COST Training School on Algebraic Reconstruction Methods in Tomography

ASTRA Exercises – Part 1

1. Test installation
Run the ASTRA sample `s004_cpu_reconstruction` to test ASTRA is working.

2. Basic operations
Define a parallel beam geometry (not too large, up to 256×256), and your favourite phantom image. Use ASTRA to perform a forward projection and a backprojection, and display the results.

3. Basic reconstructions
Using the sinogram $\mathbf{p}$ from the previous exercise, run reconstructions with the `FBP` (Filtered Backprojection) and `SIRT` algorithms in ASTRA, and display the results.

Define the residual of a reconstruction $\bar{\mathbf{x}}$ as $\mathbf{p} - \mathbf{A}\bar{\mathbf{x}}$ (where $\mathbf{A}$ is the system matrix). For the two reconstructions just computed, also compute their residuals and the (Euclidean) norm of the residuals.

4. Convergence plot
While running `SIRT`, compute the norm of the residual after each iteration. Also compute the norm of the difference with the phantom image after iteration. Plot these two series to visualize the convergence. Do the same thing for the `SART` algorithm. (Note that for SART, each iteration processes only a single projection agle.)

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ASTRA Exercises – Part 2

5. SPOT operator
Again define your favourite 2D geometry and define a `opTomo` SPOT operator. Take a phantom image, and use the SPOT operator to compute a forward projection and a backprojection, and display the results. Use the Matlab function `lsqr` to compute a reconstruction from the forward projection and the SPOT operator.

6. Fan beam geometry
Define a volume geometry of 64×64, and two `fanflat_vec` projection geometries: one for a fan beam geometry rotating around the origin, and one for a fan beam geometry with the detector shifted by 4 pixels along the direction of the detector. Perform a forward projection and a reconstruction for both of them and verify that they behave as expected.

Study what happens if you create a sinogram of a phantom using the shifted geometry, but perform the reconstruction using the original non-shifted geometry. This models misalignment in an experimental setup.

7. Null space
In this exercise you will try to see which images can be hidden from X-rays.

The SIRT algorithm discussed in the lectures, when started from initial vector $\mathbf{x}^{(0)} = 0$, computes the shortest (weighted) least-squares solution to the problem $\mathbf{Ax} = \mathbf{b}$.

In general, when started from any vector $\mathbf{x}^{(0)}$, it computes the (weighted) least-squares solution closest to $\mathbf{x}^{(0)}$. When applied to the system $\mathbf{Ax} = 0$, this can be used to compute an element of the null space of $\mathbf{A}$ closest to a given image. (Recall: $\mathbf{x}$ is in the null space of $\mathbf{A}$ if and only if we have $\mathbf{Ax} = 0$.)

Find an image of a "dangerous" object, and, using the above method, compute the element $\mathbf{y}$ in the null space of $\mathbf{A}$ closest to that image. Now add an innocent phantom image to this element $\mathbf{y}$, and compute the forward projection and reconstruction of the result.

Depending on the geometry (different numbers of projection angles, and different angular ranges), $\mathbf{y}$ might be very close to your dangerous image (in which case it is very well hidden from X-rays), or it might not resemble it at all (in which case it can't be hidden). Experiment with these parameters of the geometry.

8. Reconstruction window
Take the forward projection of a phantom image, and use SIRT from ASTRA to reconstruct the image at different reconstruction window sizes, both smaller and larger than the phantom image. Compare the results at varying window sizes and varying numbers of projection angles.

9. Discretizations
Study the effect of using a different projector (`line`, `strip`, `linear`) for the forward projection than for the reconstruction, by plotting how the error relative to the ground truth behaves while performing `SIRT` iterations. Use a very small phantom to better see these effects.

For example, `strip` for forward projection and `line` for reconstruction, compared to using `line` for both.