

Short-course:***Algebraic Iterative Reconstruction Methods – Theory and Experience***

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The goal of this short course is to present a survey of some iterative reconstruction methods for linear inverse problems that are based on the algebraic formulation of the problem, $Ax = b$, such as ART and SIRT methods as well as methods based on Krylov subspaces. We will survey the basic properties of these methods, discuss how and why they work, and demonstrate how to accelerate and stop the iterations.

We will also illustrate the use of these methods with hands-on MATLAB exercises, using existing implementations of these methods in the package AIR Tools as well as pre-defined test problems.

The lectures and exercises provide a basic understanding of the role, the advantages, and the limitations of iterative algebraic methods for imaging, and they set the stage for further developments in efficient general-purpose algorithms for large-scale linear inverse problems.

Participants are expected to have a basic knowledge of matrix computations and MATLAB, and an interest in numerical methods for solving inverse problems.

Part 1: Row Action Methods (about 2 hours + a break)

- Survey of this class of methods, which includes ART and SIRT algorithms.
- Their convergence and semi-convergence properties, without and with simple constraints.
- The choice of relaxation parameter to control the semi-convergence.
- Robust stopping rules.

Part 2: MATLAB Exercises with ART and SIRT Methods (about 3 hours)

Several MATLAB exercises using AIR Tools, exercises from [DIP], and special-designed exercises.

Part 3: Krylov Subspace Methods (about 2 hours + a break)

- Survey of this class of methods, including CGLS, GMRES, and variants
- Their convergence and semi-convergence properties
- When transpose-free methods are useful
- How noise enters the iterations and solutions
- Improving the subspace – augmentation, subspace preconditioning

Part 4: MATLAB Exercises with Krylov Subspace Methods (similar to Part 2) (about 3 hours)**Part 5: Extensions of Iterative Algebraic Methods, Current and Future Trends (1–2 hours)**

- Block rules and their role in GPU computing
- Some other ideas
- Discussion about future trends.

[PCH]: P. C. Hansen, *Discrete Inverse Problems – Insight and Algorithms*, SIAM, Philadelphia, 2010.

AIR Tools Matlab package: <http://www.imm.dtu.dk/~pch/AIRtools/>

Regularization Tools Matlab package: <http://www.imm.dtu.dk/~pch/Regutools/>