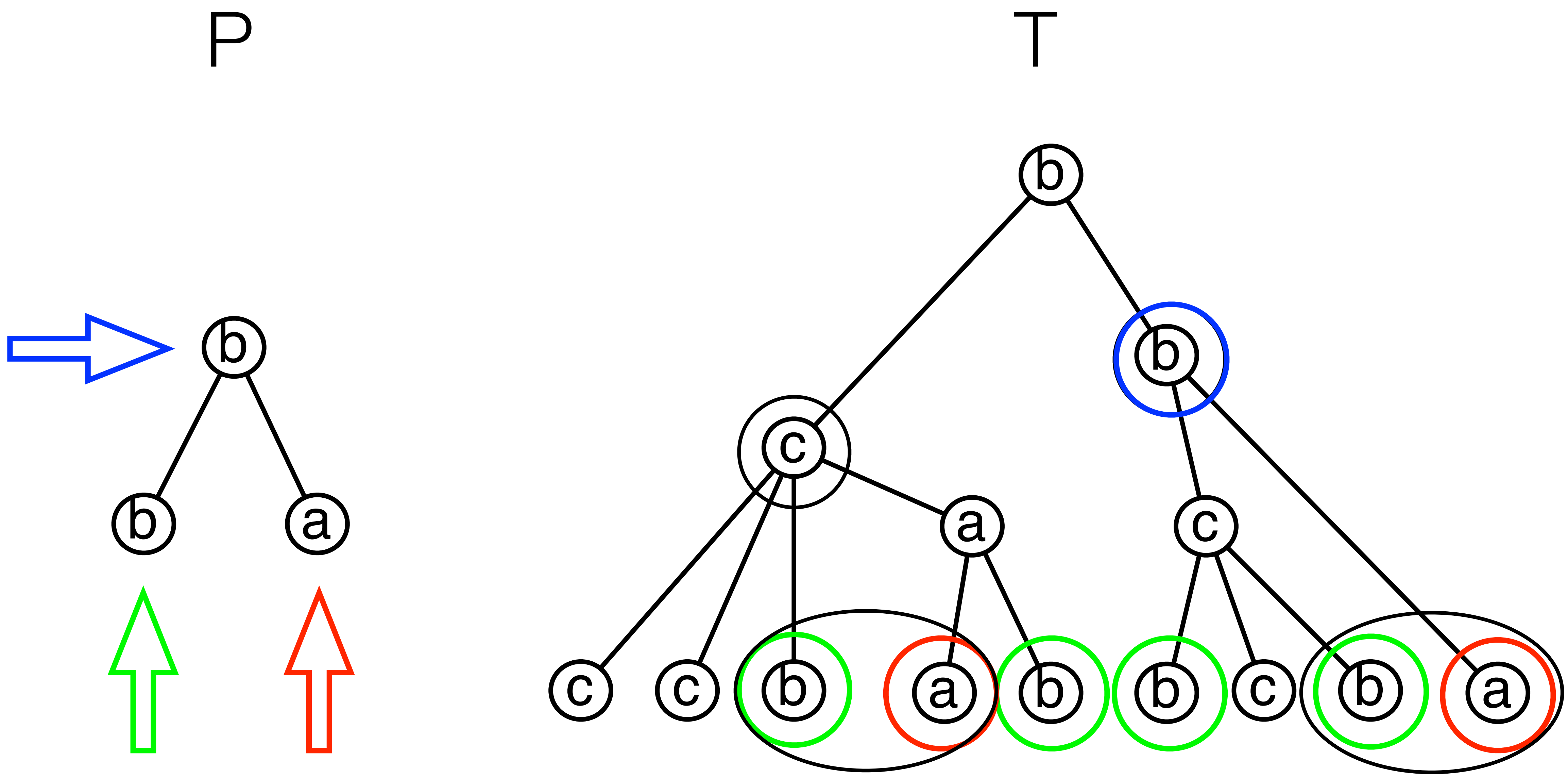

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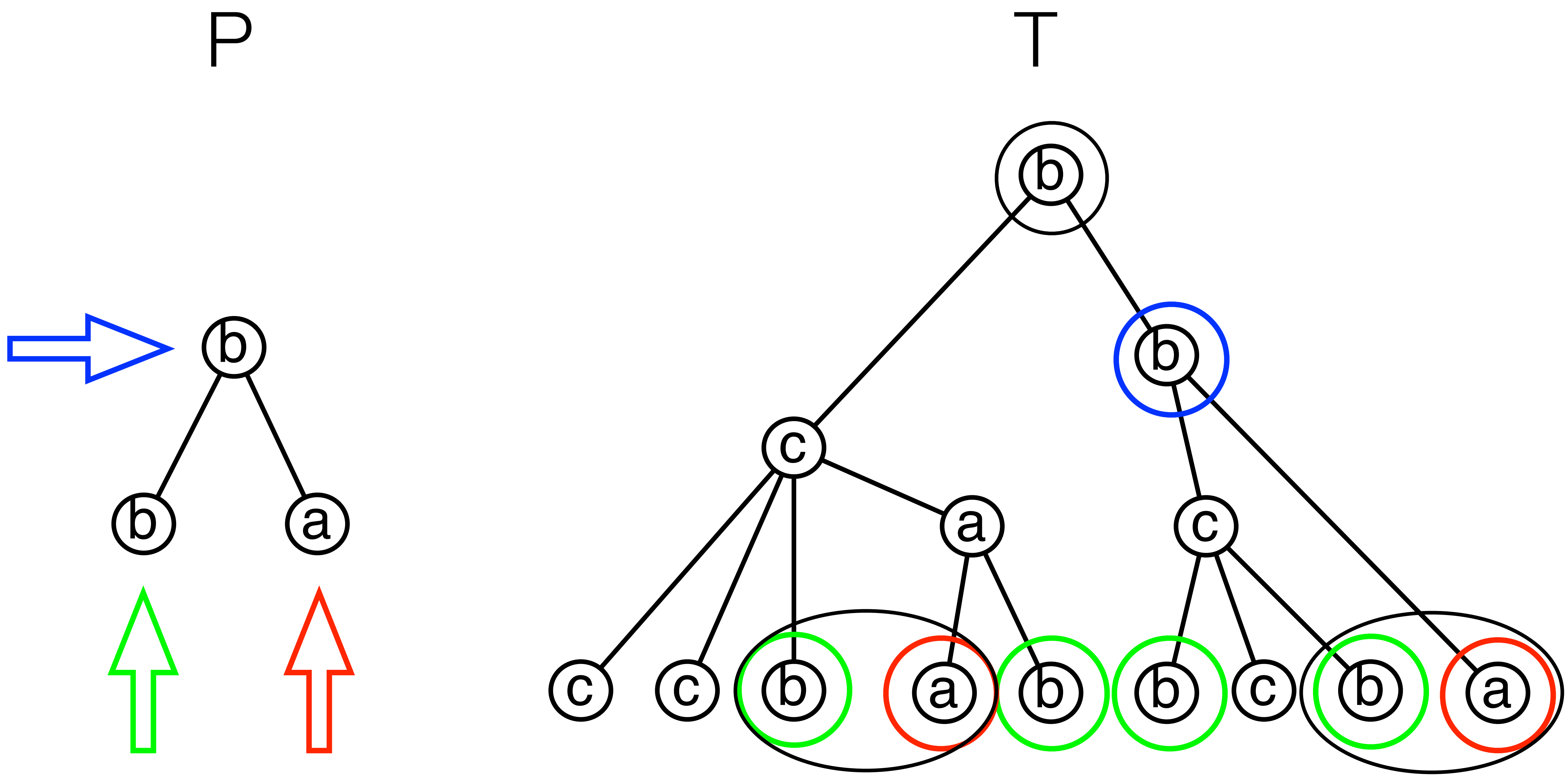

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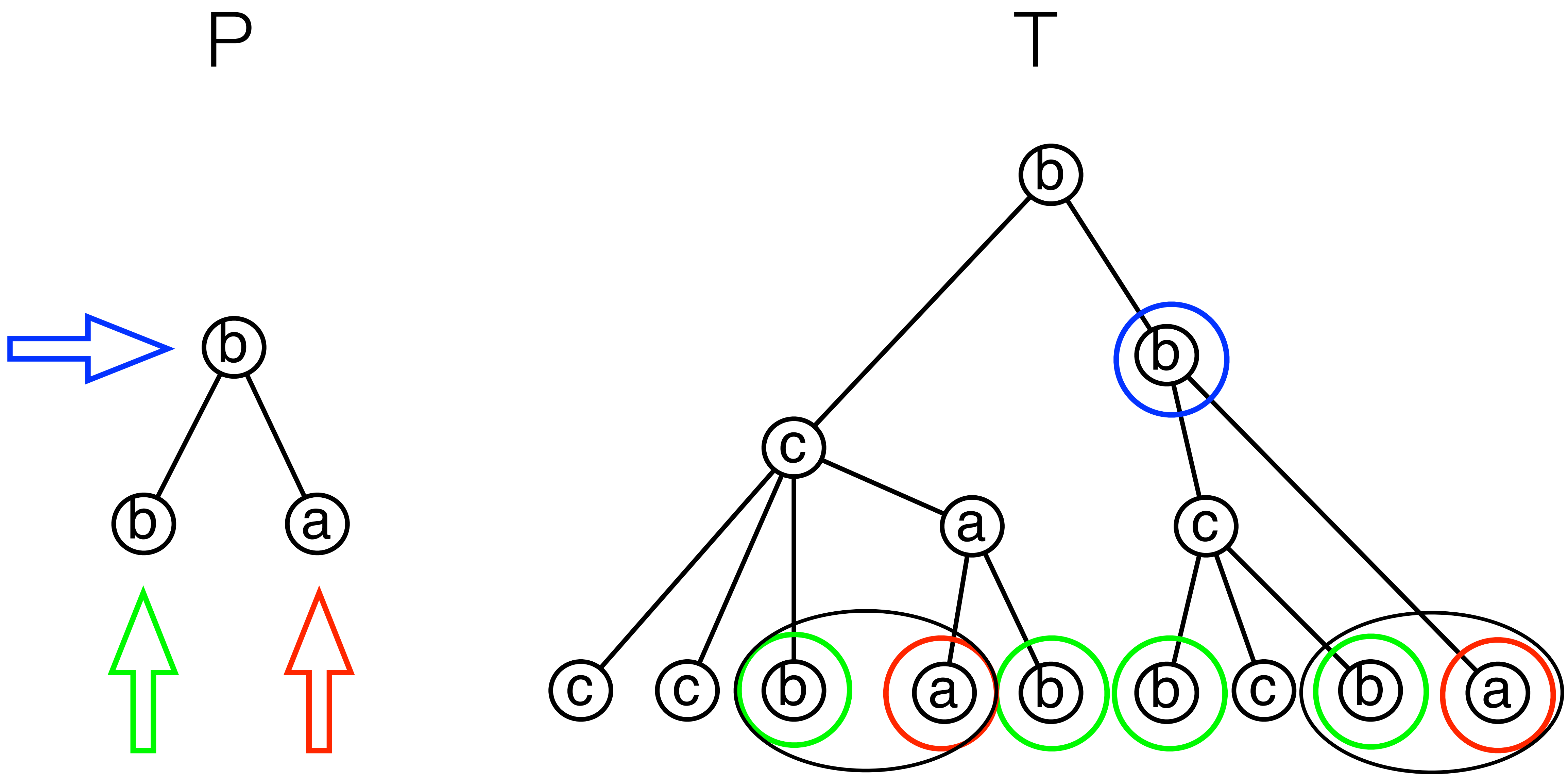


Time Complexity

- Time complexity is bounded by the time used to compute embeddings for each root-to-leaf path in P.

- => Time: $O(l_P n_T)$

Algorithm 2

- Reconsider the case when P is path:

- Let $\mathrm{firstlabel}(v, l)$ denote the nearest ancestor of node v in T labeled l.

- At each step we “essentially” compute $\mathrm{firstlabel}(v, l)$ for each node v in the active set.

Algorithm 2

- Idea: Use a fast data structure supporting $\mathsf{firstlabel}$ queries. Known as the *tree color problem*.

- Theorem [Dietz1989]: For any tree T there is a data structure using $O(n_T)$ space, $O(n_T)$ expected preprocessing time which supports $\mathsf{firstlabel}$ queries in time $O(\log \log n_T)$.

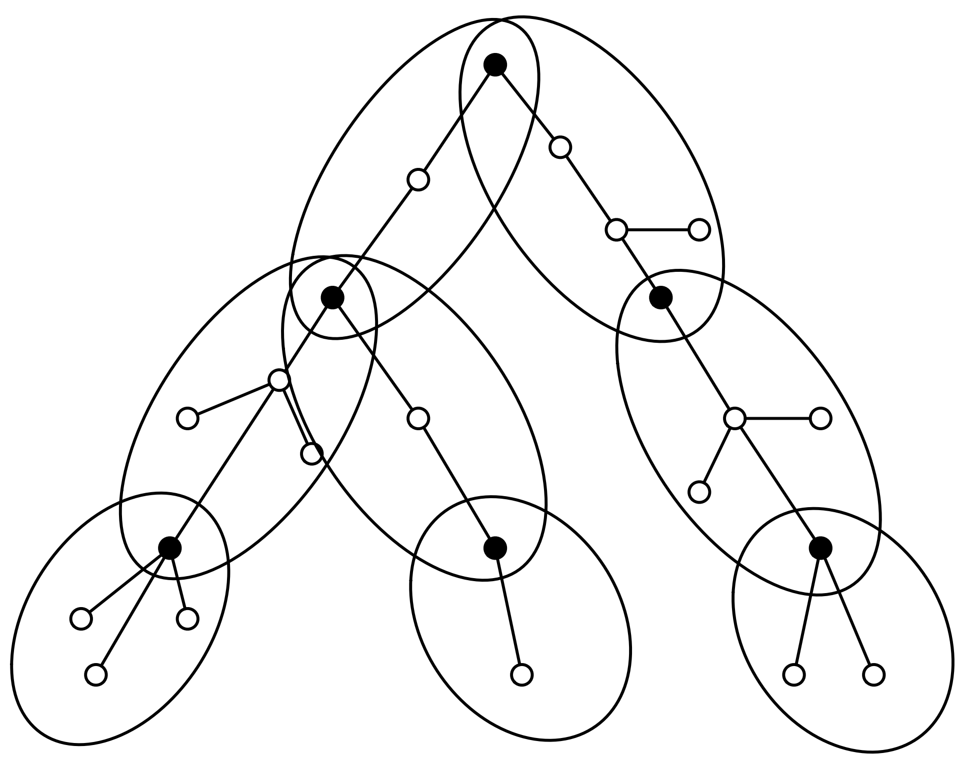
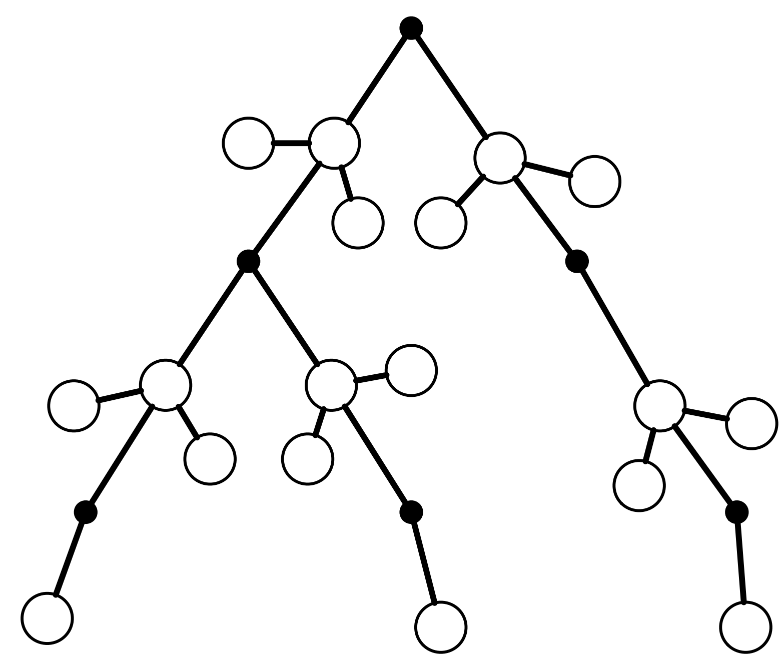
Time Complexity

- For each node in P we have an active set of size at most l_T and for each node in this active set we have to compute a $\mathsf{firstlabel}$ query.
- => Time: $O(n_P l_T \log\log n_T + n_T)$

Algorithm 3: Idea

- Divide T into $O(n_T / \log n_T)$ *micro trees* of size $O(\log n_T)$ which overlap in at most 2 nodes. Based on clustering technique from [AHLT1997].

- We represent each micro tree by a constant number of nodes in a *macro tree* and connect them according to the overlap of the micro trees.

Algorithm 3: Idea

- Active sets are represented compactly in $O(n_T / \log n_T)$ space as small bit strings for each micro tree.

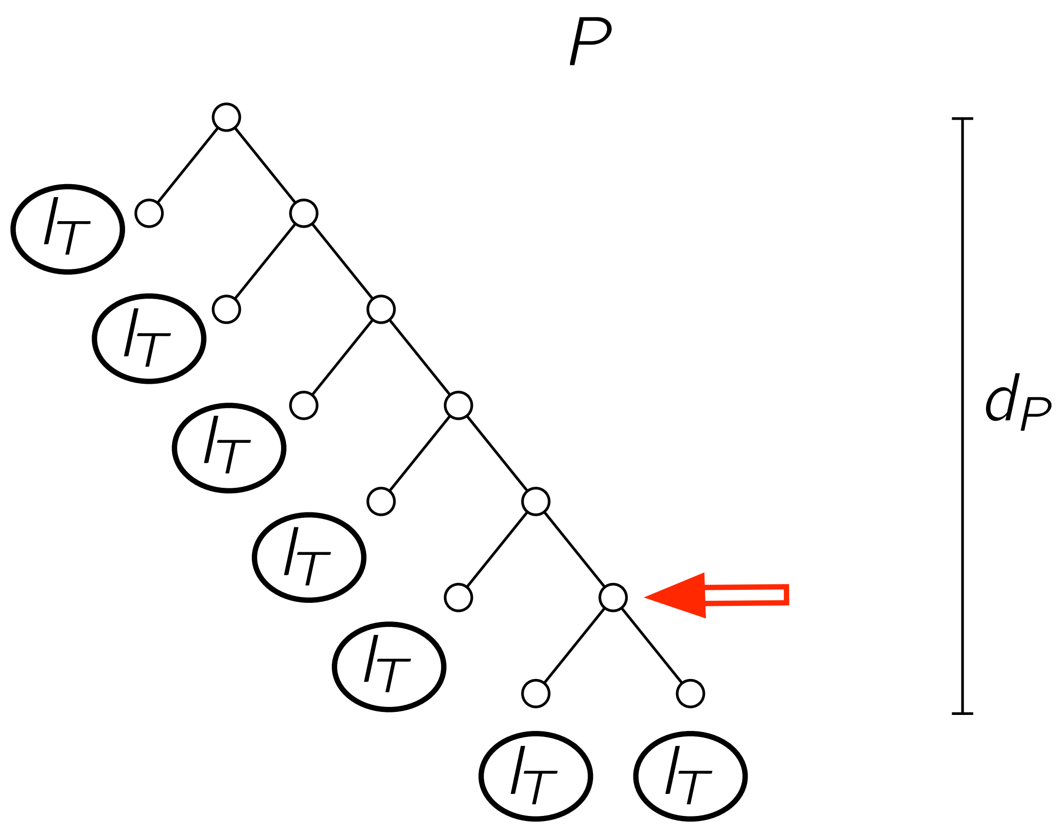
- We preprocess micro trees using a “Four Russian Technique” such that we can update the active set in constant time for each micro tree.

- Leads to an $O\left(\frac{n_P n_T}{\log n_T} + n_T \log n_T\right)$ time algorithm.

Space Complexity

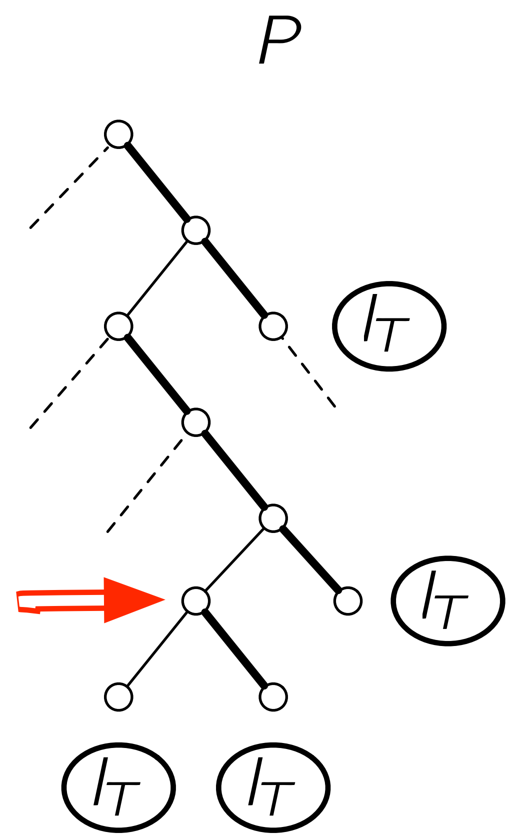
- Linear Space?
- No!

The Problem: Algorithm 1 and 2



- Storing all active sets uses $\Omega(l_T d_P)$ space.

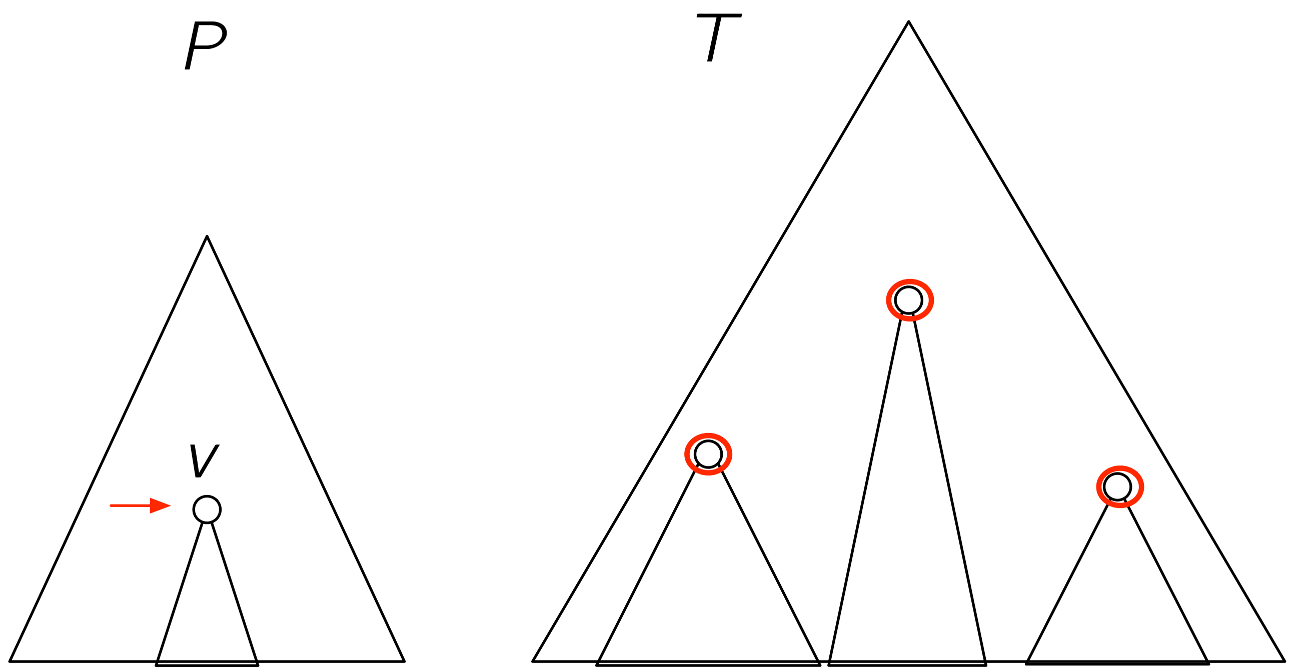
Trick 1: Recurse to subtree with the most leaves



- The number of active sets stored does not exceed $O(\log l_P)$.

- => Total space for stored active sets is $O(l_T \log l_P)$.

Trick 2: Strengthen Analysis



- Nodes in the active set for v are roots of (disjoint) subtrees that embed $P(v)$.
- => Each of these subtrees have at least $l_{P(v)}$ leaves.
- => The size of the active set for v is at most $O(l_T / l_{P(v)})$.

Space Complexity: Algorithm 1 and 2

- Trick 1 and 2 combined gives exponentially decreasing sizes of the stored active sets.

- => Total size of the stored active sets is $O(l_T)$.

- Space complexity is $O(n_P + n_T)$.

- Trick 2 shows that algorithm 2 in fact runs in $O(l_P l_T \log\log n_T + n_T)$ time.

Space Complexity: Algorithm 3

- Each active sets is represented in $O(n_T / \log n_T)$ space.

- Trick 1 gives us that the total space for the stored active sets is

$$O\left(\frac{n_T}{\log n_T} \log l_P\right) = O(n_T)$$

Summary

- Time:

$$\min \begin{cases} O(l_P n_T), \\ O(l_P l_T \log\log n_T + n_T), \\ O(\frac{n_P n_T}{\log n_T} + n_T \log n_T). \end{cases}$$

- Space: $O(n_P + n_T)$

String Matching

- Fast and Compact Regular Expression Matching. Philip Bille and Martin Farach-Colton, 2005, submitted.
- New Algorithms for Regular Expression Matching. Philip Bille, ICALP 2006.
- Improved Approximate String Matching and Regular Expression Matching on Ziv-Lempel Compressed Texts. Philip Bille and, Rolf Fagerberg, and Inge Li Gørtz, CPM 2007.

Regular Expressions

- The *regular expressions* are defined recursively:
- A character $\alpha \in \Sigma$ is a regular expression.
- If S and T are regular expressions then so is
    - the *concatenation* ST,
    - the *union* $S \mid T$, and
    - the *kleene star* S^*.

Regular Expressions

- The *language* $L(R)$ of a regular expression R is defined by:

- For any $\alpha \in \Sigma$, $L(\alpha) = \{\alpha\}$.

- For regular expressions S and T:

$$L(ST) = L(S)L(T)$$

$$L(S|T) = L(S) \cup L(T)$$

$$L(S^*) = \{\epsilon\} \cup L(S) \cup L(S)^2 \cup L(S)^3 \cup \cdots$$

Example

$$R = ac|a^*b$$

$$L(R) = \{ac, b, ab, aab, aaab, aaaab, \ldots\}$$

Regular expression Matching

- Given a regular expression R and a string Q the *regular expression matching problem* is to decide if $Q \in L(R)$.

- Example: $R = ac|a^*b$ matches $Q = aaaab$.

Applications:

- Lexical analysis phase in compilers.
- Protein searching.
- Text editing and programming languages (e.g. EMACS and Perl).

Results

Time	Space	Ref
$O(nm)$	$O(m)$	[Tho68]
$O((n + 2^m)\lceil m/w \rceil)$	$O((2^m + \sigma)\lceil m/w \rceil)$	[NR04]
$O\left(\frac{nm}{\log n} + n + m \log m\right)$	$O(n)$	[Mye92, Here]
$\begin{cases} O(n\frac{m \log w}{w} + m \log w) & \text{if } m > w \\ O(n \log m + m \log m) & \text{if } \sqrt{w} < m \leq w \\ O(\min(n + m^2, n \log m + m \log m)) & \text{if } m \leq \sqrt{w}. \end{cases}$	$O(m)$	[Here]

Practical Implications

- Except for Thompson’s algorithm all previous algorithms use large tables and perform a long series of lookups in the tables.

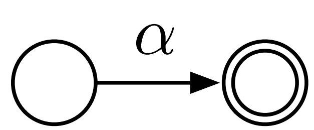
- => Many expensive cache misses.

- New algorithm does not require the large tables.

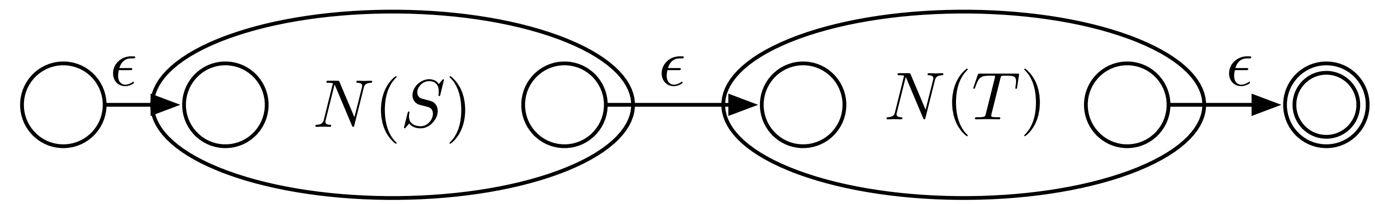
Algorithm Overview

- Construct *non-deterministic finite automata* (NFA) using Thompson's classical algorithm.
- Decompose the NFA into small subautomata.
- Simulate each subautomata using the arithmetic and logical instruction of the word RAM.
- Use the simulation of the each subautomata to simulate the entire NFA.

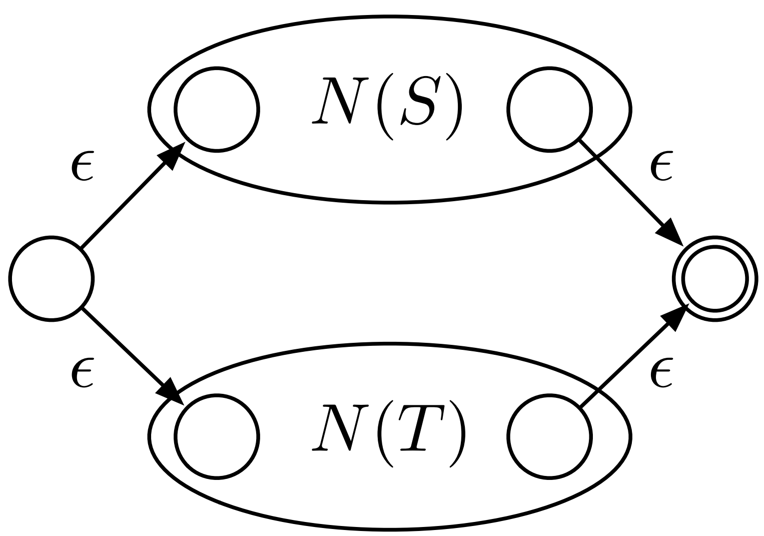
Thompson's Algorithm



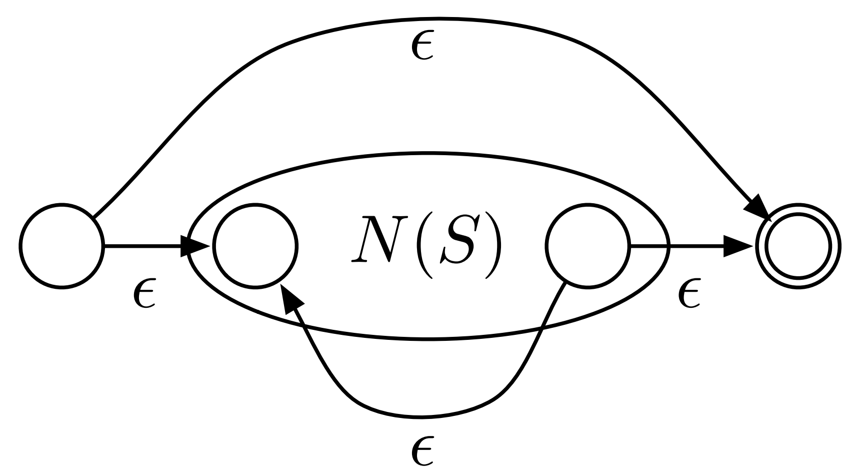
$N(\alpha)$



$N(ST)$

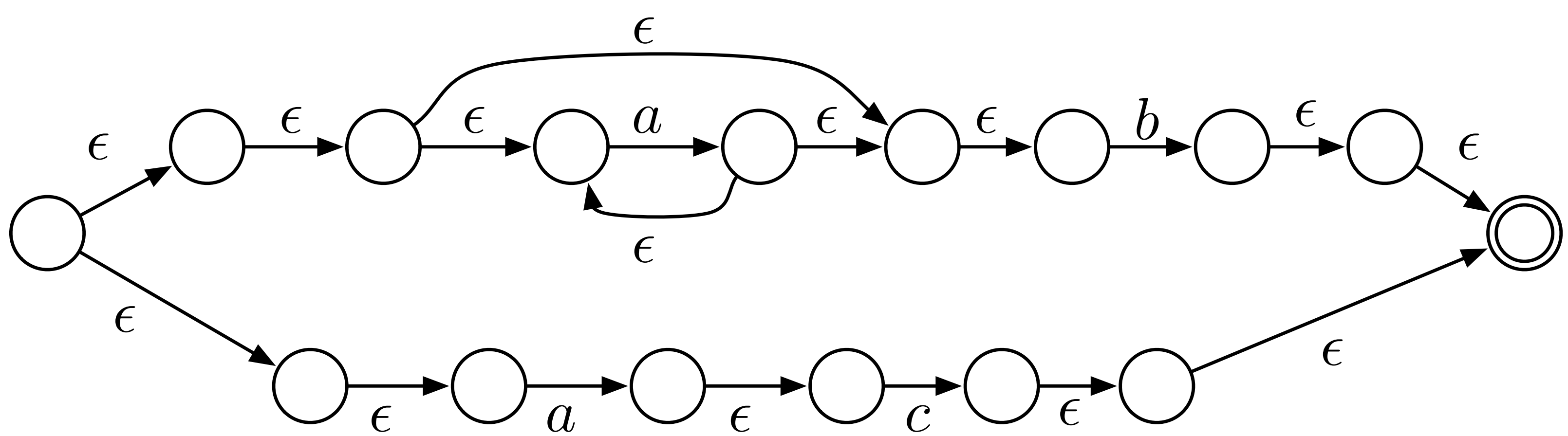


$N(S|T)$



$N(S^*)$

Thompson NFA



- *Thompson-NFA* (TNFA) for $R = ac|a^*b$.
- $N(R)$ *accepts* Q if and only if there is path from θ to ϕ that "spells" out Q .
- $Q \in L(R)$ if and only if $N(R)$ accepts Q.

Properties of TNFAs

- Linear number of states and transitions.
- Incoming transitions to a state have the same label.
- States with an incoming transition labeled $\alpha \in \Sigma$ (α-states) have exactly 1 predecessor.

Simulating TNFAs

- Let A be TNFA with m states. To test acceptance we use the following operations. For a state-set S and $\alpha \in \Sigma$:

- $\mathsf{Move}(S, \alpha)$: Find set of states reachable from S via a single α-transition.

- $\mathsf{Close}(S)$: Find set of states reachable from S via a path of ϵ-transitions.

- $O(m)$ time for both operations.

Simulating TNFAs

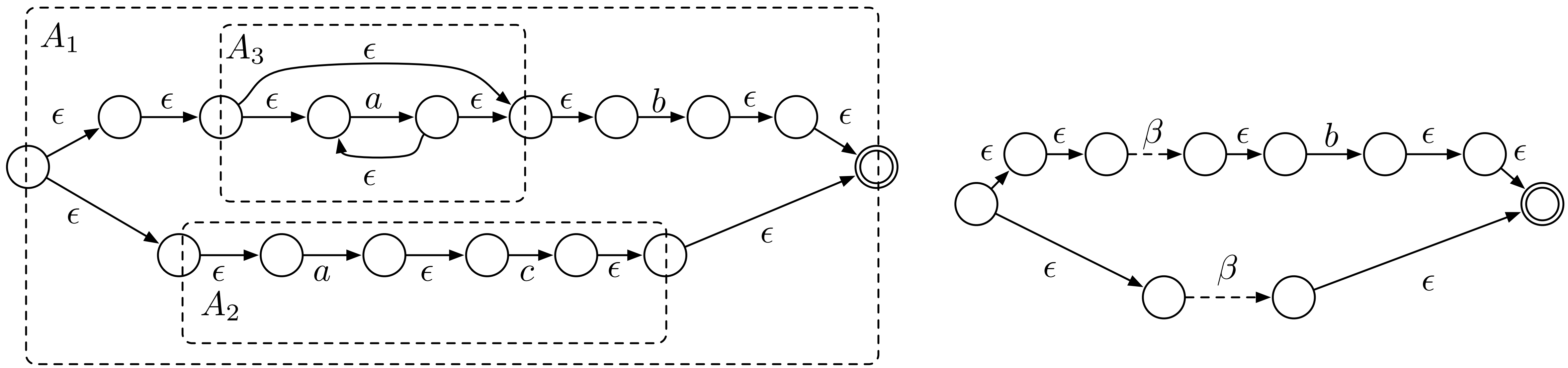
- Let Q be a string of length n.
- The *state-set simulation* of A on Q produces state-sets $S_0, S_1, \ldots, S_n$ as follows:

$$S_0 := \mathrm{Close}(\{\theta\})$$

$$S_i := \mathrm{Close}(\mathrm{Move}(S_{i-1}, Q[i]))$$

- S_i is the set of states reachable from θ through a path that spells out $Q[1..i]$.
- $Q \in L(R)$ if and only if $\phi \in S_n$.
- $O(nm)$ time and $O(m)$ space.

Four-Russian Speedup

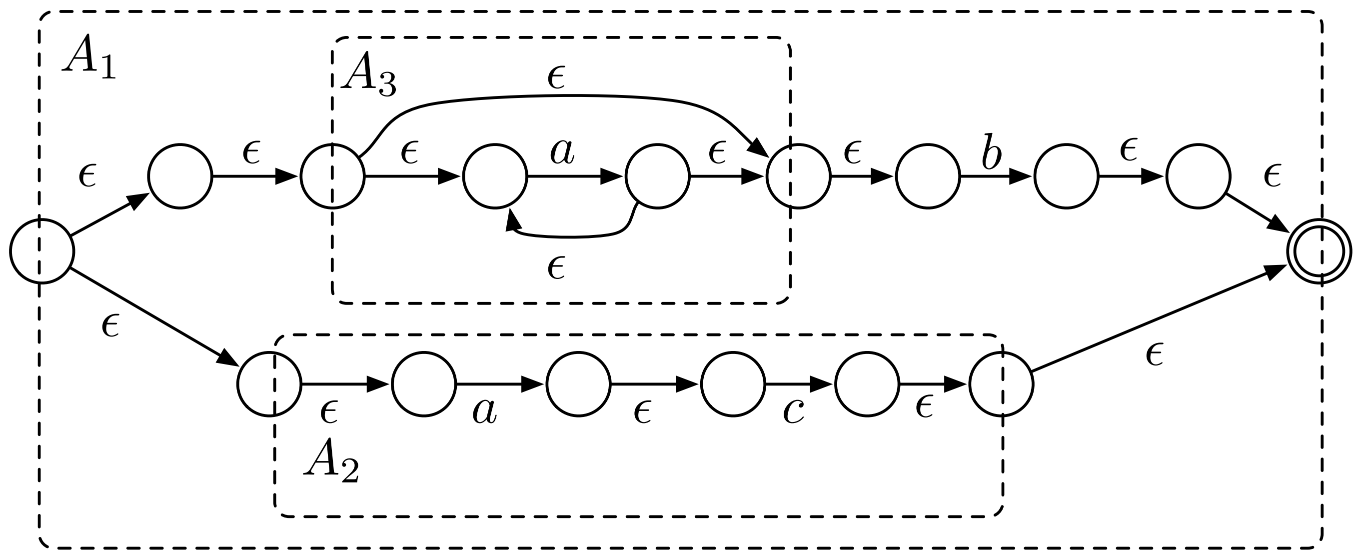


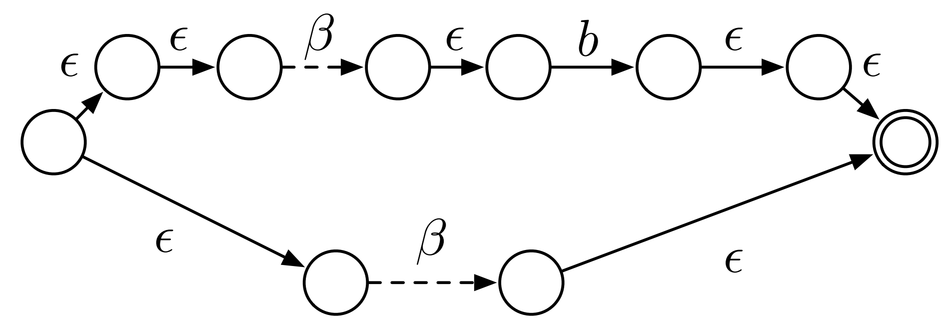
- Decompose TNFA into sub-automata with $O(\log n)$ states.

- Preprocess subautomata to get Move and Close in constant time for each. Subautomata are made "deterministic".

- => $O\left(\frac{nm}{\log n} + n + m \log m\right)$ time and $O(n)$ space algorithm [Myers92, BFC05].

Word-Level Parallel Algorithm





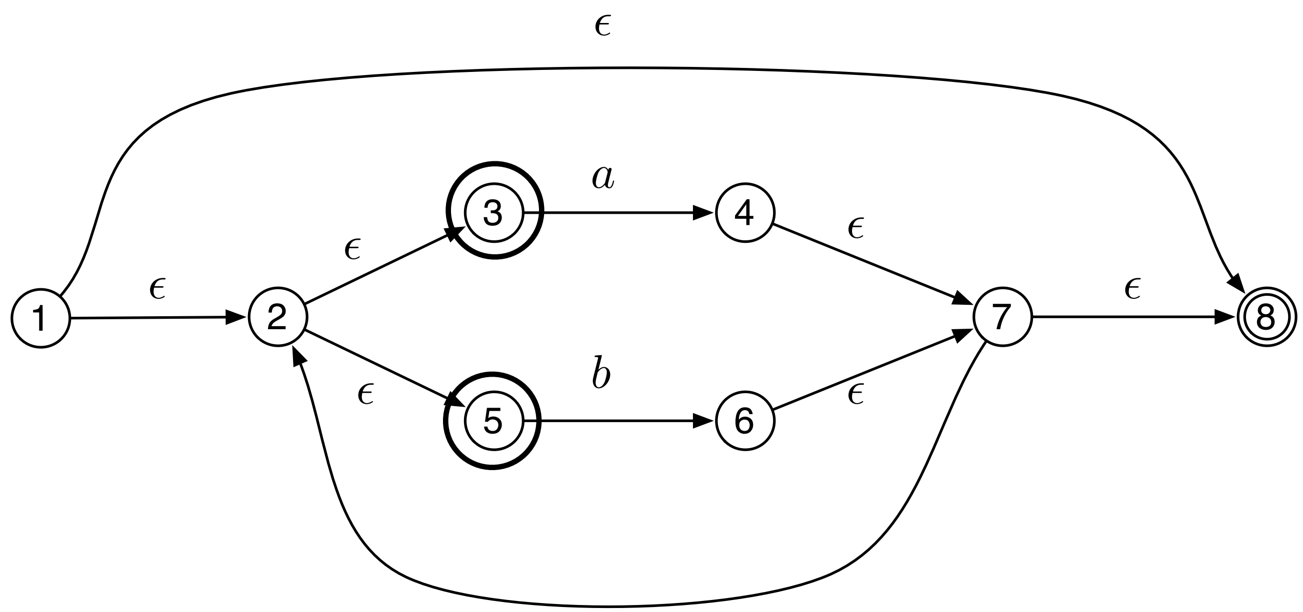
- Idea: Use essentially same decomposition into subautomata.

- Simulate Move and Close using the arithmetic and logical instructions of the word RAM.

Simple Algorithm for small TNFAs

- Suppose A is a TNFA with $m = O(\sqrt{w})$ states.
- Order the states such that the (unique) predecessor of α-state i is $i - 1$.
- Represent state-sets as a bit string.

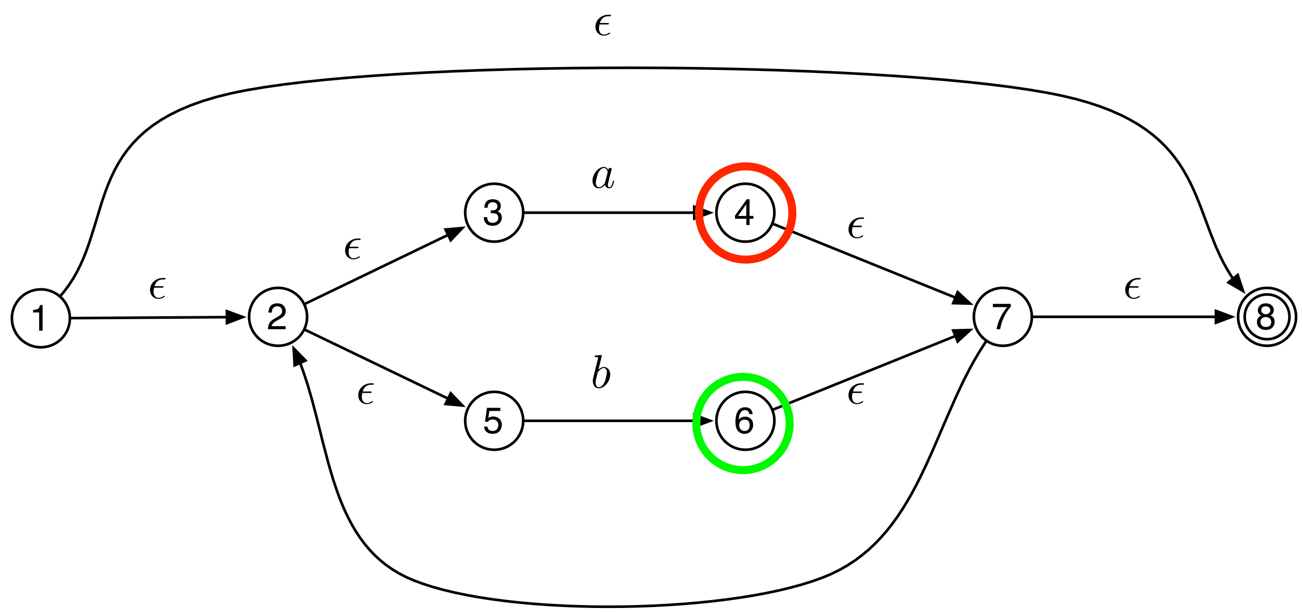
Representation of State-Sets



$S =$

1	2	3	4	5	6	7	8
0	0	1	0	1	0	0	0

Move Operation: Preprocessing

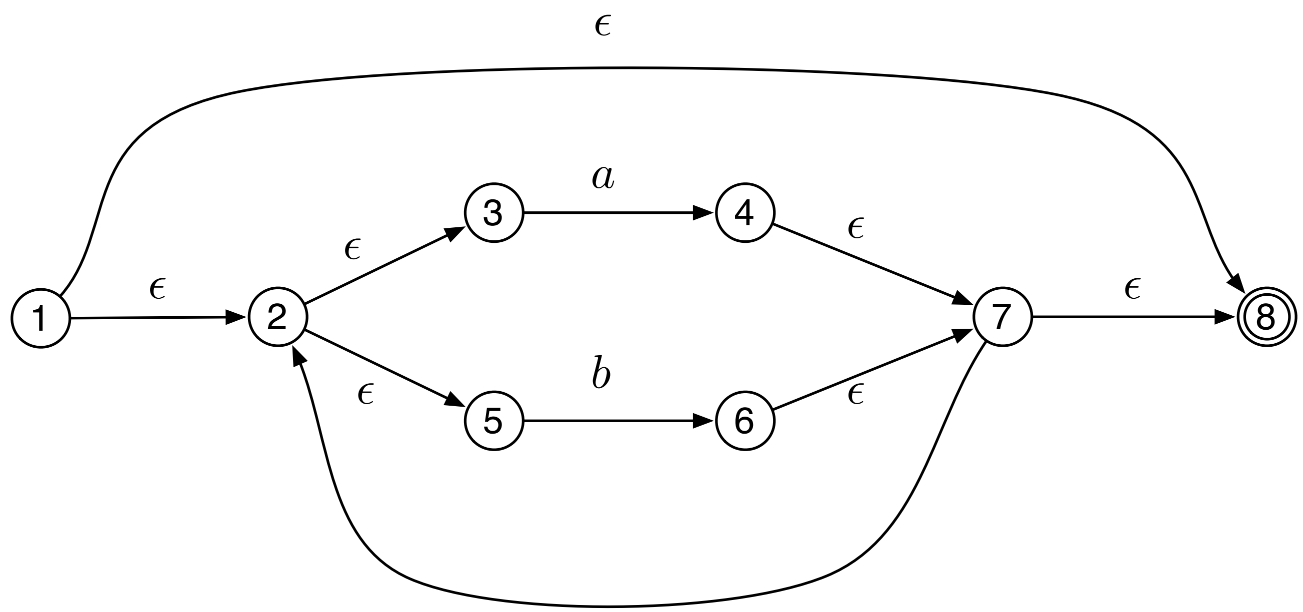


- For each $\alpha \in \Sigma$ represent α-states using a bit string:

$$\begin{array}{cc} & \begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array} \\ D_a = & \begin{array}{|c|c|c|c|c|c|c|c|} \hline 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ \hline \end{array} \end{array}$$

$$\begin{array}{cc} & \begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array} \\ D_b = & \begin{array}{|c|c|c|c|c|c|c|c|} \hline 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ \hline \end{array} \end{array}$$

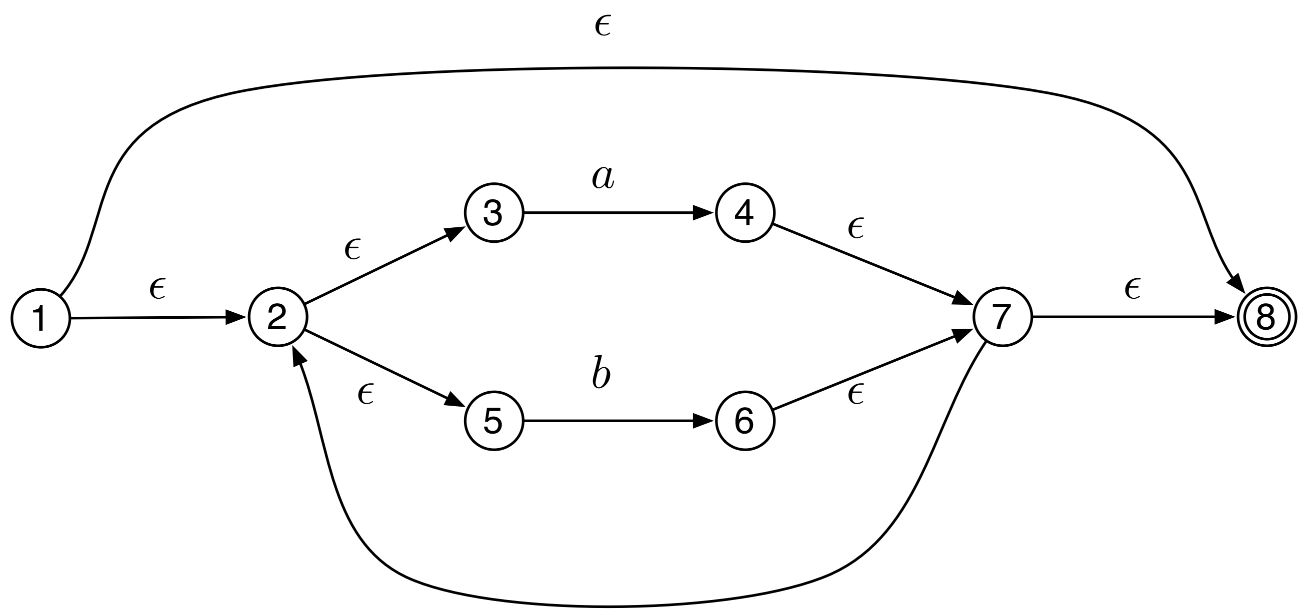
Move Operation: Simulation



- We compute $\mathsf{Move}(S, \alpha)$ as

$$S' := (S >> 1)\ \&\ D_\alpha$$

Example: $S' := (S >> 1)\ \&\ D_\alpha$



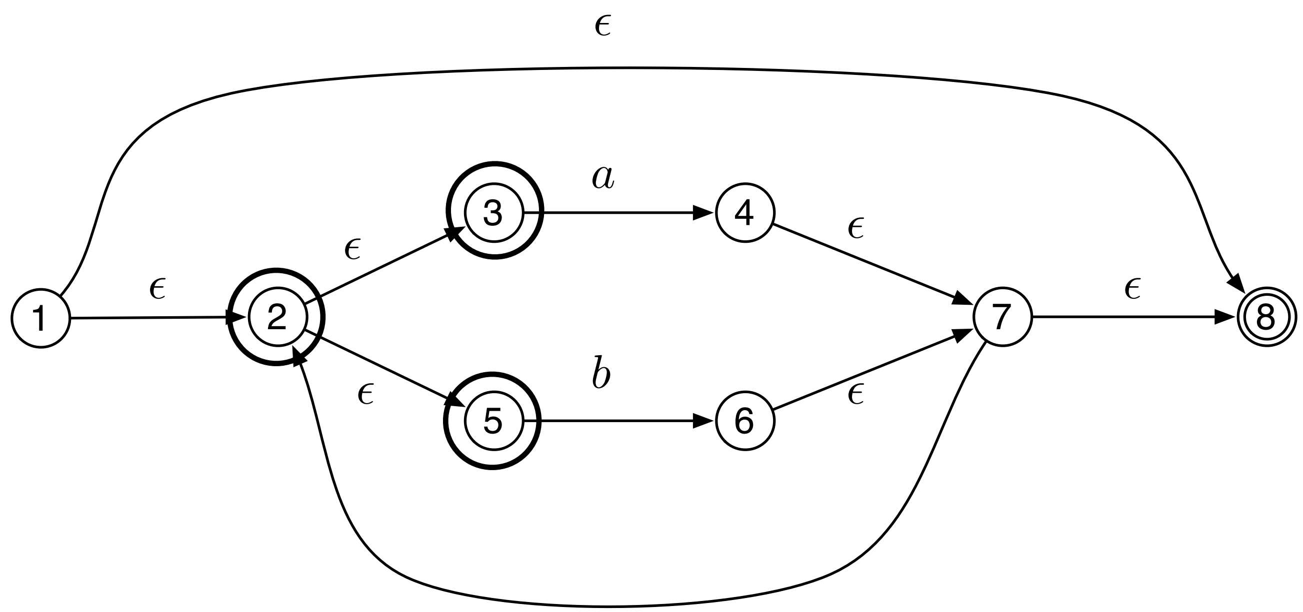
- Compute $\mathrm{Move}(S, \mathrm{a})$ where $S =$

1	2	3	4	5	6	7	8
0	1	1	0	1	0	0	0

:

	1	2	3	4	5	6	7	8
$S >> 1$	0	0	1	1	0	1	0	0
$\&\ D_\mathrm{a}$	0	0	0	1	0	0	0	0
	0	0	0	1	0	0	0	0

Example: $S' := (S >> 1)\ \&\ D_\alpha$



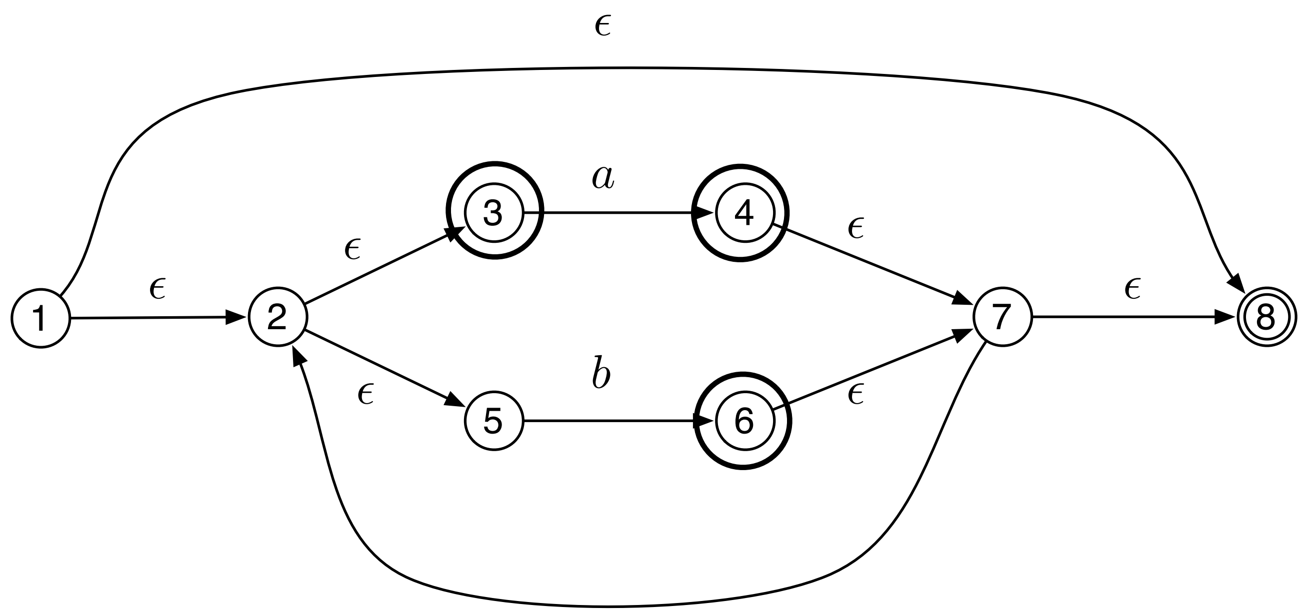
- Compute $\mathrm{Move}(S, \mathrm{a})$ where $S =$

1	2	3	4	5	6	7	8
0	1	1	0	1	0	0	0

:

	1	2	3	4	5	6	7	8
$S >> 1$	0	0	1	1	0	1	0	0
$\&\ D_\mathrm{a}$	0	0	0	1	0	0	0	0
	0	0	0	1	0	0	0	0

Example: $S' := (S >> 1)\ \&\ D_\alpha$



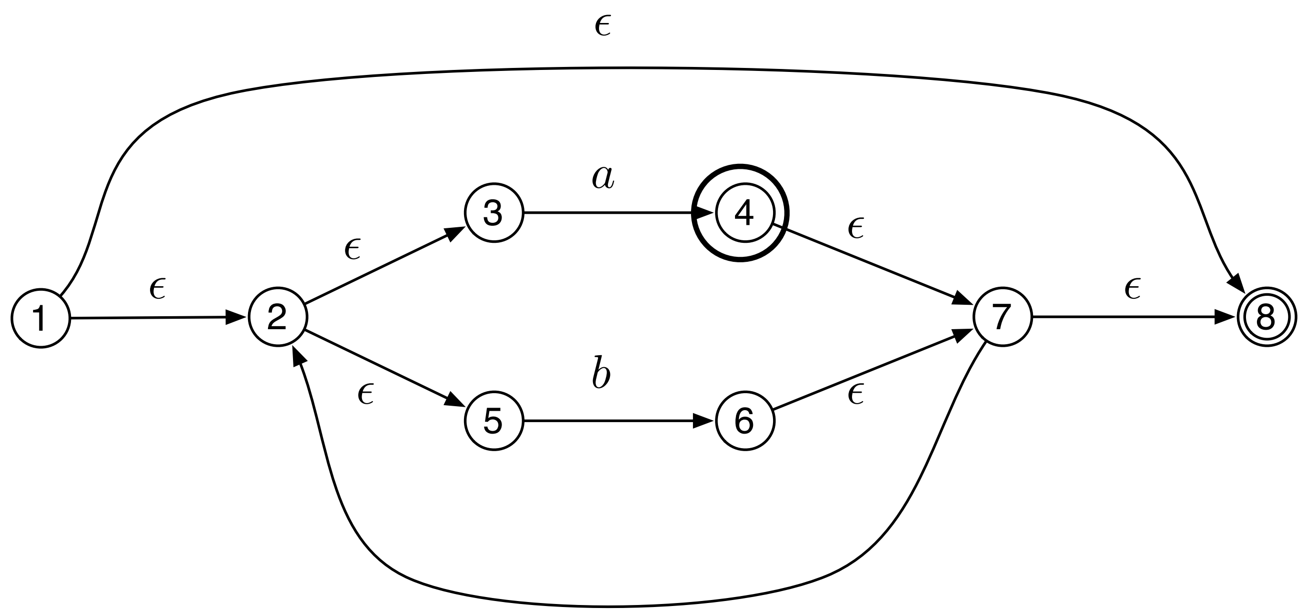
- Compute $\mathrm{Move}(S, \mathrm{a})$ where $S =$

1	2	3	4	5	6	7	8
0	1	1	0	1	0	0	0

:

	1	2	3	4	5	6	7	8
$S >> 1$	0	0	1	1	0	1	0	0
$\&\ D_\mathrm{a}$	0	0	0	1	0	0	0	0
	0	0	0	1	0	0	0	0

Example: $S' := (S >> 1)\ \&\ D_\alpha$



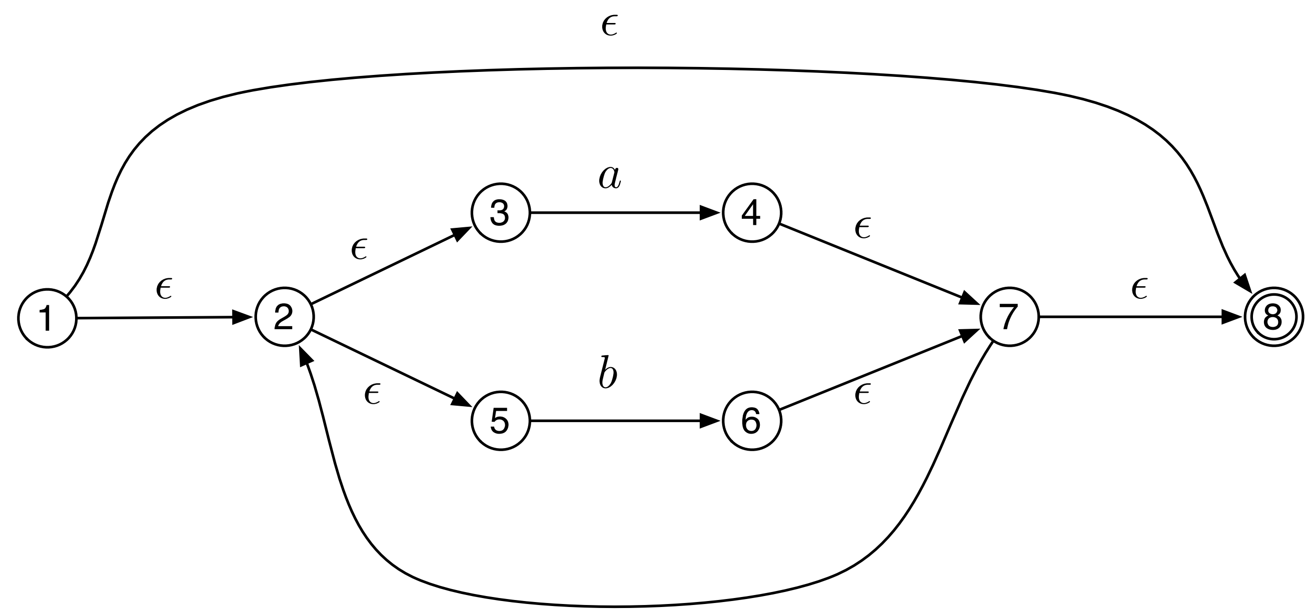
- Compute $\mathrm{Move}(S, \mathrm{a})$ where $S =$

1	2	3	4	5	6	7	8
0	1	1	0	1	0	0	0

:

	1	2	3	4	5	6	7	8
$S >> 1$	0	0	1	1	0	1	0	0
$\&\ D_{\mathrm{a}}$	0	0	0	1	0	0	0	0
	0	0	0	1	0	0	0	0

Close Operation: Preprocessing



- Encode ϵ-paths compactly:

$E =$

0	E_1	0	E_2	0	E_3	0	E_4	0	E_5	0	E_6	0	E_7	0	E_8

$E_2 =$

1	2	3	4	5	6	7	8
1	1	0	1	0	1	1	0

Close Operation: Preprocessing

- 3 constant bit strings for doing word tricks:

$$I = (10^m)^m$$

$$X = 1(0^m)^{m-1}$$

$$C = 1(0^{m-1})^{m-1}$$

Close Operation: Simulation

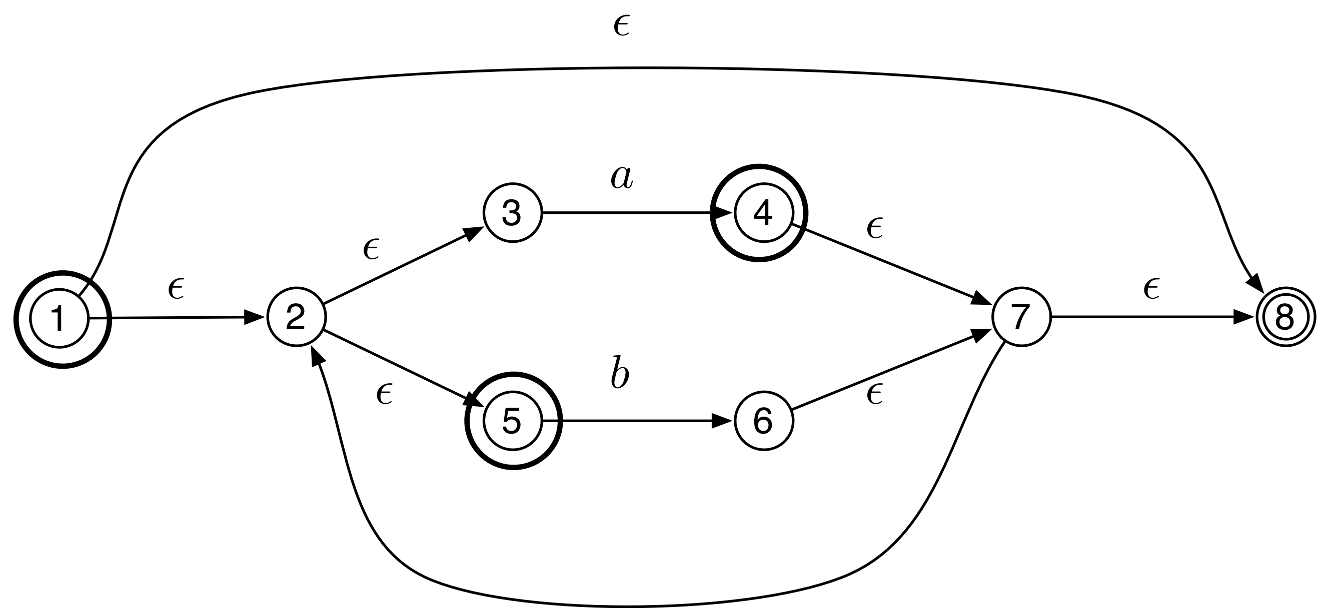
- $\mathsf{Close}(S)$ is computed as:

$$Y := (S \times X) \,\&\, E$$

$$Z := ((Y \mid I) - (I >> m)) \,\&\, I$$

$$S' := ((Z \times C) << w - m(m+1)) >> w - m$$

Example:



- Compute $\mathsf{Close}(S)$ where $S =$

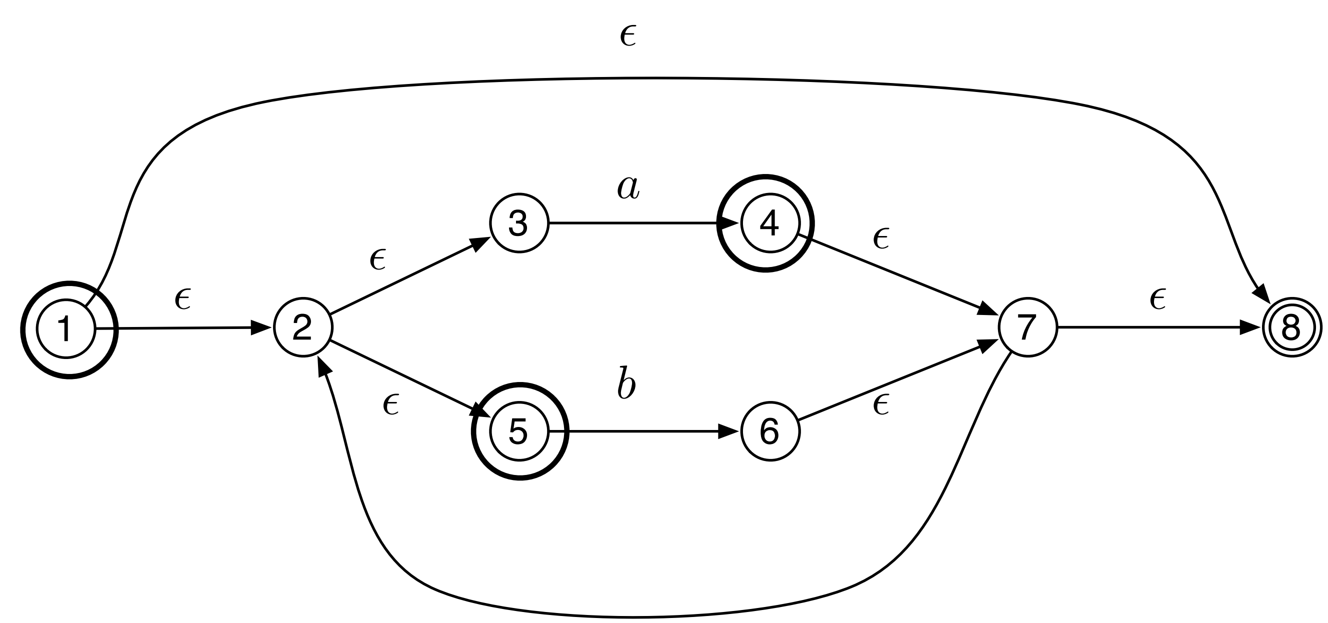
1	2	3	4	5	6	7	8
1	0	0	1	1	0	0	0

.

Step 1: $Y := (S \times X)\ \&\ E$

$S \times X =$	0	S	0	S	0	S	0	S	0	S	0	S	0	S	0	S
$\&E$	0	E_1	0	E_2	0	E_3	0	E_4	0	E_5	0	E_6	0	E_7	0	E_8
$Y =$	0	Y_1	0	Y_2	0	Y_3	0	Y_4	0	Y_5	0	Y_6	0	Y_7	0	Y_8

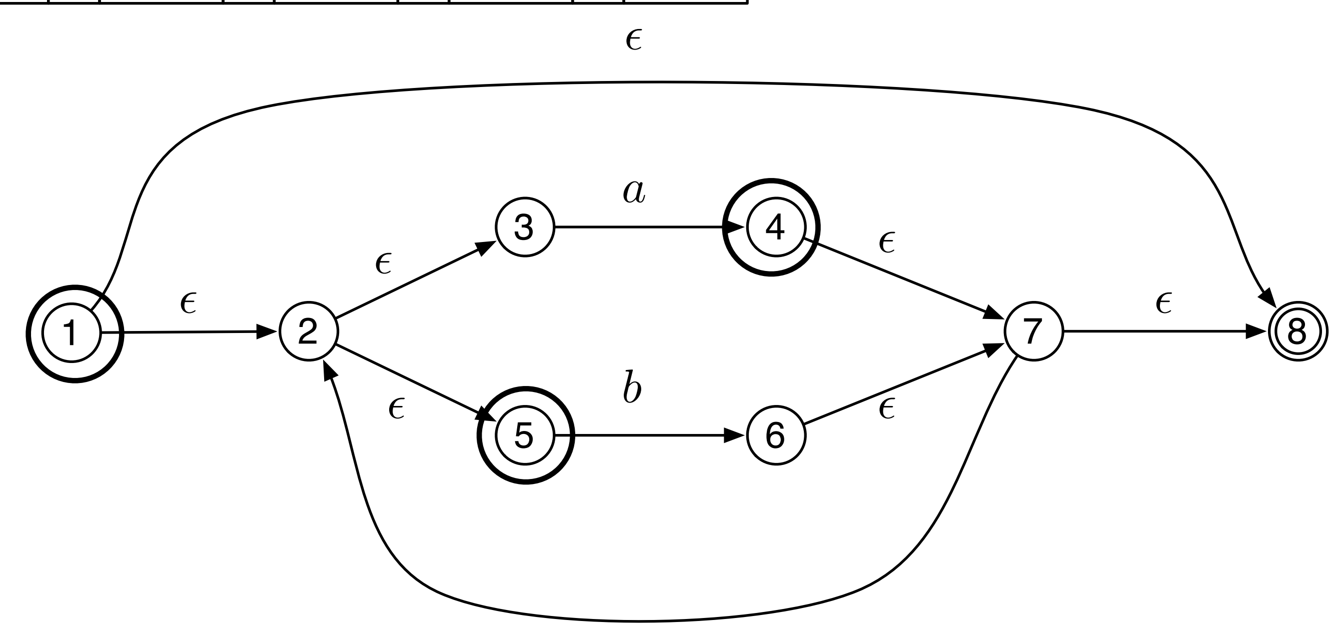
	1	2	3	4	5	6	7	8
$S =$	1	0	0	1	1	0	0	0
$\&E_2$	1	1	0	1	0	1	1	0
$Y_2 =$	1	0	0	1	0	0	0	0



Step 2: $Z := ((Y \mid I) - (I >> m))\ \&\ I$

$Y \mid I =$	1	Y_1	1	Y_2	1	Y_3	1	Y_4	1	Y_5	1	Y_6	1	Y_7	1	Y_8
$-(I >> m))$	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
	1	$*$	1	$*$	1	$*$	1	$*$	1	$*$	0	$*$	1	$*$	1	$*$
$\& I$	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0
$Z =$	1	0	1	0	1	0	1	0	1	0	0	0	1	0	1	0

		1	2	3	4	5	6	7	8
$Y_2 =$	1	1	0	0	1	0	0	0	0
$-$	0	0	0	0	0	0	0	0	1
	1	1	0	0	0	1	1	1	1



Step 3: $S' := ((Z \times C) << w - m(m+1)) >> w - m$

- $Z \times C$ produces a bit string containing the test bits of Z as a consecutive substring.

- Shifts clears remaining bits and aligns the substring.

Complexity

- Lemma: For TNFAs with $O(\sqrt{w})$ states we can support Move and Close in constant time using $O(m)$ space and $O(m^2)$ preprocessing.

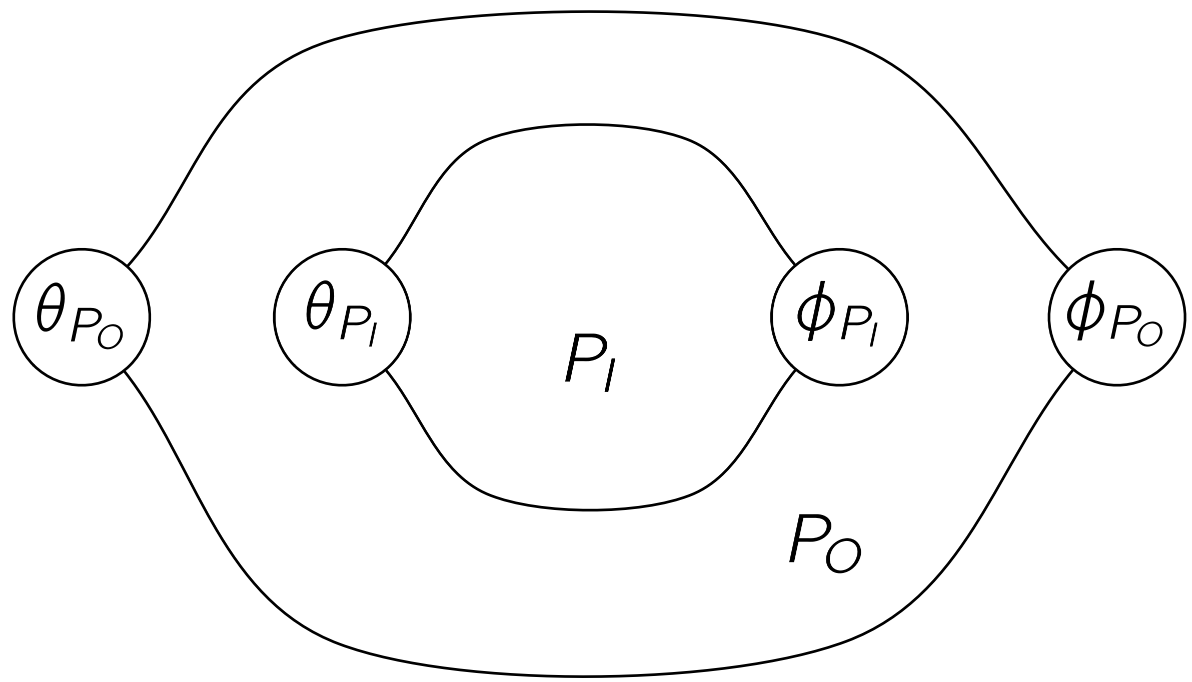
- => For string Q and regular expression R of lengths n and $m = O(\sqrt{w})$ regular expression matching can be solved in $O(n + m^2)$ time and $O(m)$ space.

Another Algorithm

- Main bottleneck: Need an $\Omega(m^2)$ length string to represent the transitive closure of ϵ-transitions.

- Idea: Compute a “good” separator for TNFAs and use a Divide-and-Conquer strategy.

Separator Property of TNFA



- There exists two states θ_{P_I} and ϕ_{P_I} whose removal partitions a TNFA into two subgraphs, P_I and P_O, of roughly equal size such that:

- Any path from P_O to P_I goes through θ_{P_I}.

- Any path from P_I to P_O goes through ϕ_{P_I}.

Recursive Closure Algorithm

1. Determine which of θ_{P_I} and ϕ_{P_I} are ϵ-reachable

2. Update the state-set accordingly.

3. Recurse in parallel on P_I and P_O.

Complexity

- Each of the $O(\log m)$ levels of recursion can be handled in parallel in constant time.

- => Lemma: For TNFAs with $m = O(w)$ states we can support Move and Close in $O(\log m)$ time using $O(m)$ space and $O(m \log m)$ preprocessing.

- => For string Q and regular expression R of lengths n and $m = O(w)$, resp., regular expression matching can be solved in time $O(n \log m + m \log m)$ and space $O(m)$.

Plug and Play

- Time:

$$\begin{cases} O(n\frac{m\log w}{w} + m\log w) & \text{if } m > w \\ O(n\log m + m\log m) & \text{if } \sqrt{w} < m \leq w \\ O(\min(n + m^2, n\log m + m\log m)) & \text{if } m \leq \sqrt{w}. \end{cases}$$

- Space: $O(m)$

Core Techniques

- Data Structures: Organize information efficiently.
    - Nearest common ancestors, firstlabel, dictionaries, dynamic perfect hashing, predecessors.
- Tree Techniques: Use combinatorial properties of trees.
    - Heavy-path decomposition, varieties of tree clusterings with or without macro trees.
- Word-Level Parallelism: Encode and simulate algorithms using arithmetic and logical instructions of the word RAM.
    - Four Russian technique, word level-parallelism.

Future Research

- Bring state-of-the-art techniques to combinatorial pattern matching and related areas. Many important problems need them!
- Use developed algorithms to improve practical applications (e.g., bioinformatics, XML data bases).
  - Word parallel regular expression matching looks promising.

Thanks!