What You Can See in Limited Data Tomography

Todd Quinto

Tufts University
Medford, Massachusetts, USA

https://math.tufts.edu/faculty/equinto/

DTU Training School: Scientific Computing for X-Ray Computed Tomography (CT), January 6, 2021

(Partial support from U.S. NSF, Otto Mønsteds Fond, Simons Foundation)
I’m happy to talk with you today! If you have the bandwidth, please keep your video on, so I see how you’re doing.
X-ray Tomography

I’m happy to talk with you today! If you have the bandwidth, please keep your video on, so I see how you’re doing.

**X-ray Tomography (CT or CAT) is the mathematics, science, and engineering used to find internal information about an object using X-ray images.**
I’m happy to talk with you today! If you have the bandwidth, please keep your video on, so I see how you’re doing.

X-ray Tomography (CT or CAT) is the mathematics, science, and engineering used to find internal information about an object using X-ray images.

Our Goals:

1. Learn a little bit of the history of CT.
I’m happy to talk with you today! If you have the bandwidth, please keep your video on, so I see how you’re doing.

**X-ray Tomography (CT or CAT)** is the mathematics, science, and engineering used to find internal information about an object using X-ray images.

**Our Goals:**

1. Learn a little bit of the history of CT.
2. Learn what limited data tomography is.
I’m happy to talk with you today! If you have the bandwidth, please keep your video on, so I see how you’re doing.

**X-ray Tomography (CT or CAT)** is the mathematics, science, and engineering used to find internal information about an object using X-ray images.

**Our Goals:**

1. Learn a little bit of the history of CT.
2. Learn what limited data tomography is.
3. Determine what features of the body will be easy to reconstruct from limited CT data, and which will be difficult.
I’m happy to talk with you today! If you have the bandwidth, please keep your video on, so I see how you’re doing.

**X-ray Tomography (CT or CAT)** is the mathematics, science, and engineering used to find internal information about an object using X-ray images.

**Our Goals:**

1. Learn a little bit of the history of CT.
2. Learn what limited data tomography is.
3. Determine what features of the body will be easy to reconstruct from limited CT data, and which will be difficult.
4. Understand, geometrically, how this depends on the data.
Some History: The first CAT Scanner
Some History: The first CAT Scanner
Some History: The first CAT Scanner

©The New Yorker
1960s: Allan Cormack, a medical physicist at Groote Schuur Hospital in Johannesburg, South Africa, saw doctors taking X-ray pictures of people all the time.
1960s: Allan Cormack, a medical physicist at Groote Schuur Hospital in Johannesburg, South Africa, saw doctors taking X-ray pictures of people all the time. He thought one could do better!
1960s: Allan Cormack, a medical physicist at Groote Schuur Hospital in Johannesburg, South Africa, saw doctors taking X-ray pictures of people all the time. He thought one could do better!

He became a professor in the Physics Department of my university (Tufts University, U.S.A.).
1960s: Allan Cormack, a medical physicist at Groote Schuur Hospital in Johannesburg, South Africa, saw doctors taking X-ray pictures of people all the time.

He thought one could do better!

He became a professor in the Physics Department of my university (Tufts University, U.S.A.).

**Allan’s big idea (from around 1960):** take X-ray pictures from different directions to give “perspective” on the patient.
Allan Cormack, a medical physicist at Groote Schuur Hospital in Johannesburg, South Africa, saw doctors taking X-ray pictures of people all the time.

He thought one could do better!

He became a professor in the Physics Department of my university (Tufts University, U.S.A.).

**Allan’s big idea (from around 1960):** take X-ray pictures from different directions to give “perspective” on the patient.

**Allan’s proof of concept:**
1960s: Allan Cormack, a medical physicist at Groote Schuur Hospital in Johannesburg, South Africa, saw doctors taking X-ray pictures of people all the time.

He thought one could do better!

He became a professor in the Physics Department of my university (Tufts University, U.S.A.).

Allan’s big idea (from around 1960): take X-ray pictures from different directions to give “perspective” on the patient.

Allan’s proof of concept:
- He developed a math algorithm to image objects from CT data. (two algorithms—Radon)
1960s: Allan Cormack, a medical physicist at Groote Schuur Hospital in Johannesburg, South Africa, saw doctors taking X-ray pictures of people all the time.

He thought one could do better!

He became a professor in the Physics Department of my university (Tufts University, U.S.A.).

Allan’s big idea (from around 1960): take X-ray pictures from different directions to give “perspective” on the patient.

Allan’s proof of concept:

- He developed a math algorithm to image objects from CT data. (two algorithms—Radon)
- He built a CT scanner, and successfully tested the algorithm using the scanner!
1960s: Allan Cormack, a medical physicist at Groote Schuur Hospital in Johannesburg, South Africa, saw doctors taking X-ray pictures of people all the time. He thought one could do better!

He became a professor in the Physics Department of my university (Tufts University, U.S.A.).

Allan’s big idea (from around 1960): take X-ray pictures from different directions to give “perspective” on the patient.

Allan’s proof of concept:

- He developed a math algorithm to image objects from CT data. (two algorithms—Radon)
- He built a CT scanner, and successfully tested the algorithm using the scanner!

Allan won the 1979 Nobel Prize in Medicine! (early AM)...taught!
Cormack’s CT Scanner

Allan + Scanner

His original calculations
Cormack’s CT Scanner

Allan + Scanner

His original calculations

Cost: DKK 2400

Nobel Prize!!
Modern GE scanner

GE Reconstruction
Modern GE scanner

Cost: DKK 12,000,000

GE Reconstruction
The Mathematical Model of X-ray CT

$f$ a function in the plane representing the density of an object
$L$ a line in the plane over which the photons travel.

The X-ray (Radon) Transform:

\[
\mathcal{R} f(L) = \int_{x \in L} f(x) \, ds
\]

–The ’amount’ of material on the line the X-rays traverse.

Tomographic Data: $\mathcal{R} f(L)$
The Mathematical Model of X-ray CT and the Goal

\( f \) a function in the plane representing the density of an object

\( L \) a line in the plane over which the photons travel.

The X-ray (Radon) Transform:

\[
\mathcal{R}f(L) = \int_{x \in L} f(x) \, ds
\]

–The ‘amount’ of material on the line the X-rays traverse.

**The goal:** Recover a picture of the body (values of \( f(x) \)), from X-ray CT data over a finite number of lines.
The Mathematical Model of X-ray CT and the Goal

\( f \) a function in the plane representing the density of an object

\( L \) a line in the plane over which the photons travel.

The X-ray (Radon) Transform:

\[
\mathcal{R} f(L) = \int_{x \in L} f(x) \, ds
\]

–The ’amount’ of material on the line the X-rays traverse.

**The goal:** Recover a picture of the body (values of \( f(x) \)), from X-ray CT data over a finite number of lines.

With **complete data** (lines throughout the object in fairly evenly spaced directions), good reconstruction methods exist (e.g., Filtered Backprojection [Natterer, Natterer-Wübbling]).
**Parallel Beam Scanning Geometry**

**The angle:** \( \theta \in [0^\circ, 360^\circ] \quad \vec{\theta} = (\cos(\theta), \sin(\theta)) \)

**The lines over which X-rays travel:** \( L_{\theta,s} \) is the line perpendicular to \( \theta \) and \( s \) units from the origin (in the opposite direction of \( \theta \) if \( s < 0 \)) (~fan beam but simpler)

\[
\theta \in [0^\circ, 360^\circ]
\]

\[
\vec{\theta} = (\cos(\theta), \sin(\theta))
\]

\[
L_{\theta,s}
\]

\[
\text{Note } L_{\theta+180^\circ,s} = L_{\theta,s}
\]
Parallel Beam Scanning Geometry

The angle: \( \theta \in [0^\circ, 360^\circ] \quad \vec{\theta} = (\cos(\theta), \sin(\theta)) \)

The lines over which X-rays travel: \( L_{\theta,s} \) is the line perpendicular to \( \theta \) and \( s \) units from the origin (in the opposite direction of \( \theta \) if \( s < 0 \))  

\( \sim \) fan beam but simpler

Note \( L_{\theta+180^\circ,-s} = L_{\theta,s} \)

Moral

Each line can be parameterized by a unique \((\theta, s) \in [0^\circ, 180^\circ] \times [-1, 1]\).  

Or redundantly by two \((\theta, s) \in [0^\circ, 360^\circ] \times [-1, 1]\).  

X-ray Tomographic Data

**The object:**  \( f \) is the density function of an object in the plane–inside the unit disk (radius 1 centered at \((0, 0)\)).

**Tomographic data:**  \( \mathcal{R}f(\theta, s) = \int_{x \in L_{\theta,s}} f(x) \, ds \) is calculated using X-rays traveling along the line \( L_{\theta,s} \).

In practice: a finite number of evenly distributed lines.

The **Data Domain** is the set of lines \( L_{\theta,s} \) over which data are taken–equivalently, the set of \((\theta, s)\) parameterizing those lines.
Complete X-ray Tomographic Data

**The object:** $f$ is the density function of an object in the plane—inside the unit disk (radius 1 centered at $(0, 0)$).

**Tomographic data:** $\mathcal{R} f(\theta, s) = \int_{x \in L_{\theta,s}} f(x) ds$ is calculated using X-rays traveling along the line $L_{\theta,s}$.

**Complete Tomographic Data:** X-ray data are given over all lines going through the body (e.g., $L_{\theta,s}$ for $(\theta, s) \in [0^\circ, 180^\circ] \times [-1, 1]$).

**The Data Domain** is the set of lines $L_{\theta,s}$ over which data are taken—equivalently, the set of $(\theta, s)$ parameterizing those lines.
The object: \( f \) is the density function of an object in the plane–inside the unit disk (radius 1 centered at \((0, 0)\)).

Tomographic data: \( \mathcal{R}f(\theta, s) = \int_{x \in L_{\theta,s}} f(x) \, ds \) is calculated using X-rays traveling along the line \( L_{\theta,s} \).

**Complete Tomographic Data:** X-ray data are given over all lines going through the body (e.g., \( L_{\theta,s} \) for \((\theta, s) \in [0^\circ, 180^\circ] \times [-1, 1]\)). In practice: a finite number of evenly distributed lines.

The Data Domain is the set of lines \( L_{\theta,s} \) over which data are taken–equivalently, the set of \((\theta, s)\) parameterizing those lines.
Limited and Complete X-ray Tomographic Data

The object: $f$ is the density function of an object in the plane–inside the unit disk (radius 1 centered at $(0, 0)$).

**Tomographic data:** $R f(\theta, s) = \int_{x \in L_{\theta, s}} f(x) \, ds$ is calculated using X-rays traveling along the line $L_{\theta, s}$.

**Complete Tomographic Data:** X-ray data are given over all lines going through the body (e.g., $L_{\theta, s}$ for $(\theta, s) \in [0^\circ, 180^\circ] \times [-1, 1]$). In practice: a finite number of evenly distributed lines.

**Limited Data Tomography:** When data over some lines are missing.
Limited and Complete X-ray Tomographic Data

The object: $f$ is the density function of an object in the plane—inside the unit disk (radius 1 centered at $(0,0)$).

**Tomographic data:** $\mathcal{R}f(\theta, s) = \int_{x \in L_{\theta,s}} f(x) \, ds$ is calculated using X-rays traveling along the line $L_{\theta,s}$.

**Complete Tomographic Data:** X-ray data are given over all lines going through the body (e.g., $L_{\theta,s}$ for $(\theta, s) \in [0^\circ, 180^\circ] \times [-1, 1]$). In practice: a finite number of evenly distributed lines.

**Limited Data Tomography:** When data over some lines are missing.

The **Data Domain** is the set of lines $L_{\theta,s}$ over which data are taken—equivalently, the set of $(\theta, s)$ parameterizing those lines.
Example of Limited Data Tomography

**Limited angle X-ray CT:** the scanner cannot move all the way around the object—it images the object from lines in a limited range of angles:
**Example of Limited Data Tomography**

**Limited angle X-ray CT:** the scanner cannot move all the way around the object—it images the object from lines in a limited range of angles:

*The Data Domain:* $S = [a^\circ, b^\circ] \times [-1, 1]$ ($0 < a < b < 180$), lines $L_{\theta,s}$ for $(\theta, s) \in S$ (more generally $0 < b - a < 180$).
**Example of Limited Data Tomography**

**Limited angle X-ray CT:** the scanner cannot move all the way around the object—it images the object from lines in a limited range of angles:

The Data Domain: \( S = [a^\circ, b^\circ] \times [-1, 1] \) (\( 0 < a < b < 180 \)),
lines \( L_{\theta,s} \) for \((\theta, s) \in S\) (more generally \( 0 < b - a < 180 \)).

**The missing data:** data *not* on lines \( L_{\theta,s} \) for \((\theta, s) \in S\).
**Example of Limited Data Tomography**

**Limited angle X-ray CT:** the scanner cannot move all the way around the object—it images the object from lines in a limited range of angles:

*The Data Domain:* $S = [a^\circ, b^\circ] \times [-1, 1]$ ($0 < a < b < 180$), lines $L_{\theta,s}$ for $(\theta, s) \in S$ (more generally $0 < b - a < 180$).

**The missing data:** data *not* on lines $L_{\theta,s}$ for $(\theta, s) \in S$.

**Example:** The set of $(\theta, s)$ over which data are taken, the *data domain*, includes only horizontal-ish lines—$L_{\theta,s}$ with $(\theta, s) \in S = [45^\circ, 135^\circ] \times [-1, 1]$.

Horizontal-*ish* lines are in the data domain, (shown in $(\theta, s)$ space on right).
Example of Limited Data Tomography

**Limited angle X-ray CT:** the scanner cannot move all the way around the object—it images the object from lines in a limited range of angles:

*The Data Domain:* \( S = [a^\circ, b^\circ] \times [-1, 1] \) \((0 < a < b < 180)\),
lines \( L_{\theta,s} \) for \((\theta, s) \in S\) \((\text{more generally } 0 < b - a < 180)\).

**The missing data:** data *not* on lines \( L_{\theta,s} \) for \((\theta, s) \in S\).

**Example:** The set of \((\theta, s)\) over which data are taken, the *data domain*, includes only horizontal-ish lines—\( L_{\theta,s} \) with \((\theta, s) \in S = [45^\circ, 135^\circ] \times [-1, 1]\).

Vertical-\(ish\) lines are missing (data domain shown in \((\theta, s)\) space on right).
Limited Angle CT in Dental Imaging

Dental Scanner–head goes in “Π”  Jaw showing X-ray projection angles

http://www.siltanen-research.net
Limited Angle CT in Luggage Testing

Luggage Scanner

Scanner moves above and below suitcase

Sample Luggage scan

Analogic COBRA carry-on luggage scanner
FBP for complete data: \( f = \frac{1}{4\pi} R^* \Lambda (Rf) \)

- \( R^* \) is the backprojection operator, \( \Lambda \) the filter, \( Rf \) the data.

- We know the data only for \( (\theta, s) \in S \).
- So, we mask the data off of \( S \) the data domain—set the data we don’t have to zero, and then do FBP on this completed data!

- The mask for limited data: 
  \[
  \chi_S(\theta, s) = \begin{cases} 
  1 & (\theta, s) \in S \\
  0 & (\theta, s) \notin S
  \end{cases}
  \]
- Then we do FBP on this completed data:
  \[
  f \sim \frac{1}{4\pi} R^* \Lambda (\chi_S Rf)
  \]
Limited Data FBP

FBP for complete data: \( f = \frac{1}{4\pi} R^* \Lambda (Rf) \)

- \( R^* \) is the backprojection operator, \( \Lambda \) the filter, \( Rf \) the data.

(FBP for Limited Data)

- In general, limited data are given over some data domain \( S \subseteq [0^\circ, 360^\circ] \times [-1, 1] \).
Limited Data FBP

FBP for complete data: \( f = \frac{1}{4\pi} \mathcal{R}^* \Lambda (\mathcal{R} f) \)

- \( \mathcal{R}^* \) is the backprojection operator, \( \Lambda \) the filter, \( \mathcal{R} f \) the data.

(FBP for Limited Data)

- *In general, limited data are given over some data domain \( S \subseteq [0^\circ, 360^\circ] \times [-1, 1] \).*
- *We know the data only for \((\theta, s) \in S\).*

Note: by multiplying \( \mathcal{R} f \) by \( \chi_S \), we set the data off of \( S \) — the data we don't have—to zero.
**Limited Data FBP**

**FBP for complete data:** \( f = \frac{1}{4\pi} \mathcal{R}^* \Lambda (\mathcal{R}f) \)

- \( \mathcal{R}^* \) is the backprojection operator, \( \Lambda \) the filter, \( \mathcal{R}f \) the data.

**(FBP for Limited Data)**

- *In general, limited data are given over some data domain* \( S \subset [0^\circ, 360^\circ] \times [-1, 1] \).
- *We know the data only for* \( (\theta, s) \in S \).
- *So, we mask the data off of* \( S \) *the data domain*—*set the data we don’t have to zero, and then do FBP on this completed data!*

![Diagram](image)
Limited Data FBP

**FBP for complete data:** \( f = \frac{1}{4\pi} \mathcal{R}^* \Lambda (\mathcal{R}f) \)

- \( \mathcal{R}^* \) is the backprojection operator, \( \Lambda \) the filter, \( \mathcal{R}f \) the data.

**(FBP for Limited Data)**

- *In general, limited data are given over some data domain* \( S \subset [0^\circ, 360^\circ] \times [-1, 1] \).
- *We know the data only for* \( (\theta, s) \in S \).
- *So, we mask the data off of* \( S \), *the data domain*—*set the data we don’t have to zero, and then do FBP on this completed data!*

- **The mask for limited data:** \( \chi_S(\theta, s) = \begin{cases} 1 & (\theta, s) \in S \\ 0 & (\theta, s) \notin S \end{cases} \).
Limited Data FBP

**FBP for complete data:** \( f = \frac{1}{4\pi} \mathcal{R}^* \Lambda (\mathcal{R} f) \)
- \( \mathcal{R}^* \) is the backprojection operator, \( \Lambda \) the filter, \( \mathcal{R} f \) the data.

(FBP for Limited Data)
- In general, limited data are given over some data domain \( S \subset [0^\circ, 360^\circ] \times [-1, 1] \).
- We know the data only for \( (\theta, s) \in S \).
- So, we mask the data off of \( S \) the data domain—set the data we don’t have to zero, and then do FBP on this completed data!
- **The mask for limited data:** \( \chi_S(\theta, s) = \begin{cases} 1 & (\theta, s) \in S \\ 0 & (\theta, s) \notin S \end{cases} \).
- Then we do FBP on this completed data!

\[ f \sim \frac{1}{4\pi} \mathcal{R}^* \Lambda (\chi_S \mathcal{R} f) \]
Limited Data FBP

**FBP for complete data:** \( f = \frac{1}{4\pi} \mathcal{R}^* \Lambda (\mathcal{R}f) \)

- \( \mathcal{R}^* \) is the backprojection operator, \( \Lambda \) the filter, \( \mathcal{R}f \) the data.

(FBP for Limited Data)

- **In general, limited data are given over some data domain** \( S \subseteq [0^\circ, 360^\circ] \times [-1, 1] \).
- **We know the data only for** \( (\theta, s) \in S \).
- **So, we mask the data off of** \( S \) **the data domain**—set the data we don’t have to zero, and then do FBP on this completed data!

- **The mask for limited data:** \( \chi_S(\theta, s) = \begin{cases} 1 & (\theta, s) \in S \\ 0 & (\theta, s) \notin S \end{cases} \).

- **Then we do FBP on this completed data!**

\[
\begin{align*}
\mathcal{R}^* \Lambda (\chi_S \mathcal{R}f) & \sim \frac{1}{4\pi} \mathcal{R}^* \Lambda (\chi_S \mathcal{R}f)
\end{align*}
\]

**Note:** by multiplying \( \mathcal{R}f \) by \( \chi_S \), we set the data off of \( S \)—the data we don’t have—to zero.
Example: Limited Angle FBP

FBP with general data domain $S$.

$$f(x) \sim \frac{1}{4\pi} \mathcal{R}^* \Lambda (\chi_S \mathcal{R} f), \quad \chi_S(\theta, s) = \begin{cases} 1 & (\theta, s) \in S \\ 0 & (\theta, s) \notin S \end{cases}.$$ 

$$= \frac{1}{4\pi} \int_{0^\circ}^{360^\circ} \Lambda (\chi_S \mathcal{R} f)(\theta, x \cdot \overline{\theta}) d\theta.$$ 

$$\overrightarrow{O} = (\cos \theta, \sin \theta)$$

$$x \cdot \overrightarrow{O} \sim x_1 \cos \theta + x_2 \sin \theta.$$
Example: Limited Angle FBP

FBP with general data domain $S$.

$$f(x) \sim \frac{1}{4\pi} \mathcal{R}^* \Lambda (\chi_S \mathcal{R}f), \quad \chi_S(\theta, s) = \begin{cases} 1 & (\theta, s) \in S \\ 0 & (\theta, s) \notin S \end{cases}. $$

$$= \frac{1}{4\pi} \int_{0^\circ}^{360^\circ} \Lambda (\chi_S \mathcal{R}f) (\theta, x \cdot \overline{\theta}) d\theta. $$

The data domain for limited angle CT is: $$S = [a^\circ, b^\circ] \times [-1, 1],$$
The data domain for limited angle CT is: \( S = [a^\circ, b^\circ] \times [-1, 1] \), so the limited angle FBP becomes:
Example: Limited Angle FBP

FBP with general data domain $S$.

\[ f(x) \sim \frac{1}{4\pi} \mathcal{R}^* \Lambda (\chi_S \mathcal{R} f), \quad \chi_S(\theta, s) = \begin{cases} 1 & (\theta, s) \in S \\ 0 & (\theta, s) \notin S \end{cases}. \]

\[
= \frac{1}{4\pi} \int_{0^\circ}^{360^\circ} \Lambda (\chi_S \mathcal{R} f) (\theta, x \cdot \bar{\theta}) \, d\theta.
\]

The data domain for limited angle CT is: $S = [a^\circ, b^\circ] \times [-1, 1]$, so the limited angle FBP becomes:

\[ f \sim \frac{1}{4\pi} \int_{a^\circ}^{b^\circ} \Lambda (\mathcal{R} f) (\theta, x \cdot \bar{\theta}) \, d\theta. \]
Example: Limited Angle FBP

**FBP with general data domain $S$.**

\[
 f(x) \sim \frac{1}{4\pi} \mathcal{R}^* \Lambda (\chi_S \mathcal{R} f), \quad \chi_S(\theta, s) = \begin{cases} 
 1 & (\theta, s) \in S \\
 0 & (\theta, s) \notin S 
\end{cases}.
\]

\[
 = \frac{1}{4\pi} \int_{0^\circ}^{360^\circ} \Lambda (\chi_S \mathcal{R} f) (\theta, x \cdot \bar{\theta}) \, d\theta.
\]

The data domain for limited angle CT is: $S = [a^\circ, b^\circ] \times [-1, 1]$, so the limited angle FBP becomes:

\[
 f \sim \frac{1}{4\pi} \int_{a^\circ}^{b^\circ} \Lambda (\mathcal{R} f) (\theta, x \cdot \bar{\theta}) \, d\theta.
\]

- By integrating from $a$ to $b$, we reconstruct using only data in the data domain, $S = [a^\circ, b^\circ] \times [-1, 1]$. 
Example: Limited Angle FBP

FBP with general data domain $S$.

$$f(x) \sim \frac{1}{4\pi} R^* \Lambda (\chi_S R f), \quad \chi_S(\theta, s) = \begin{cases} 
1 & (\theta, s) \in S \\
0 & (\theta, s) \notin S .
\end{cases}$$

$$= \frac{1}{4\pi} \int_{0^\circ}^{360^\circ} \Lambda (\chi_S R f)(\theta, x \cdot \bar{\theta}) d\theta.$$

The data domain for limited angle CT is: $S = [a^\circ, b^\circ] \times [-1, 1]$, so the limited angle FBP becomes:

$$f \sim \frac{1}{4\pi} \int_{a^\circ}^{b^\circ} \Lambda (R f)(\theta, x \cdot \bar{\theta}) d\theta.$$ 

- By integrating from $a$ to $b$, we reconstruct using only data in the data domain, $S = [a^\circ, b^\circ] \times [-1, 1]$. We do standard FBP on data that is masked (is set to zero) off of $S$. 

 Tufts University
Our Goal: learn what object boundaries can be reconstructed from limited data.
Our Goal: learn what object boundaries can be reconstructed from limited data.

In many tomography problems, the shapes/boundaries of features in the test object are diagnostically important:

- Cracks in industrial objects,
- Illegal stuff in carry-on luggage,
Our Goal: learn what object boundaries can be reconstructed from limited data.

In many tomography problems, the shapes/boundaries of features in the test object are diagnostically important:

- Cracks in industrial objects,
- Illegal stuff in carry-on luggage,
- Cavities in teeth,
- Blockages in blood vessels,
- Uneven boundaries in some tumors.
Our Goal: learn what object boundaries can be reconstructed from limited data.

In many tomography problems, the shapes/boundaries of features in the test object are diagnostically important:

- Cracks in industrial objects,
- Illegal stuff in carry-on luggage,
- Cavities in teeth,
- Blockages in blood vessels,
- Uneven boundaries in some tumors.

So, knowing the boundaries of structures in the test object is important.
Our Goal: learn what object boundaries can be reconstructed from limited data.

In many tomography problems, the shapes/boundaries of features in the test object are diagnostically important:

- Cracks in industrial objects,
- Illegal stuff in carry-on luggage,
- Cavities in teeth,
- Blockages in blood vessels,
- Uneven boundaries in some tumors.

- So, knowing the boundaries of structures in the test object is important.
- We don’t always need to know the exact density values of the object.
Our Goal: learn what object boundaries can be reconstructed from limited data.

In many tomography problems, the shapes/boundaries of features in the test object are diagnostically important:

- Cracks in industrial objects,
- Illegal stuff in carry-on luggage,
- Cavities in teeth,
- Blockages in blood vessels,
- Uneven boundaries in some tumors.

- So, knowing the boundaries of structures in the test object is important.
- We don’t always need to know the exact density values of the object.
- Algorithms such as limited data FBP can be useful!
Which **features** of the body are **sharpest** in this X-ray image?

My Answer: The **edges/boundaries** of the bones!

Answer: The beams tangent to the edges (boundaries) of the bones!

Now see why mathematically.
Which features of the body are sharpest in this X-ray image?

My Answer: The edges/boundaries of the bones!
Which features of the body are sharpest in this X-ray image?
My Answer: The the edges/boundaries of the bones!

Which X-ray beams show edges (boundaries) (pic→)?
Which features of the body are sharpest in this X-ray image?
My Answer: The edges/boundaries of the bones!

Which X-ray beams show edges (boundaries) (pic—→)?
Answer: The beams tangent to the edges (boundaries) of the bones!

Now see why mathematically.
CT data of a disk of radius 1 over vertical lines

The CT data has a "corner" (graph not smooth) at any line tangent to the boundary of the disk.

So the boundary will be easy to see in the data.
CT data of a disk of radius 1 over vertical lines

The CT data has a "corner" (graph not smooth) at any line tangent to the boundary of the disk.

So the boundary will be easy to see in the data.

$RF(0, -1)$
CT data of a disk of radius 1 over vertical lines

The CT data has a "corner" (graph not smooth) at any line tangent to the boundary of the disk. So the boundary will be easy to see in the data.
CT data of a disk of radius 1 over vertical lines

The CT data has a "corner" (graph not smooth) at any line tangent to the boundary of the disk. So the boundary will be easy to see in the data.
CT data of a disk of radius 1 over vertical lines

The CT data has a “corner” (graph not smooth) at any line tangent to the boundary of the disk. So the boundary will be easy to see in the data.
CT data of a disk of radius 1 over vertical lines

- The CT data has a “corner” (graph not smooth) at any line tangent to the boundary of the disk.
CT data of a disk of radius 1 over vertical lines

- The CT data has a “corner” (graph not smooth) at any line tangent to the boundary of the disk.
- So the boundary will be easy to see in the data.
In limited data CT, data over some lines are missing.

- Data not smooth $\rightarrow$ easy to see the feature that caused it in this data.

```
not smooth at tangent line
```

- Data smooth $\rightarrow$ features on that line are "washed out" in this data.

```
smooth at other lines
```

If a boundary in the object is tangent to a line in the data domain, then it will be easy to see in the data (like the chest X-ray!).

$
\begin{align*}
6 &\text{ easy to reconstruct from limited data.} \\
\text{If a boundary in the object is not tangent to any line in the data domain, then it will be hard to see in the data.} &\text{ hard to reconstruct from limited data.}
\end{align*}
$

Moral

```
Boundary tangent to some line in the data domain $\rightarrow$ boundary easy to reconstruct.
```

```
Boundary tangent to no line in the data domain $\rightarrow$ boundary hard to reconstruct.
```
In limited data CT, data over some lines are missing.

- Data not smooth → easy to see the feature that caused it in this data.
- Data is smooth → features on that line are “washed out” in this data.

Moral

- Boundary tangent to some line in the data domain → boundary easy to reconstruct.
- Boundary tangent to no line in the data domain → boundary hard to reconstruct.
In limited data CT, data over some lines are missing.

- Data not smooth → easy to see the feature that caused it in this data.
- Data is smooth → features on that line are “washed out” in this data.

If a boundary in the object is tangent to a line in the data domain, then it will be easy to see in the data (like the chest X-ray!).

Boundary tangent to no line in the data domain → boundary hard to reconstruct.
In limited data CT, data over some lines are missing.

- Data not smooth → easy to see the feature that caused it in this data.
- Data is smooth → features on that line are “washed out” in this data.

If a boundary in the object is tangent to a line in the data domain, then it will be easy to see in the data (like the chest X-ray). ∴ easy to reconstruct from limited data.

Boundary tangent to no line in the data domain → boundary hard to reconstruct.
In limited data CT, data over some lines are missing.

- **Data not smooth** → easy to see the feature that caused it in this data.
- **Data is smooth** → features on that line are “washed out” in this data.

If a boundary in the object is tangent to a line in the data domain, then it will be easy to see in the data (like the chest X-ray).

∴ easy to reconstruct from limited data.

If a boundary in the object is *not* tangent to *any* line in the data domain, then it will be had to see in the data.

> **Boundary tangent to no line in the data domain** → boundary hard to reconstruct.
In limited data CT, data over some lines are missing.

- Data not smooth $\rightarrow$ easy to see the feature that caused it in this data.
- Data is smooth $\rightarrow$ features on that line are “washed out” in this data.

If a boundary in the object is tangent to a line in the data domain, then it will be easy to see in the data (like the chest X-ray!).
∴ easy to reconstruct from limited data.

If a boundary in the object is *not* tangent to any line in the data domain, then it will be hard to see in the data.
∴ hard to reconstruct from limited data.
In limited data CT, data over some lines are missing.

- Data not smooth $\rightarrow$ easy to see the feature that caused it in this data.
- Data is smooth $\rightarrow$ features on that line are “washed out” in this data.

If a boundary in the object is tangent to a line in the data domain, then it will be easy to see in the data (like the chest X-ray!).
\[ \therefore \text{easy to reconstruct from limited data.} \]

If a boundary in the object is \textit{not} tangent to \textit{any} line in the data domain, then it will be had to see in the data.
\[ \therefore \text{hard to reconstruct from limited data.} \]

\textbf{Moral}

- Boundary tangent to \textit{some line in the data domain} $\rightarrow$ \textit{boundary easy to reconstruct.}
In limited data CT, data over some lines are missing.  

- Data not smooth → easy to see the feature that caused it in this data.  
- Data is smooth → features on that line are “washed out” in this data.  

If a boundary in the object is tangent to a line in the data domain, then it will be easy to see in the data (like the chest X-ray!).  
∴ easy to reconstruct from limited data.  

If a boundary in the object is not tangent to any line in the data domain, then it will be hard to see in the data.  
∴ hard to reconstruct from limited data.  

**Moral**  
- Boundary tangent to some line in the data domain → boundary easy to reconstruct.  
- Boundary tangent to no line in the data domain → boundary hard to reconstruct.
In limited data CT, data over some lines are missing.

- Data not smooth → easy to see the feature that caused it in this data.
- Data is smooth → features on that line are “washed out” in this data.

If a boundary in the object is tangent to a line in the data domain, then it will be easy to see in the data (like the chest X-ray!)
- easy to reconstruct from limited data.

If a boundary in the object is not tangent to any line in the data domain, then it will be hard to see in the data.
- hard to reconstruct from limited data.

**Moral**

- Boundary tangent to some line in the data domain → boundary easy to reconstruct.
- Boundary tangent to no line in the data domain → boundary hard to reconstruct.
Example

Limited angle CT data of a disk over lines $L_{\theta,s}$ with $(\theta, s) \in [45^\circ, 135^\circ] \times [-1, 1]$

[Frikel, Q 2013] *Left:* disk,  *Right:* Limited data FBP reconstruction
The data domain: all lines with $L_{\theta,s}$ with $(\theta, s) \in [45^\circ, 135^\circ] \times [-1, 1]$.

Which object boundaries are visible in the reconstruction?

Answer: the horizontal-ish boundaries.

We claimed that, if a line in the data domain is tangent to a boundary, that boundary will be easy to see in the reconstruction from that data. Is that true in this picture? YES!

Moral: A boundary of an object will be visible in the reconstruction from limited data if it is tangent to a line in the data domain.
The data domain: all lines with $L_{\theta,s}$ with $(\theta, s) \in [45^\circ, 135^\circ] \times [-1, 1]$.

Which object boundaries are visible in the reconstruction? 

Answer: the horizontal-ish boundaries.
The data domain: all lines with $L_{\theta,s}$ with $(\theta, s) \in [45^\circ, 135^\circ] \times [-1, 1]$.

Which object boundaries are visible in the reconstruction?

*Answer:* the horizontal-ish boundaries.

We claimed that, if a line in the data domain is tangent to a boundary, that boundary will be easy to see in the reconstruction from that data.

Is that true in this picture?
The data domain: all lines with $L_{\theta,s}$ with $(\theta, s) \in [45^\circ, 135^\circ] \times [-1, 1]$.

Which object boundaries are visible in the reconstruction? *Answer:* the horizontal-ish boundaries.

We claimed that, if a line in the data domain is tangent to a boundary, that boundary will be easy to see in the reconstruction from that data.

Is that true in this picture?
The data domain: all lines with \( L_{\theta,s} \) with \((\theta, s) \in [45^\circ, 135^\circ] \times [-1, 1] \).

Which object boundaries are visible in the reconstruction?  
*Answer:* the horizontal-ish boundaries.

We claimed that, if a line in the data domain is tangent to a boundary, that boundary will be easy to see in the reconstruction from that data.

Is that true in this picture? *YES!*
The data domain: all lines with $L_{\theta,s}$ with $(\theta, s) \in [45^\circ, 135^\circ] \times [-1, 1]$.

Which object boundaries are visible in the reconstruction? 
Answer: the horizontal-ish boundaries.

We claimed that, if a line in the data domain is tangent to a boundary, that boundary will be easy to see in the reconstruction from that data.

Is that true in this picture? YES!

Moral

A boundary of an object will be visible in the reconstruction from limited data if it is tangent to a line in the data domain.
The data domain: all lines with \( L_{\theta, s} \) with 
\((\theta, s) \in [45^\circ, 135^\circ] \times [-1, 1].\)

Which boundaries of the disk are not visible in the reconstruction?

Answer: the vertical-ish boundaries.

We claimed that, if no line in the data domain is tangent to a
boundary, that boundary will be hard to see in the
reconstruction. Is that true in this picture? YES!

Moral
A boundary will be difficult to see in the reconstruction from
limited data if no line in the data domain is tangent to it.
The data domain: all lines with $L_{\theta,s}$ with $(\theta, s) \in [45^\circ, 135^\circ] \times [-1, 1]$. Which boundaries of the disk are not visible in the reconstruction? 

*Answer:* the vertical-ish boundaries.
The data domain: all lines with \( L_{\theta,s} \) with 
\[(\theta, s) \in [45^\circ, 135^\circ] \times [-1, 1].\]

Which boundaries of the disk are not visible in the reconstruction?

*Answer:* the vertical-ish boundaries.

We claimed that, if no line in the data domain is tangent to a boundary, that boundary will be hard to see in the reconstruction.

*Is that true in this picture?*
The data domain: all lines with $L_{\theta,s}$ with $(\theta, s) \in [45^\circ, 135^\circ] \times [-1, 1]$.

Which boundaries of the disk are not visible in the reconstruction?

Answer: the vertical-ish boundaries.

We claimed that, if no line in the data domain is tangent to a boundary, that boundary will be hard to see in the reconstruction.

Is that true in this picture? YES!
The data domain: all lines with $L_{\theta,s}$ with 
$(\theta, s) \in [45^\circ, 135^\circ] \times [-1, 1]$.

Which boundaries of the disk are not visible in the reconstruction?

Answer: the vertical-ish boundaries.

We claimed that, if no line in the data domain is tangent to a boundary, that boundary will be hard to see in the reconstruction.

Is that true in this picture? YES!

Moral

A boundary will be difficult to see in the reconstruction from limited data if no line in the data domain is tangent to it.
The data domain: all lines with $L_{\theta,s}$ with $(\theta, s) \in [45^\circ, 135^\circ] \times [-1, 1]$.

How do the streak lines relate to the object? They are tangent to the object.

How do the streaks relate to the data domain? They are along lines $L_{\theta,s}$.

What are the values of $\theta$ for the streak lines? Either $45^\circ$ or $135^\circ$ – they represent lines at the ends (i.e., boundary) of the data set.

Moral: Streaks in the reconstruction come from lines at the boundary of the data domain (min. or max. of $\theta$) that are tangent to the object.
The data domain: all lines with $L_{\theta,s}$ with $(\theta, s) \in [45^\circ, 135^\circ] \times [-1, 1]$.

How do the streak lines relate to the object? They are tangent to the object.

How do the streaks relate to the data domain? They are along lines $L_{\theta,s}$.

What are the values of $\theta$ for the streak lines? Either $45^\circ$ or $135^\circ$ – they represent lines at the ends (i.e., boundary) of the data set.

Moral: Streaks in the reconstruction come from lines at the boundary of the data domain (min. or max. of $\theta$) that are tangent to the object.
The data domain: all lines with $L_{\theta,s}$ with $(\theta, s) \in [45^\circ, 135^\circ] \times [-1, 1]$.

How do the streak lines relate to the object? They are tangent to the object.

How do the streaks relate to the data domain? They are along lines $L_{\theta,s}$! What are the value of $\theta$ for the streak lines? 45°, 135°.
The data domain: all lines with $L_{\theta,s}$ with $(\theta, s) \in [45^\circ, 135^\circ] \times [-1, 1]$.

How do the streak lines relate to the object? They are tangent to the object.

How do the streaks relate to the data domain? They are along lines $L_{\theta,s}$! What are the value of $\theta$ for the streak lines? Either $45^\circ$ or $135^\circ$
**The data domain:** all lines with $L_{\theta,s}$ with
$(\theta, s) \in [45^\circ, 135^\circ] \times [-1, 1]$.

How do the streak lines relate to the object? They are tangent to the object.

How do the streaks relate to the data domain? They are along lines $L_{\theta,s}$! What are the value of $\theta$ for the streak lines? Either $45^\circ$ or $135^\circ$—they represent lines at the ends (i.e., boundary) of the data set.
The data domain: all lines with $L_{\theta,s}$ with $(\theta, s) \in [45^\circ, 135^\circ] \times [-1, 1]$.

How do the streak lines relate to the object? They are tangent to the object.

How do the streaks relate to the data domain? They are along lines $L_{\theta,s}$! What are the value of $\theta$ for the streak lines? Either $45^\circ$ or $135^\circ$—they represent lines at the ends (i.e., boundary) of the data set.

Moral

_Streaks in the reconstruction come from lines at the boundary of the data domain (min. or max. of $\theta$) that are tangent to the object._
Moral (up to now) for Limited Data Tomography

- A boundary of the object is *(should be)* visible in the reconstruction if:

- A boundary of the object is *(should be)* invisible (not seen) in the reconstruction if:

- A streak artifact can occur in the reconstruction on a line if:

This is valid for general limited data problems by deep mathematics + a precise concept of singularity–microlocal analysis [3].
Moral (up to now) for Limited Data Tomography

- A boundary of the object is (should be) *visible* in the reconstruction if:
  it is *tangent* to a line in the data domain.

- A boundary of the object is *invisible* (not seen) in the reconstruction if:
  it is not tangent to any line in the data domain.

- A streak artifact can occur in the reconstruction on a line if:
  the streak is tangent to the object and on a line at the boundary of the data domain.

This is valid for general limited data problems by deep mathematics + a precise concept of singularity–microlocal analysis [3].
A boundary of the object is *(should be)* *visible* in the reconstruction if:

- it is *tangent* to a line in the data domain.

A boundary of the object is *(should be)* *invisible* (not seen) in the reconstruction if:

- the streak is tangent to the object and on a line at the boundary of the data domain.
Moral (up to now) for Limited Data Tomography

▶ A boundary of the object is (should be) *visible* in the reconstruction if:
  it is *tangent* to a line in the data domain.

▶ A boundary of the object is (should be) *invisible* (not seen) in the reconstruction if:
  it is *not tangent* to *any* line in the data domain.

This is valid for general limited data problems by deep mathematics + a precise concept of singularity–microlocal analysis [3].
Moral (up to now) for Limited Data Tomography

- A boundary of the object is *(should be)* visible in the reconstruction if:
  it is *tangent* to a line in the data domain.

- A boundary of the object is *(should be)* invisible (not seen) in the reconstruction if:
  it is *not tangent* to *any* line in the data domain.

- A streak artifact can occur in the reconstruction on a line if:
Moral (up to now) for Limited Data Tomography

- A boundary of the object is *(should be)* visible in the reconstruction if:
  it is *tangent* to a line in the data domain.

- A boundary of the object is *(should be)* invisible (not seen) in the reconstruction if:
  it is *not tangent* to *any* line in the data domain.

- A streak artifact can occur in the reconstruction on a line if:
  the streak is tangent to the object *and* on a line at the boundary of the data domain.
A boundary of the object is *(should be) visible* in the reconstruction if:
it is *tangent* to a line in the data domain.

A boundary of the object is *(should be) invisible* (not seen) in the reconstruction if:
it is *not tangent* to any line in the data domain.

A streak artifact can occur in the reconstruction on a line if:
the streak is tangent to the object *and* on a line at the boundary of the data domain.

This is valid for general limited data problems by deep mathematics + a precise concept of singularity–microlocal analysis [3].
Exercise (In Class/Breakout Rooms)

Brain phantom [radiopedia.org], FBP reconstruction [Frikel, Q 2013]

(a) Which features of the brain are visible in the reconstruction?
(b) Which are invisible?
(c) Are there added streak artifacts?
(d) Use this information to determine the data domain for this reconstruction.
Artifact Reduction

$f(x) \sim \frac{1}{4\pi} \mathcal{R}^* \Lambda (\chi_S \mathcal{R} f), \quad \chi_S(\theta, s) = \begin{cases} 1 & (\theta, s) \in S \\ 0 & (\theta, s) \notin S \end{cases}$. 
Artifact Reduction

\[ f(x) \sim \frac{1}{4\pi} \mathcal{R}^* \Lambda (\chi_S \mathcal{R} f), \quad \chi_S(\theta, s) = \begin{cases} 1 & (\theta, s) \in S \\ 0 & (\theta, s) \notin S \end{cases}. \]

- By multiplying by \( \chi_S \), we restrict to data in the given data domain, \( S \).
Artifact Reduction

\[ f(x) \sim \frac{1}{4\pi} \mathcal{R}^* \Lambda (\chi_S \mathcal{R}f), \quad \chi_S(\theta, s) = \begin{cases} 1 & (\theta, s) \in S \\ 0 & (\theta, s) \notin S \end{cases}. \]

- By multiplying by \( \chi_S \), we restrict to data in the given data domain, \( S \).
- The streaks occur along lines at the ends of the data domain.
Artifact Reduction

\[ f(x) \sim \frac{1}{4\pi} \mathcal{R}^* \Lambda (\chi_S \mathcal{R} f), \quad \chi_S(\theta, s) = \begin{cases} 1 & (\theta, s) \in S \\ 0 & (\theta, s) \notin S \end{cases}. \]

- By multiplying by \( \chi_S \), we restrict to data in the given data domain, \( S \).

- *The streaks occur along lines at the ends of the data domain.*

- *The cause of streaks:* the sharp cutoff in the mask at the ends of the data domain.
Artifact Reduction

$$f(x) \sim \frac{1}{4\pi} \mathcal{R}^* \Lambda (\chi_S \mathcal{R} f), \quad \chi_S(\theta, s) = \begin{cases} 1 & (\theta, s) \in S \\ 0 & (\theta, s) \notin S \end{cases}.$$ 

- By multiplying by $\chi_S$, we restrict to data in the given data domain, $S$.

- The streaks occur along lines at the ends of the data domain.

- The cause of streaks: the sharp cutoff in the mask at the ends of the data domain.

- The solution: Make a smooth, gradual cutoff in the mask.
Artifact Reduction

\[
\begin{align*}
\chi_S(\theta, s) &= \begin{cases} 
1 & (\theta, s) \in S \\
0 & (\theta, s) \notin S
\end{cases}
\end{align*}
\]

- By multiplying by \( \chi_S \), we restrict to data in the given data domain, \( S \).
- The streaks occur along lines at the ends of the data domain.
- The cause of streaks: the sharp cutoff in the mask at the ends of the data domain.
- The solution: Make a smooth, gradual cutoff in the mask.
- Replace \( \chi_S \) by a smooth function \( \psi(\theta, s) \) that is 1 on most of \( S \) (e.g., for limited angle equal to 1 on most of \([a, b]\) and smoothly goes to 0 at \( a, b \)).
Artifact Reduction

\[
f(x) \sim \frac{1}{4\pi} \mathcal{R}^* \Lambda (\chi_S \mathcal{R} f), \quad \chi_S(\theta, s) = \begin{cases} 
1 & (\theta, s) \in S \\
0 & (\theta, s) \notin S 
\end{cases}.
\]

- By multiplying by \( \chi_S \), we restrict to data in the given data domain, \( S \).
- The streaks occur along lines at the ends of the data domain.
- The cause of streaks: the sharp cutoff in the mask at the ends of the data domain.
- The solution: Make a smooth, gradual cutoff in the mask.
- Replace \( \chi_S \) by a smooth function \( \psi(\theta, s) \) that is 1 on most of \( S \) (e.g., for limited angle equal to 1 on most of \( [a, b] \) and smoothly goes to 0 at \( a, b \)).

Artifact reduced limited angle FBP:

\[
f \sim \frac{1}{4\pi} \mathcal{R}^* \Lambda (\psi \mathcal{R} f)
\]
Sample Reconstruction [Frikel Q 2013]

FBP reconstruction: \( \varepsilon = 0^\circ \)

FBP reconstruction: \( \varepsilon = 20^\circ \)

Reconstruction

Artifact reduced reconstruction
ROI Tomography: tomography using only lines that pass through a small part of the object to reconstruct that part of the object. This is often because the object is too large or we are interested in only imaging that small part of the object.

Figure: Skyscan Micro-CT Scanner

Figure: Object in scanner
ROI Tomography: tomography using only lines that pass through a small part of the object to reconstruct that part of the object. This is often because the object is too large or we are interested in only imaging that small part of the object.

Figure: Skyscan Micro-CT Scanner

Figure: Object in scanner

ROI CT is used for nondestructive evaluation of parts of small objects.
The data domain for ROI CT: $S = [0^\circ, 180^\circ] \times [-r, r]$ where $r < 1$ is the radius of the ROI. 

The mask: $\chi_S$.

**Figure:** The Shepp Logan Phantom + ROI  

**Figure:** Complete Data Sinogram and ROI Sinogram
Exercise (Breakout rooms!)

Let’s say you have a ROI data domain of an object.

1. According to the theory, what object boundaries would be easy to reconstruct from the ROI data inside the ROI?
2. According to the theory, what types of object boundaries would be difficult to reconstruct from the ROI data inside the ROI?
3. According to the theory, what object boundaries would be easy to reconstruct from the ROI data outside the ROI?
4. According to the theory, what types of object boundaries would be difficult to reconstruct from the ROI data outside the ROI?
5. Did you observe this in the ROI reconstruction exercise using iRadon?
Artifact Curves

Moral

Artifacts can occur on curves generated from lines in the boundary of the data set. We have a formula for them in [3]!
Exterior Tomography: only rays through an outer annulus of object are measured, not the rays through its center.
Exterior Tomography: only rays through an outer annulus of object are measured, not the rays through its center.

Exterior CT is used for nondestructive evaluation (NDE) of rockets because industrial X-ray CT scanners can’t penetrate the thick central part of the rocket, but they can penetrate the outside annulus. Often scientists are interested in cracks, etc., in the rocket shell, anyway.
The data domain for exterior CT: If the central disk has radius $r < 1$, then $S = [0^\circ, 180^\circ] \times ([-1, -r] \cup [r, 1])$.

The mask: $\chi_S$. 
Exercise (Breakout rooms!)

Use what we’ve learned to answer the following questions about an exterior reconstruction of this phantom [Q1988]

(a) What boundaries should be easy to see in an exterior reconstruction of the phantom?

(b) What boundaries should be difficult to see in an exterior reconstruction of the phantom?

(c) Could there be artifact curves? (HINT: think about artifacts in ROI CT.)
Exercise (If time–Breakout rooms!)

Defects in rocket shells are generally along the circumference direction of the shell.
Exercise (If time–Breakout rooms!)

Defects in rocket shells are generally along the circumference direction of the shell.

1. **Would exterior CT be a good modality for such defects?**
2. **According to the theory, what types of defects would be easy to reconstruct from exterior CT?**
3. **According to the theory, what types of defects would be difficult to reconstruct from exterior CT?**
4. **Do you think there could be added artifacts in reconstructions from exterior data? Why or why not?**
Visible boundary: boundary tangent to a line in the data domain.

Invisible boundary: boundary tangent to no line in the data domain.

Added Artifacts: streaks on lines at the boundary of the data domain that are tangent to the object.
  More subtle added artifacts can occur on lines or curves generated by lines at the boundary of the data domain.
Visible boundary: boundary tangent to a line in the data domain.

Invisible boundary: boundary tangent to no line in the data domain.

Added Artifacts: streaks on lines at the boundary of the data domain that are tangent to the object.
   - More subtle added artifacts can occur on lines or curves generated by lines at the boundary of the data domain.

Artifact reduction: smooth the mask at the ends of the data domain. (ROI/Exterior simple extensions work well (limited angle???)�.)
Summary I

- **Visible boundary:** boundary tangent to a line in the data domain.

- **Invisible boundary:** boundary tangent to *no* line in the data domain.

- **Added Artifacts:** streaks on lines at the boundary of the data domain that are tangent to the object.
  - More subtle added artifacts can occur on lines or curves generated by lines at the boundary of the data domain.

- **Artifact reduction:** smooth the mask at the ends of the data domain. (ROI/Exterior simple extensions work well (limited angle???).)

- We make this characterization mathematically precise using the Fourier transform and microlocal analysis. See:

The analysis of visible and invisible singularities is intrinsic to limited data CT [2].
The analysis of visible and invisible singularities is intrinsic to limited data CT [2].

The artifact characterization in [3] applies to backprojection algorithms for many limited data tomography problems in X-ray CT, sonar, radar, seismics. . .
The analysis of visible and invisible singularities is intrinsic to limited data CT [2].

The artifact characterization in [3] applies to **backprojection algorithms** for many limited data tomography problems in X-ray CT, sonar, radar, seismics. . .

Backprojection is useful in general and easy to program. However, it isn’t perfect for limited data problems.
The analysis of visible and invisible singularities is intrinsic to limited data CT [2].

The artifact characterization in [3] applies to backprojection algorithms for many limited data tomography problems in X-ray CT, sonar, radar, seismics. . .

Backprojection is useful in general and easy to program. However, it isn’t perfect for limited data problems.

Other effective algorithms:

- Use a priori info about the object (general shape, . . . ) and iterative methods [PCH]
- Develop and carefully implement inversion formulas or fill in data cleverly (e.g., [Louis, Katsevich]).
- Use deep learning (e.g., [Bubba et al.]) and good training sets.
The analysis of visible and invisible singularities is intrinsic to limited data CT [2].

The artifact characterization in [3] applies to backprojection algorithms for many limited data tomography problems in X-ray CT, sonar, radar, seismics. . .

Backprojection is useful in general and easy to program. However, it isn’t perfect for limited data problems.

Other effective algorithms:

- Use a priori info about the object (general shape, . . .) and iterative methods [PCH]
- Develop and carefully implement inversion formulas or fill in data cleverly (e.g., [Louis, Katsevich]).
- Use deep learning (e.g., [Bubba et al.]) and good training sets.

THANKS FOR YOUR ATTENTION!
References to the work in the talk:

For Further Reading II


General references:


For Further Reading III

*Introductory*


For Further Reading IV

Limited Data, Local CT, and Lambda CT


Microlocal references:

www.springer.com/978-1-4939-0789-2

