Inverse problems in security screening

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Security screening

- In security screening we need to detect *threat objects*
- For airport *hold luggage* screening this is bombs.
- For passengers and cabin luggage it is a variety of weapons and bomb components
- In airports screening is *regulation driven*
- In other contexts less regulated.
- In this talk I will cover x-ray baggage screening and metal detectors

Outline

1. The problem of X-ray CT for airport security
2. Sketch of multi-surface rebinning reconstruction algorithm
3. Firing order and algebraic reconstruction
4. Unstable uniqueness
5. Scattering and Montecarlo simulation.

Airport baggage screening

- Hold luggage has to be checked for bombs Explosive + detonator + actuator
- Mostly bags are x-rayed (radiograph) and an operator looks at the picture
- Explosives can be made in a thin sheet or concealed by denser objects
- Three dimensional images allow the operator to identify objects better and improve automatic screening
- Some airports have hospital style X-ray CT scanners. Slow and expensive.
- Rapiscan have developed a fast X-ray CT machine for airport baggage, but it is not the usual geometry...

Some pictures

A GE CTX conventional CT machine

A Rapiscan RTT 80 real time tomography system

CLICK FOR ANIMATION OF GEOMETRY

A case study in Industrial Mathematics I

- (2006) Rapiscan, specifically RTT inventor Dr Ed Morton, approached us initially for a ‘scoping exercise’ for the reconstruction problem for the RTT. They had an early prototype system
- We found that there was nothing published on reconstruction for this type of system
- (2007) We applied for a grant (EPSRC Maths for Industry and Business) with matching funding from Rapiscan.
- While waiting for that we implemented a ‘fast and dirty’ method
- Got the grant and recruited a team. None had worked on CT before!
- Three main aspects to the work
    1. Reconstruction algorithm
    2. Scatter correction and modelling
    3. Optimal firing order (not necessarily sequential)
- (2010) Reconstruction algorithms implemented, patented, and tested. Implementation in real system.

A case study in Industrial Mathematics II

- Scatter modelling successful and mathematician who did this seconded to Rapiscan to transfer expertise
- Firing order implemented and tested on simulated data, and patent filed.
- (2013) RTT 110 get regulatory approval
- (2014) RTT 110 systems in production and operational at airports

What is known about X-ray reconstruction?

The X-ray transform of a function f on $\mathbb{R}^3$ is

$$Xf(\xi, x) = \int_{-\infty}^{\infty} f(x + \xi t)dt$$

the integral over all lines. Note lines in $\mathbb{R}^3$ form a 4-manifold (as wlog $x \in \xi^{\perp}, |\xi| = 1$) so formally overdetermined.
By contrast lines in $\mathbb{R}^2$ form a 2-manifold so the 2D "Radon transform" correctly determined. There is an efficient explicit inversion for this case called 'filtered back projection'.
In the 3D case clearly some three dimensional submanifolds of lines are sufficient. For example for some η consider all lines perpendicular to η. In each plane perpendicular to η one can invert using the 2D Radon transform inversion.
The Radon plane transform of a function on $\mathbb{R}^3$ is

$$Rf(\xi, s) = \int_{x \cdot \xi = s} f(x)dx$$

Again this has an explicit inversion and can be useful to analyse the x-ray transform.

Helical cone beam data

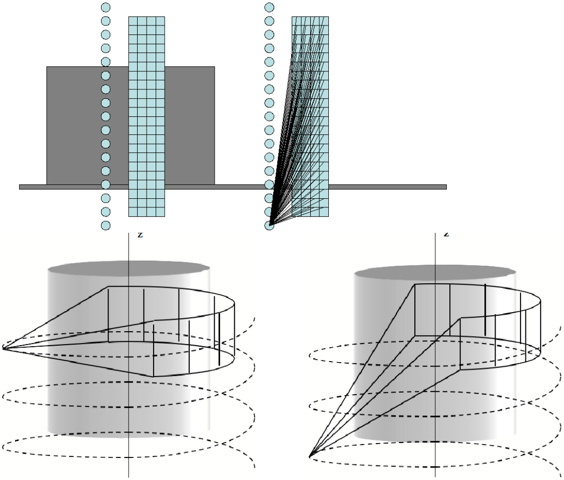
In practical situations often the X-ray source travels on a curve relative to the object (support of f). If we can then use a 2D detector array to measure integrals of lines through that curve. There are several reconstruction algorithms for specific configurations with a large enough detector array and sources on a helix. For example Katsevich gives an exact reconstruction algorithm related to the 2D filtered back projection reconstruction. It can be implemented as a relatively fast algorithm. It requires a specific "window of data" but does not use all the data from a rectangular detector (so less stable).

Rebinning methods

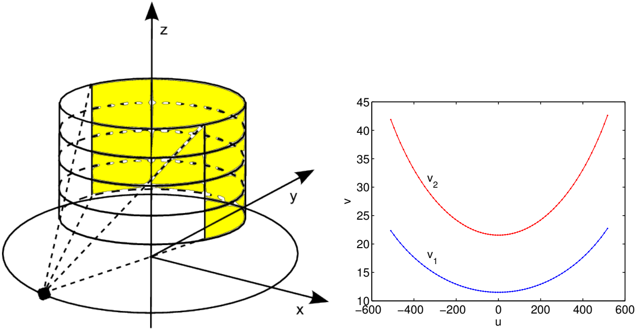
One approximate algorithm used is to interpolate or “re-bin” the data for lines in (tilted) planes, then use 2D reconstruction on each plane. This amounts to *interpolation* the rays on the planes by those we do measure. One can also find a family of non-flat surfaces that lie closer to lines that are measured. One then uses 2D reconstruction on the projection of these surfaces on to a plane. This is called **surface rebinning** [DNK1]. The optimal surface is found by a fixed-point iteration that minimizes the least squares distance between teh rays and the surface.
None of these methods are much help for the RTT. The ring of detectors is offset from the ring of sources. So while we can measure with sources on a helix we do not measure enough of the rays as the sensor is truncated.

RTT offset geometry

RTT System Geometry



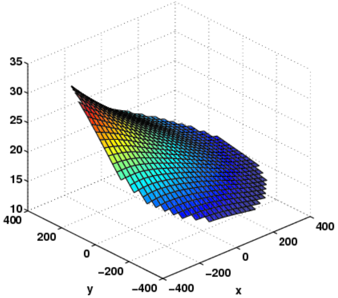
Virtual detector plane and active area



Left Illustration of the offset geometry with an exaggerated *z*-scale. In the RTT80 the source ring has a larger radius than the detector ring. The yellow area indicates where detectors are active for the indicated source. **Right** The image of the active cylindrical detector on the virtual flat panel detector containing the *z* axis. As usual *u* and *v* are coordinates of the virtual detector, with the origin being the orthogonal projection of the the active source on the virtual detector plane. The functions v1,v2 are the lower and upper bounds, of the active area of the virtual flat detector, respectively.

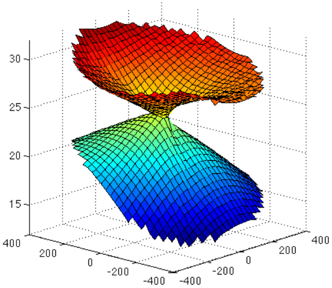
Surface rebinning for the RTT

For the conventional geometry even the optimal rebinning surface is quite close to flat so not much advantage. For the RTT geometry one can find a surface close to rays and rebinning on this surface works to some extent. But as rays are only going “up” (in our picture) so rays used for interpolation are not so close to this surface..



Multi-sheet surface rebinning for the RTT

Marta Betcke [BL,BL1] came up with the idea of finding a surface with two sheets. Roughly half of each line is close to one sheet and half to the other.



Applying 2D reconstruction to the projection on to the (horizontal) plane gives us the sum of the values of f on the two sheets of the surface above each point.
But as each point in the object is on both an upward and a downward sheet of the surface we have sparse simultaneous equations for the values of f at each point (still fast)

Multi-sheet rebinning details

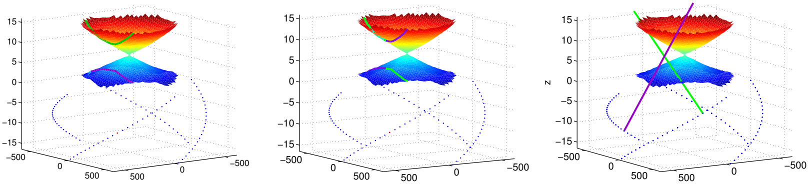
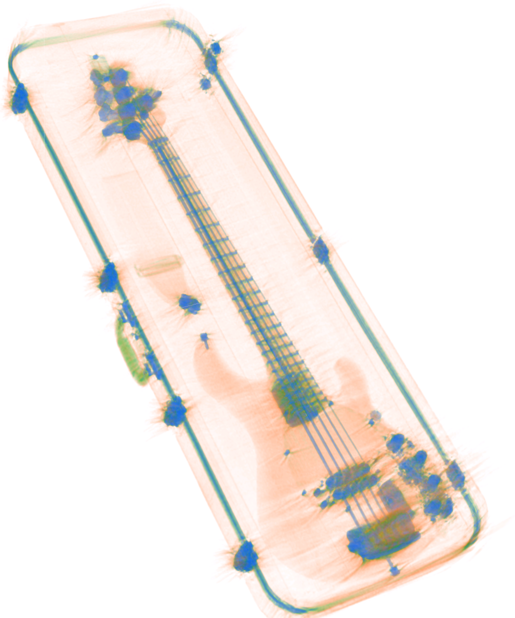


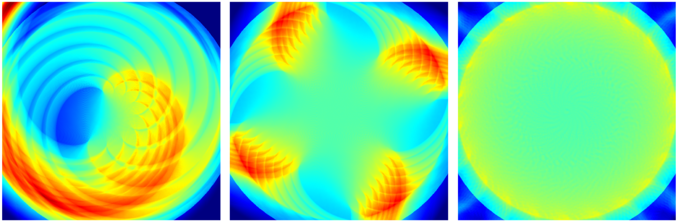
Illustration of the two-sheet surface rebinning idea. Two-sheet surface with highlighted (a) fan beam transform on the two-sheet surface (purple) and (green). (b) mixed fan beam transforms (green) and (purple). (c) rebinned cone beam rays (green) and (purple)

It works!



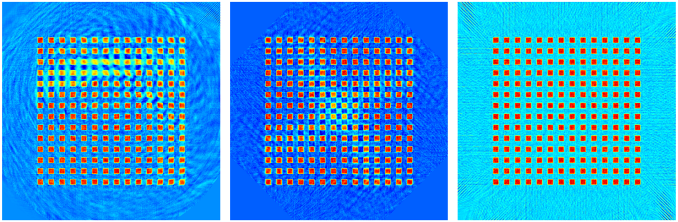
Firing order

The electronically switch sources mean we can fire in any order. We can think of the source trajectory as a helix, or a multi threaded helix, or just as a sampling pattern on the cylinder. One way to compare how good the firing order is is to look at the density of rays hitting each pixel. Shown here for sequential firing (left), the pattern currently in use in the RTT (middle) and the optimal spacing 35 (right). (details see [TL])



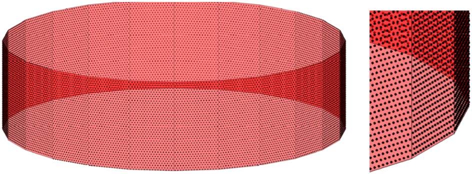
Algebraic Reconstruction

If we do not need the fast reconstruction our multisheet rebinning offers we can simply solve a sparse system of equations (eg with Conjugate Gradient Least Squares). This works for any firing order and allows us to study the condition number. Firing every 35th source in the RTT80 (with 768 sources) gives the best condition number as well as ray density. Again sequential firing (left), the pattern currently in use in the RTT (middle) and the optimal spacing 35 (right).



Lattice sampling

While a source firing order might be thought of as a source trajectory, such as a (multi) helix in fact it is a discrete set on a cylinder. We can arrange the sources to be roughly uniformly distributed on a (polygonal) cylinder (in fact this is what our optimal firing order achieves).



Uniqueness but instability

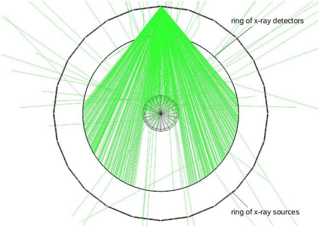
Often in inverse problems we look at a continuum limit in which we have an infinite number of measurements, and then ask if the solution is unique in this case. For the RTT with this lattice sampling idea we measure sample of an *four-dimensional* data set. We can consider this as a truncated data set for the Radon plane transform, with the angle normal ξ to the axial direction in an interval. The Fourier slice theorem for the Radon plane transform gives

$$\widehat{Rf(\xi,\sigma)} = 2\pi \hat{f}(\sigma\xi)$$

where the first FT is with respect to the s variable only, with frequency var σ.
For f compactly supported $\hat{f}$ is analytic (Paley-Wiener), so we see knowing $\hat{f}$ on an open set determines f uniquely but the reconstruction is unstable. This is typical of limited data problems in CT.

Modelling scatter

The RTT configuration means that detectors are not collimated, so we measured scattered x-rays. We need to model this to correct for scatter. Or possibly in the future use scattered photons to help detect bombs. We use Geant4 Monte Carlo simulations to model scatter details see [MWL]



References for X-ray CT

K Alexander Katsevich (2002), Theoretically Exact Filtered Backprojection-Type Inversion Algorithm for Spiral CT, SIAM Journal on Applied Mathematics, 62 (6), 2012-2026

DNK1 M Defrise, F Noo, H Kudo (2001) Rebinning-based algorithms for helical cone-beam CT Phys. Med. Biol. 46 (2001) 29112937 Marta Betcke, William Lionheart, Edward Morton (2009) US Patent Application 61225257 "Method for Image Reconstruction by Using Multi-Sheet Surface Rebinning"

BL MM Betcke, WRB Lionheart, (2013). Multi-sheet surface rebinning methods for reconstruction from asymmetrically truncated cone beam projections: I. Approximation and optimality. Inverse Problems 29, 115003

BL1 MM Betcke, WRB Lionheart, (2013). Multi-sheet surface rebinning methods for reconstruction from asymmetrically truncated cone beam projections: II. Axial deconvolution. Inverse Problems 29, 115004-115004,

TL William M. Thompson and William R. B. Lionheart, Optimisation of the Source Firing Pattern for Real Time

People screening

- Metal detectors detect highly conductive objects and can detect threat objects such as guns and knives
- They cannot detect ceramic or titanium knives


- All practical (not single use) guns have a steel barrel
- Other people scanning technologies for non-conductive threats include: x-ray backscatter, mm wave (active and passive), ultrawide band rf.

Walkthrough metal detectors

Ceia (left) and Rapiscan (right)

Multistatic measurement

Currently deployed technology excites one coil and measures on one coil. The coil pair with the largest signal indicates approximate height of metal object. The new idea is to measure on several coils when each drive coil is excited and reconstruct the location of the object (like magnetic induction tomography)

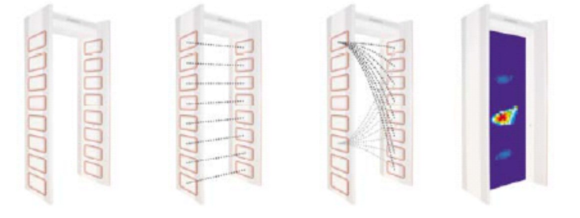


Figure 1. *EM Tomography and WTMD's*

Location and characterization

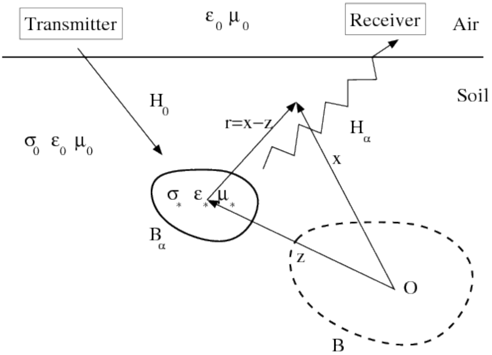
Locating a few metal objects is not so hard as the perturbation in the megnetic field is approximately like that due to a dipole source at the object.

With a bit more work we can distinguish between different combinations of conductive and ferromagnetic objects, and tell if an object is long and thin, flat or roughly spherical, and which way it is pointing.

Also useful for detecting anti-personel land mines and unexploded ordinance (UXO).

Illustration of magnetic induction for metal detection

Consider a single transmitter-receiver coil arrangement



- $B_\alpha = \alpha B + \mathbf{z}$;
- $\sigma_* \gg \sigma_0$ and therefore $\sigma_0 \approx 0$ in soil;
- $\mathbf{H}_0$ generated by an alternating current source $\mathbf{J}_0$ satisfying $\nabla \cdot \mathbf{J}_0 = 0$;
- $\sqrt{\epsilon_* \mu_*} \alpha \omega \ll 1$ and eddy current approximation applies.

Eddy current equations

The incident field $\mathbf{H}_0$ and $\mathbf{E}_0$ satisfy

$$\begin{aligned} \nabla \times \mathbf{E}_0 &= \mathrm{i}\omega\mu_0 \mathbf{H}_0 && \text{in } \mathbb{R}^3, \\ \nabla \times \mathbf{H}_0 &= \mathbf{J}_0 && \text{in } \mathbb{R}^3. \end{aligned}$$

and interaction fields $\mathbf{E}_\alpha$, $\mathbf{H}_\alpha$

$$\begin{aligned} \nabla \times \mathbf{E}_\alpha &= \mathrm{i}\omega\mu_\alpha \mathbf{H}_\alpha && \text{in } \mathbb{R}^3, \\ \nabla \times \mathbf{H}_\alpha &= \sigma_\alpha \mathbf{E}_\alpha + \mathbf{J}_0 && \text{in } \mathbb{R}^3, \\ \mathbf{E}_\alpha(\mathbf{x}) = O(|\mathbf{x}|^{-1}),\ \mathbf{H}_\alpha(\mathbf{x}) &= O(|\mathbf{x}|^{-1}) && \text{as } |\mathbf{x}| \to \infty, \end{aligned}$$

where

$$\mu_\alpha = \begin{cases} \mu_* & \text{in } B_\alpha \\ \mu_0 & \text{in } \mathbb{R} \setminus B_\alpha \end{cases}, \sigma_\alpha = \begin{cases} \sigma_* & \text{in } B_\alpha \\ 0 & \text{in } \mathbb{R} \setminus B_\alpha \end{cases}.$$

The object is conducting and non–ferrous, $\mu_r := \mu_*/\mu_0 = O(1)$ and $\nu := \alpha^2/(2\Delta^2) = O(1)$ as $\alpha \to 0$, where $\delta = \sqrt{2/(\sigma_*\mu_0\omega)}$ is the skin depth.

<u>Goal</u>: To find an asymptotic expansion for $(\mathbf{H}_\alpha - \mathbf{H}_0)(\mathbf{x})$ as $\alpha \to 0$, which describes the shape and material properties of the hidden

Engineering prediction

Ideally, one would like a cheap way of detecting the shape, material properties and location of the hidden conducting object from $(\mathbf{H}_\alpha - \mathbf{H}_0)(\mathbf{x})$.

Engineers predict $\hat{\mathbf{q}} \cdot (\mathbf{H}_\alpha - \mathbf{H}_0)(\mathbf{x})$ to be proportional to linear combinations of components of $\mathbf{H}_0^T$ and $\mathbf{H}_0^M$

- $\mathbf{H}_0^T$ Incident field generated by the transmitter coil, evaluated at the target;
- $\mathbf{H}_0^M$ Incident field generated by the receiver coil, evaluated at the target;

and assume that this $\mathbf{H}^T.\mathbf{H}^M$ sensitivity can be expressed as

$$\hat{\mathbf{q}} \cdot (\mathbf{H}_\alpha - \mathbf{H}_0)(\mathbf{x}) \approx \mathbf{H}^T \cdot \mathbf{m} = \mathbf{H}^T \cdot (\mathcal{M}\mathbf{H}^M),$$

- Can a symmetric polarisation tensor $\mathcal{M}$ be found that only on the shape and material properties of the object? If so, this might offer a cheap means of detection.

Ammari et al 2013 [ACCGV] proved that there was such a formula but using a rank 4 tensor depending on the object but independent of position,
Ledger and Lionheart 2014 [LL] prove that due to symmetries the rank 4 tensor reduces to a symmetric rank 2-tensor $\mathcal{M}$ that can be calculated by solving a transmission problem for a system of PDEs. We calculated this for several examples using finite elements and compared the actual perturbed fields with the predictions from this asymptotic expansion.

References for metal detectors

NW S.J. Norton and I.J. Won,Identification of buried unexploded ordnance from broadband electromagnetic induction data, IEEE Transaction on Geoscience and Remote Sensing, vol. 39, 2001.

ACCGV H. Ammari, J. Chen, Z. Chen, J. Garnier, and D. Volkov, Target detection and characterization from electromagnetic induction data, Journal de Mathématiques Pures et Appliqués, , 2013.

LL P.D. Ledger and W.R.B Lionheart, Characterising the shape and material properties of hidden targets from magnetic induction data, IEEE Trans Mag, under review 2014.