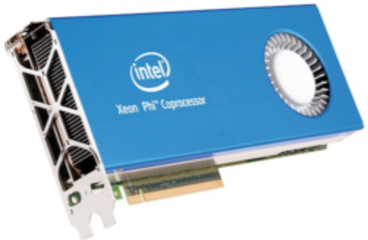

DTU

Block algebraic methods for CT and their performance

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DTU DCC Computing Center

GPUlab DTU Informatics

DTU Compute
Department of Applied Mathematics and Computer Science

Contributions from students etc.

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MULTICORE PERFORMANCE OF BLOCK ALGEBRAIC ITERATIVE RECONSTRUCTION METHODS*

HANS HENRIK B. SØRENSEN[†] AND PER CHRISTIAN HANSEN[†]

Abstract. Algebraic iterative methods are routinely used for solving the ill-posed sparse linear

Block Algebraic Methods for 3D Image Reconstructions on GPUs

Kenneth Kjær Nielsen

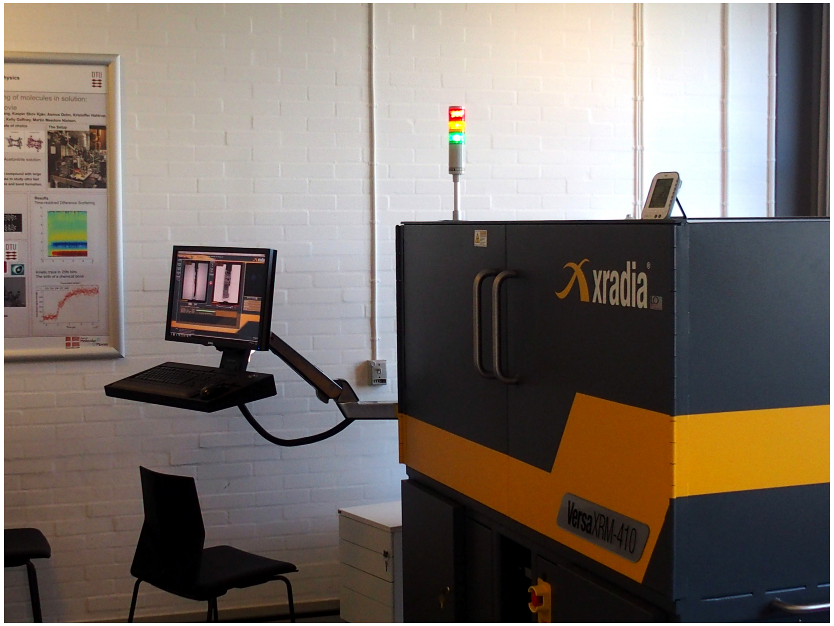
Kongens Lyngby 2014

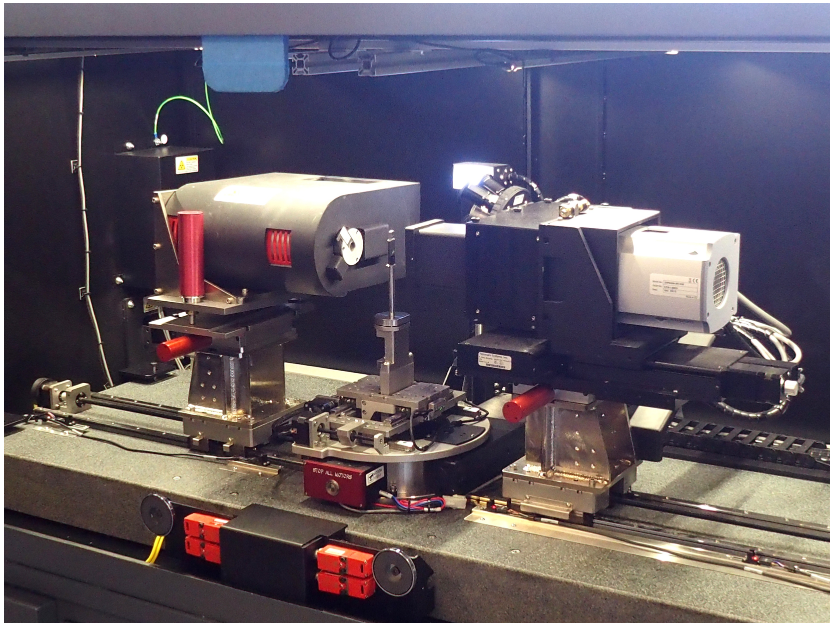
High-Performance Computing for Block-Iterative Tomography Reconstructions

Mads Friis Hansen

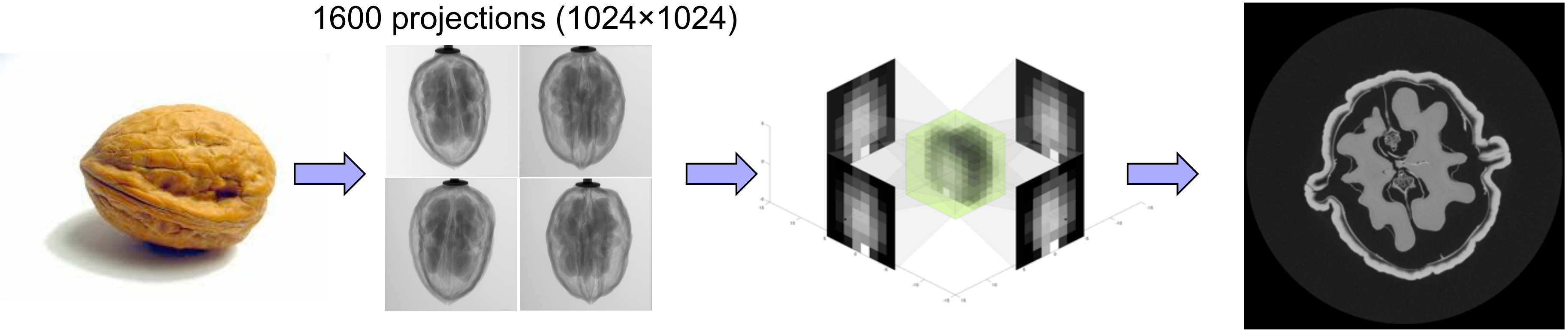
Kongens Lyngby 2016

The Walnut case





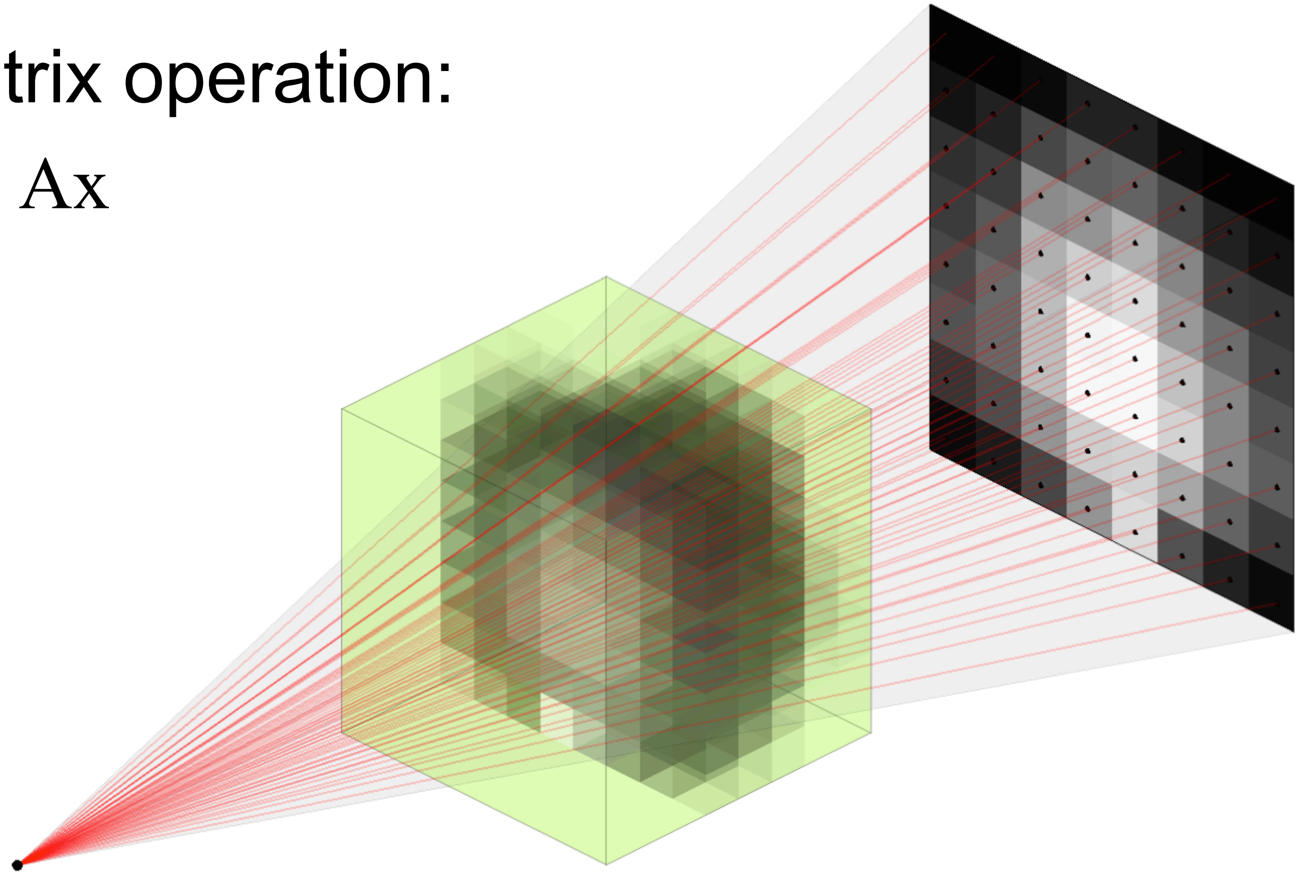
1600 projections (1024×1024)



Forward projection

- Matrix operation:

  $$y = Ax$$

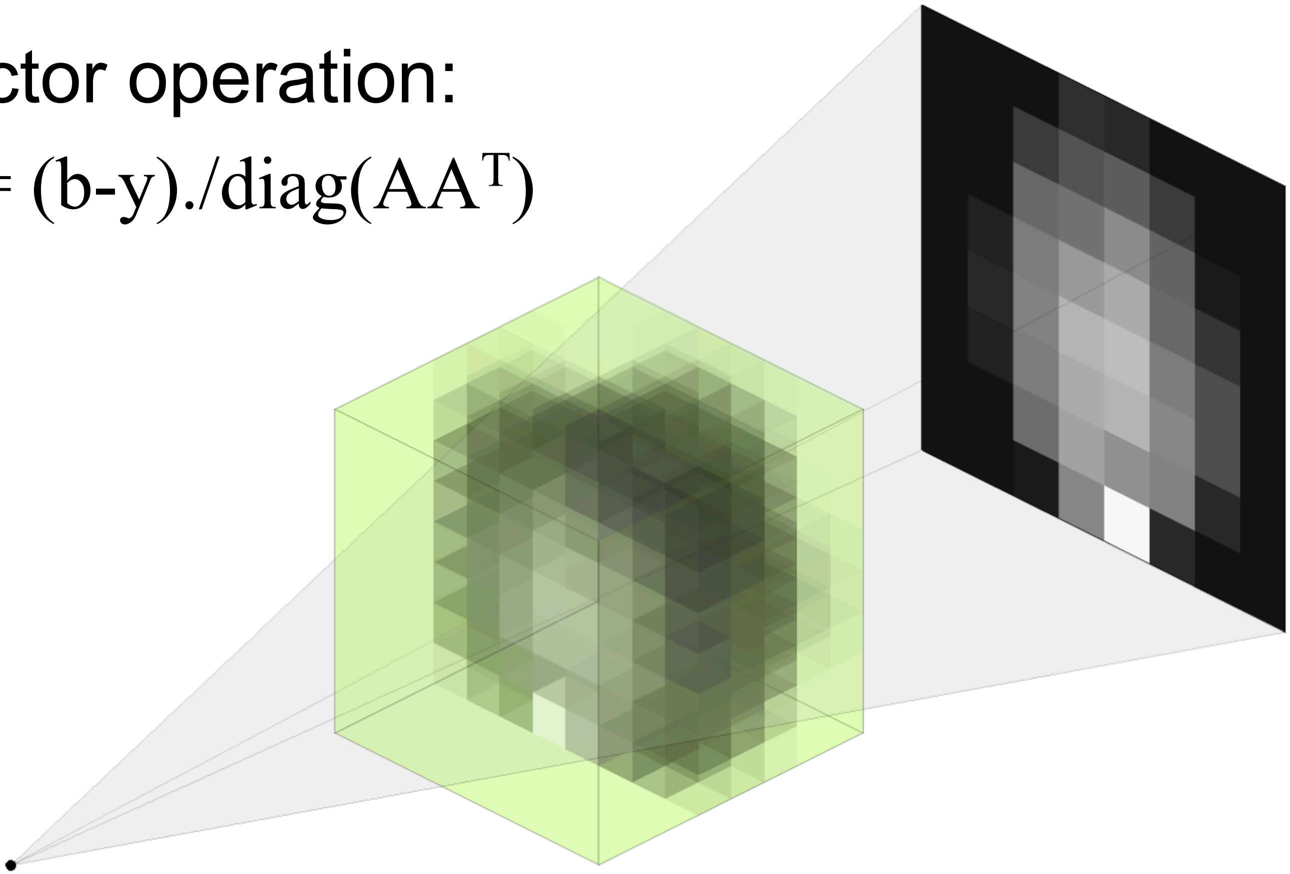


- Several different types:
  - Line, splatting, footprint, interpolation, ext. rays

Correction

- Vector operation:

  $y := (b-y)./\mathrm{diag}(AA^T)$

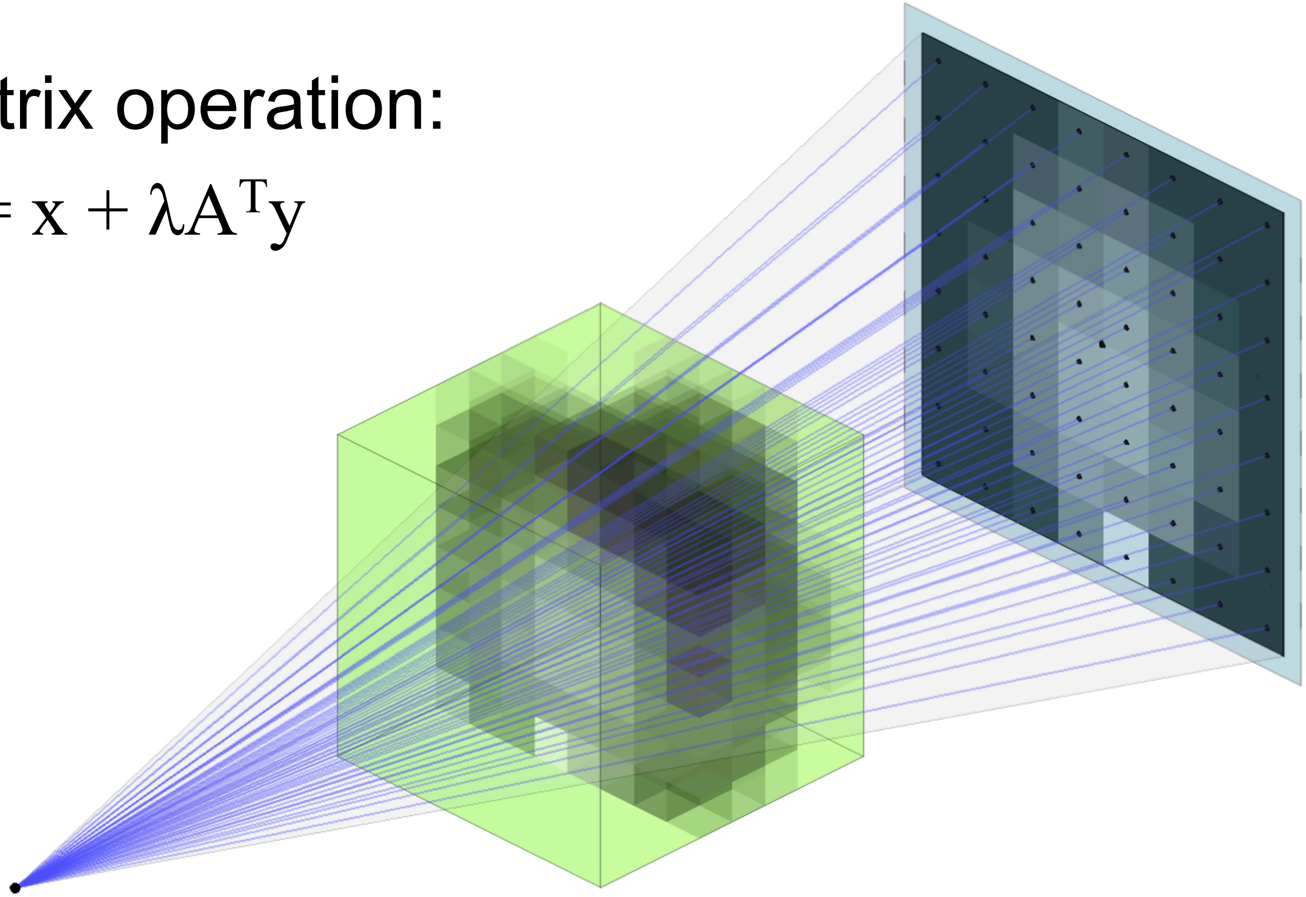


- AA^T can be pre-calculated (as forward projection).

Back projection

- Matrix operation:

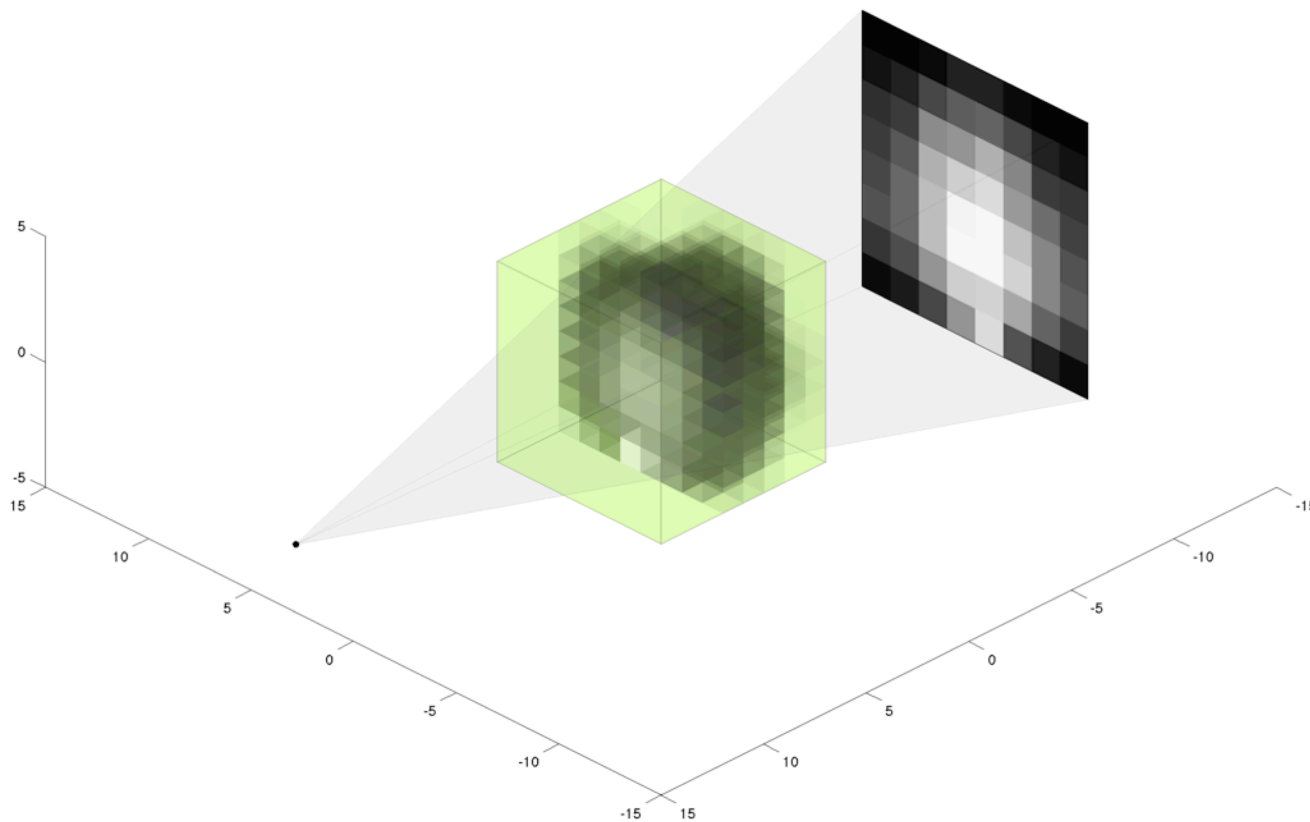
  $$x := x + \lambda A^T y$$



- Several different types:
  - Line, splatting, footprint, interpolation, ext. rays

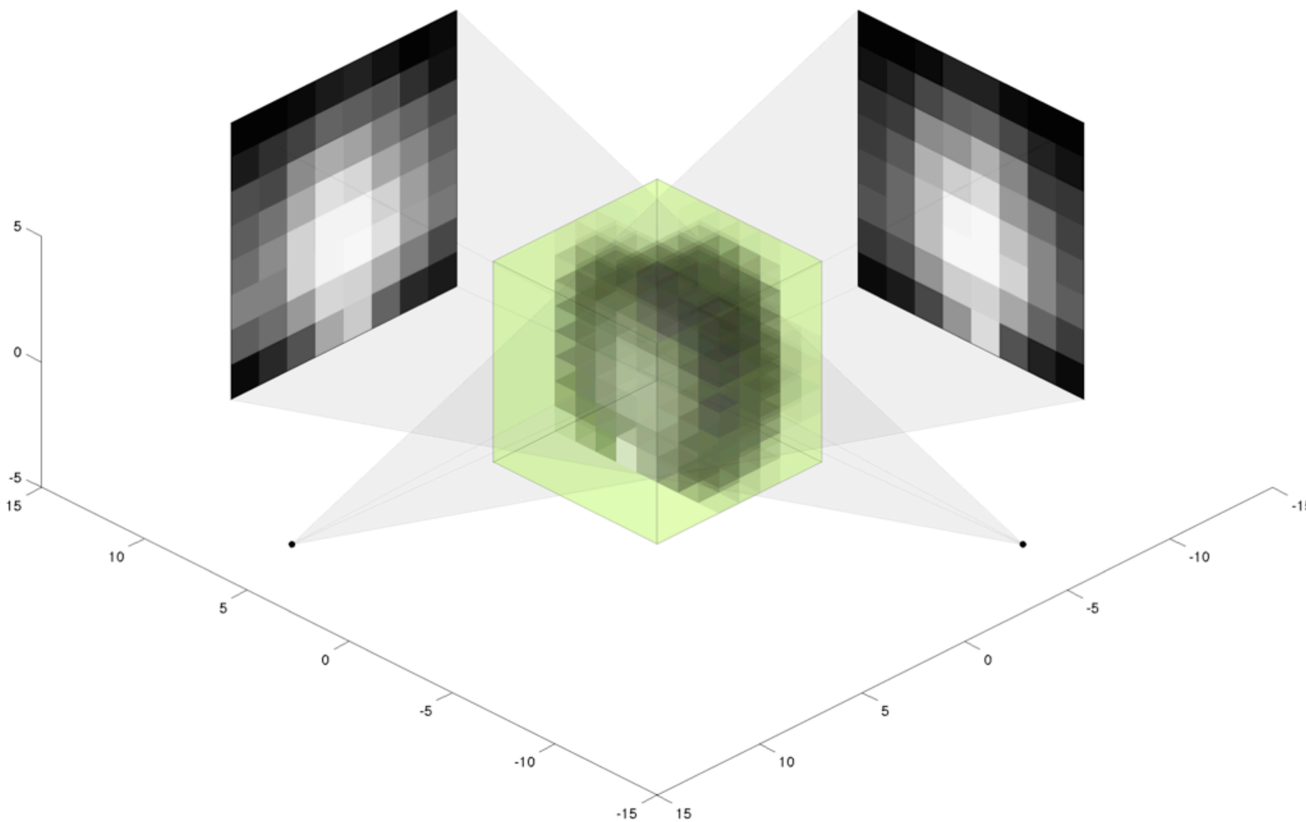
SIRT

- Forward projections ($\mathbf{A}\mathbf{x}$)



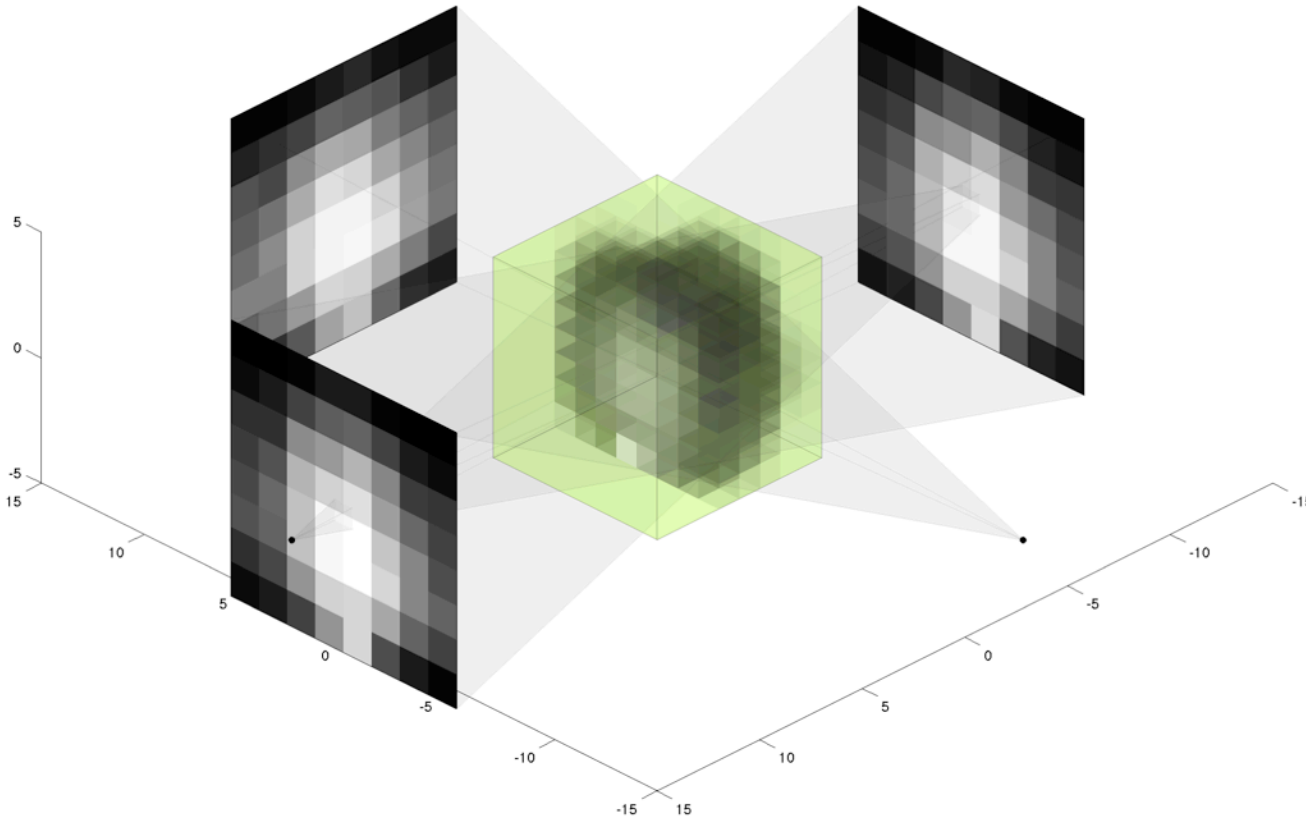
SIRT

- Forward projections ($\mathbf{A}\mathbf{x}$)



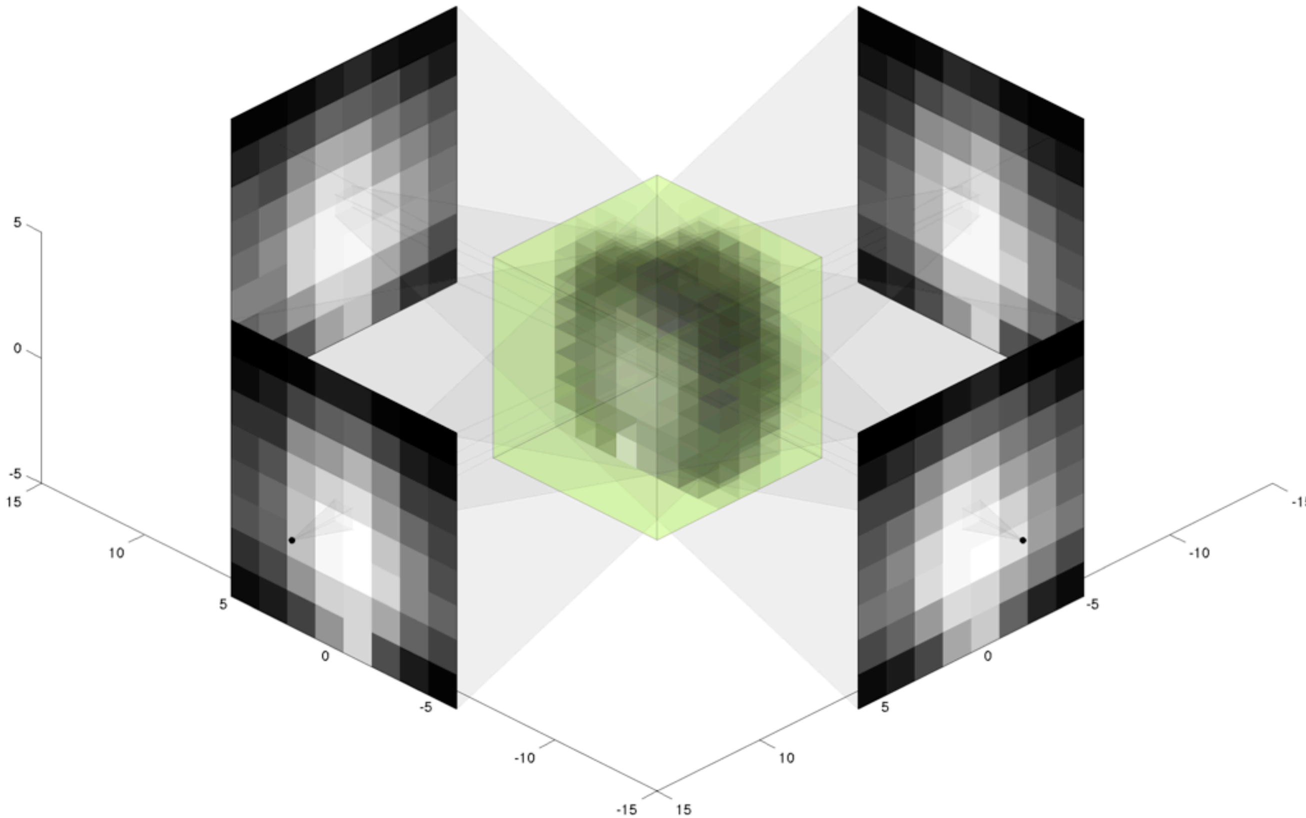
SIRT

- Forward projections ($\mathbf{A}x$)



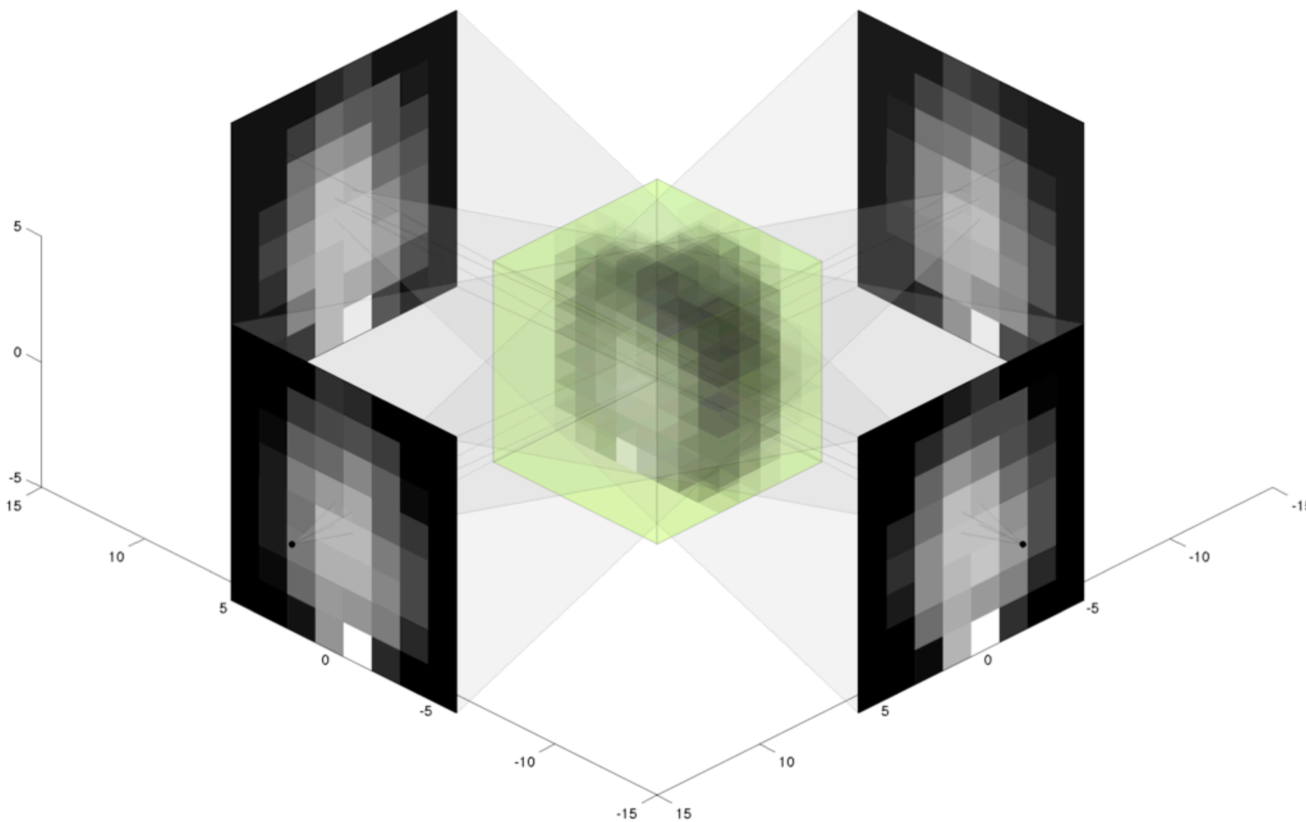
SIRT

- Forward projections ($\mathbf{A}x$)



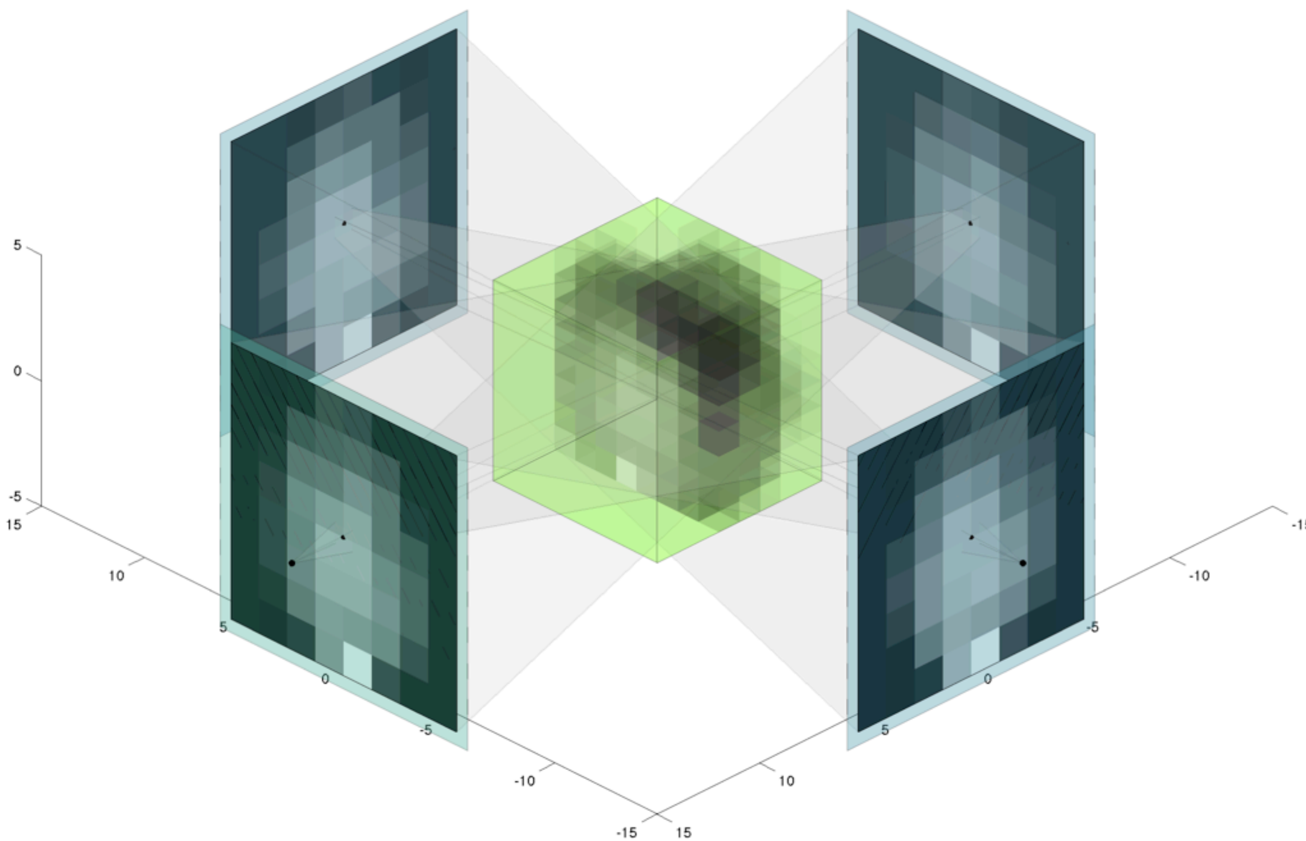
SIRT

- Correction ($y = (b\text{-}Ax)\ ./\mathrm{diag}(AA^T)$)



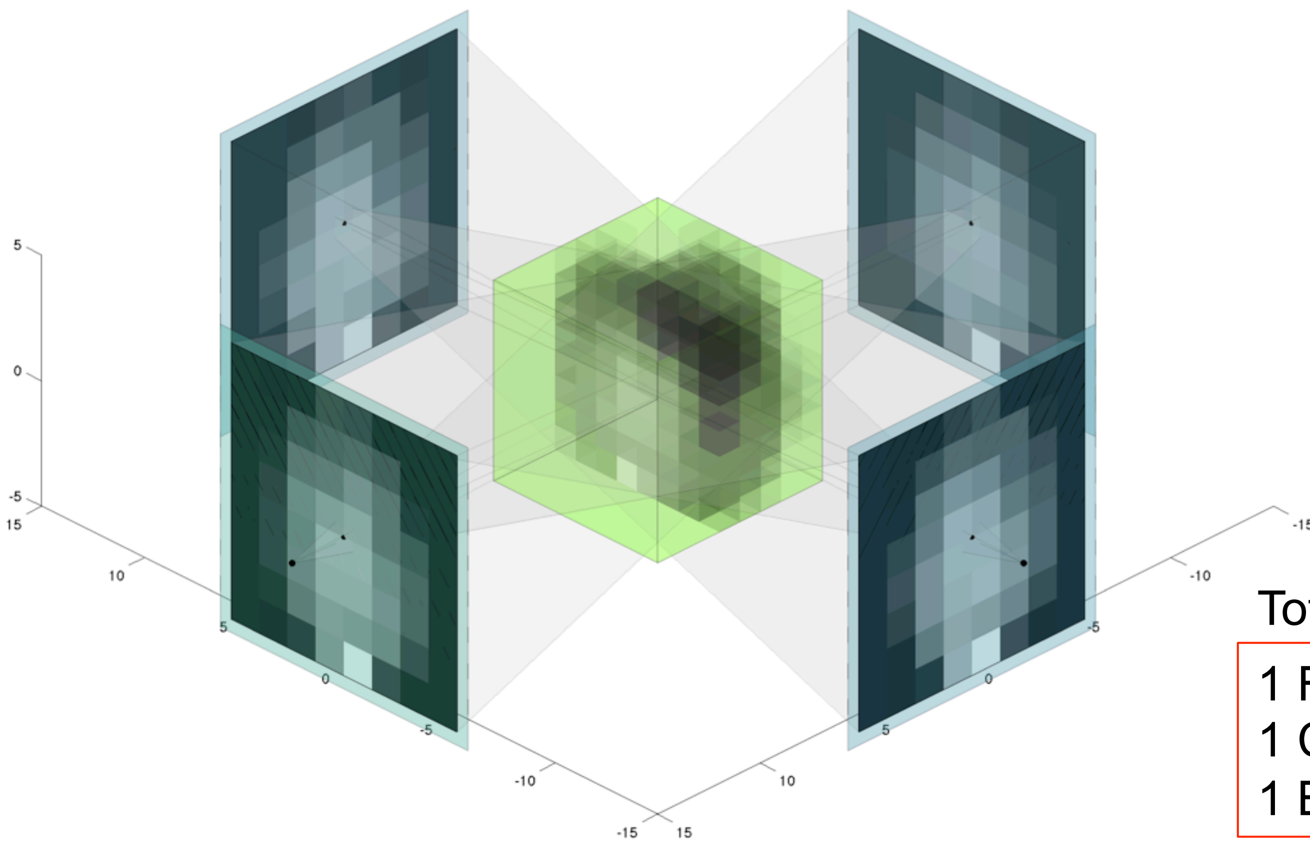
SIRT

- Back projection ($x := x + \lambda A^T y$)



SIRT

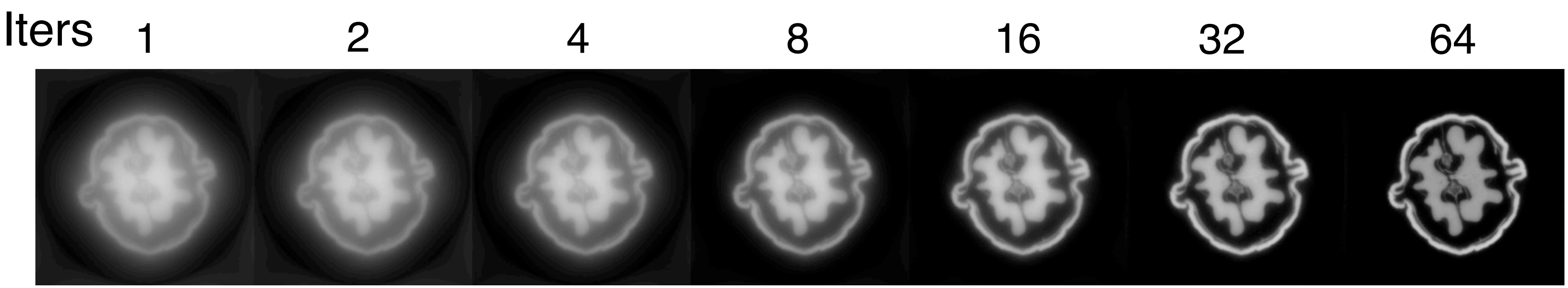
- Back projection $(x := x + \lambda A^T y)$



Total:

1 FP x 4
1 Correction x 4
1 BP x 4

SIRT reconstruction in ASTRA

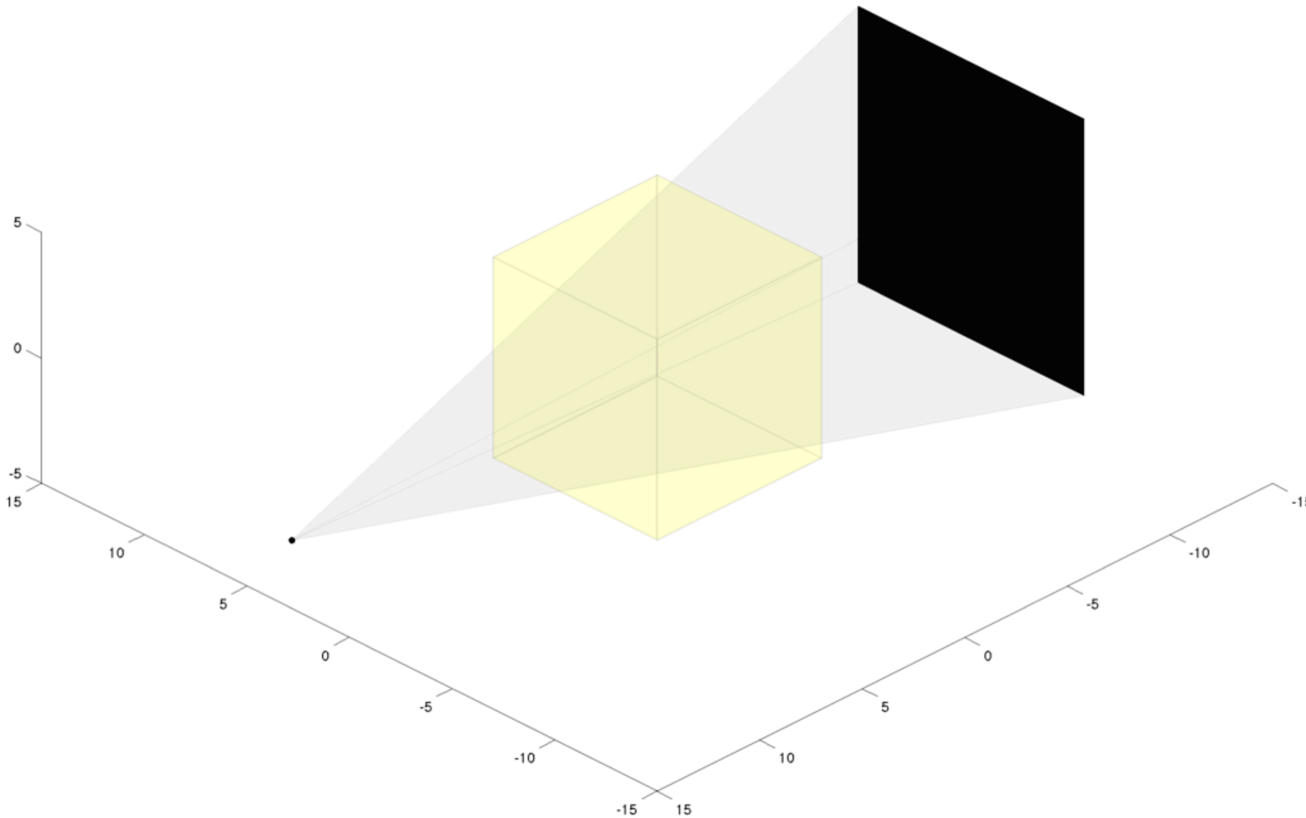



```
% Set up a GPU reconstruction
cfg = astra_struct('SIRT3D_CUDA');
cfg.ReconstructionDataId = rec_id;
cfg.ProjectionDataId = proj_id;
cfg.option.GPUindex = 0;
% Create the algorithm object
alg_id = astra_mex_algorithm('create', cfg);
% Run 64 SIRT iterations
astra_mex_algorithm('iterate', alg_id, 64);
% Get the result
rec = astra_mex_data3d('get', rec_id);
```

Block Methods

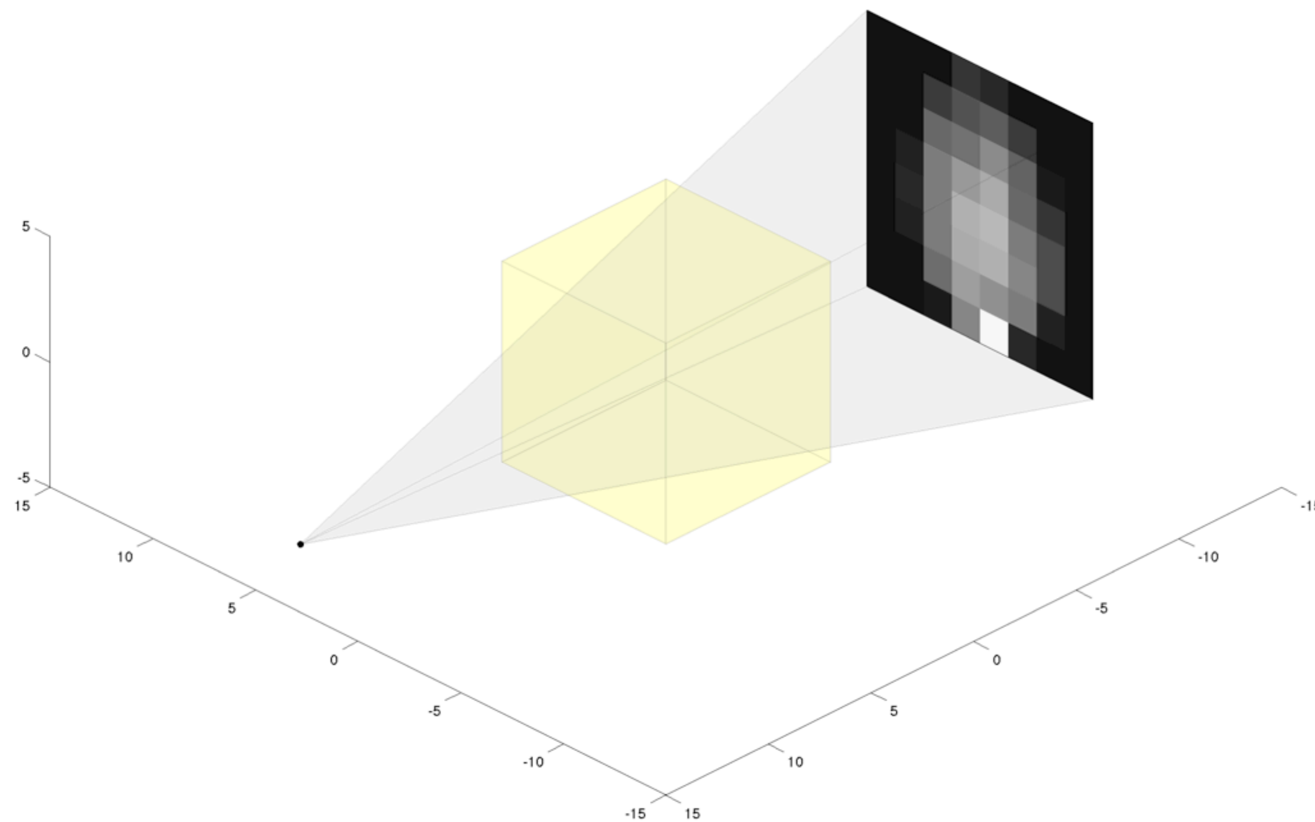
Block method (one per projection)

- Forward projection (A_0x)



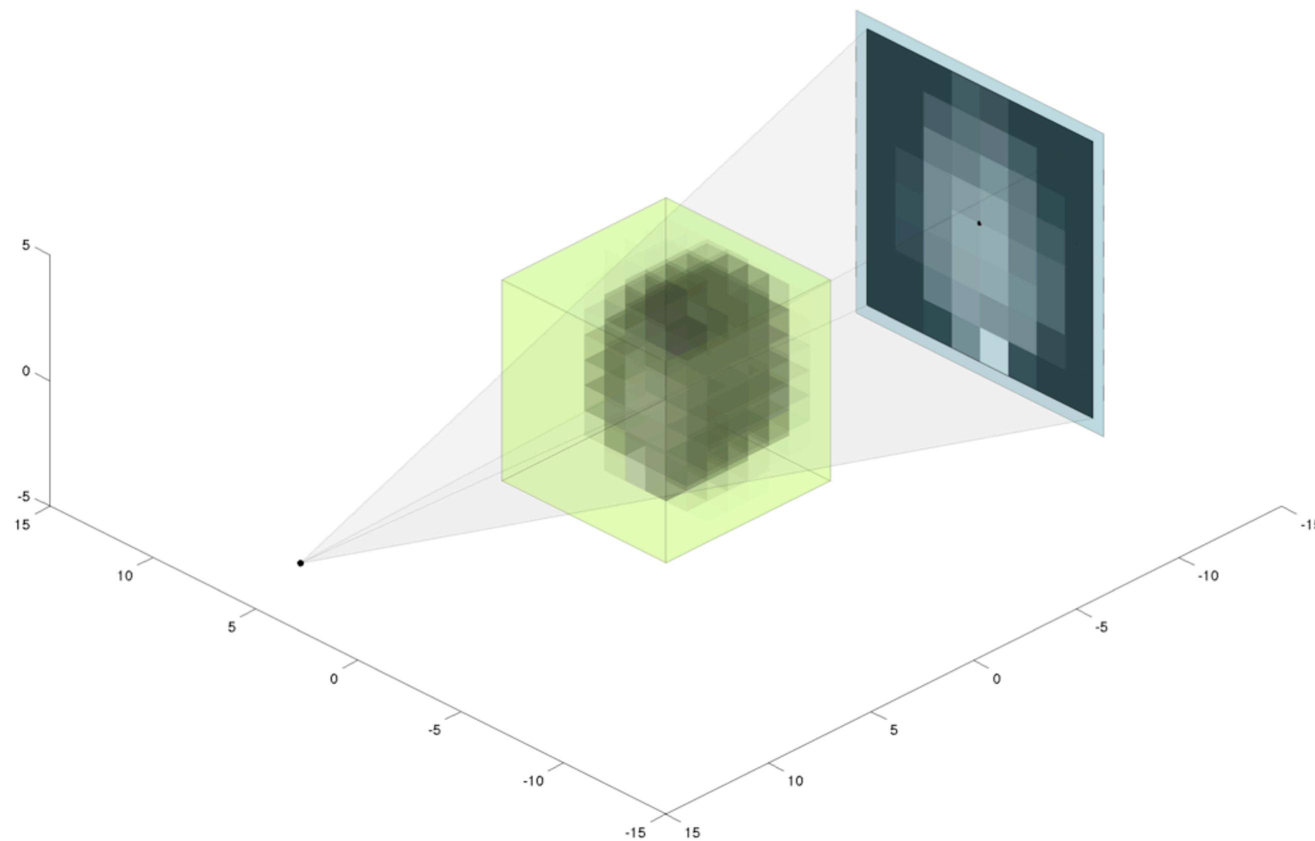
Block method (one per projection)

- Correction ($y = (b-A_0x)$ ./diag($A_0A_0^T$))



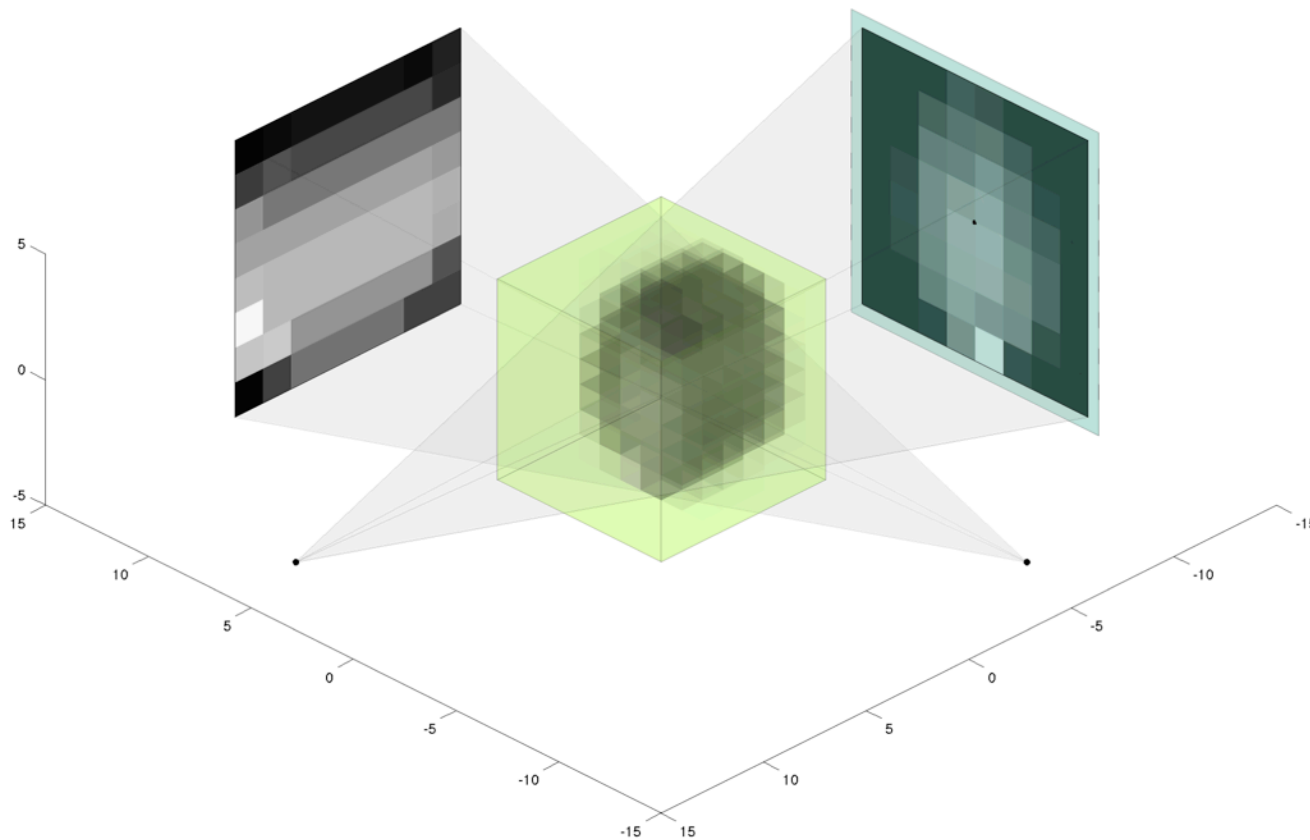
Block method (one per projection)

- Back projection ($x := x + \lambda A_0^T y$)



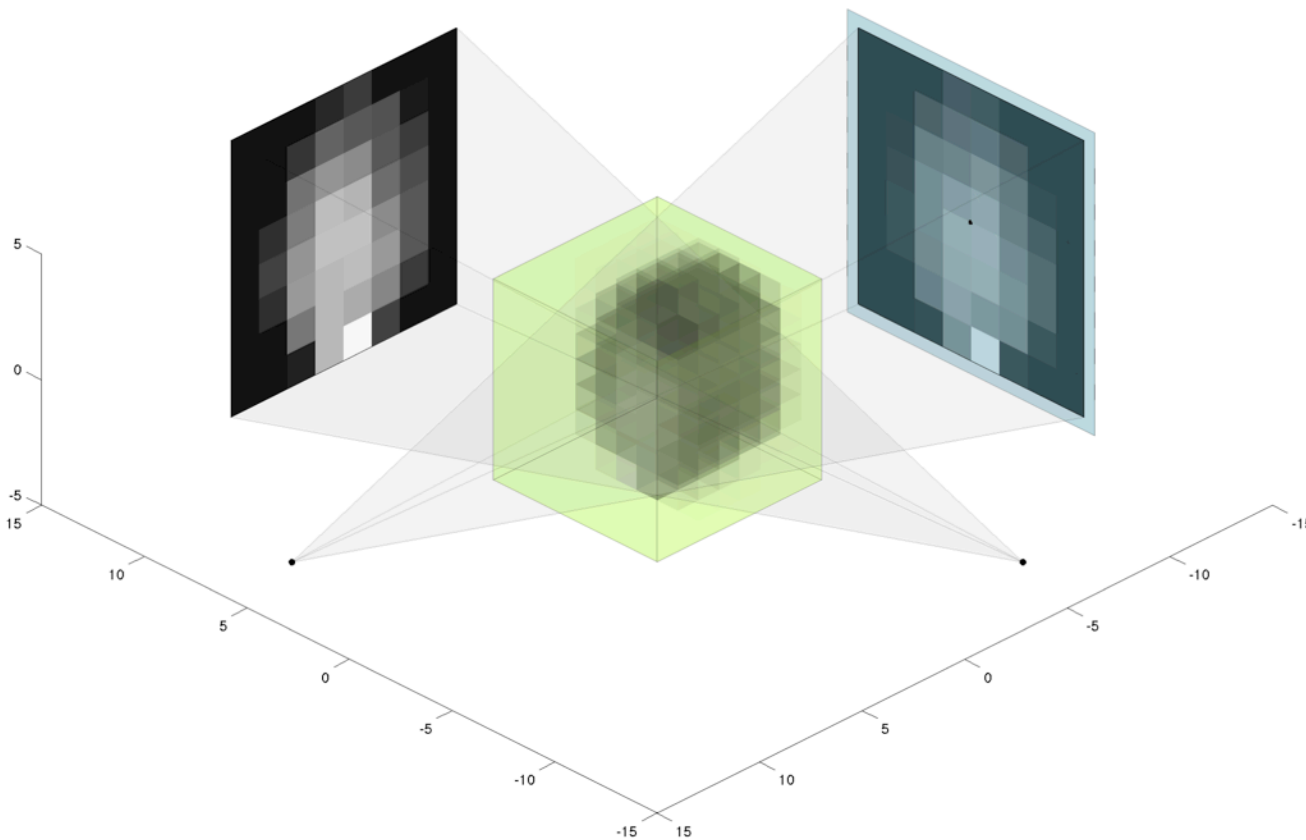
Block method (one per projection)

- Forward projection ($A_1 x$)



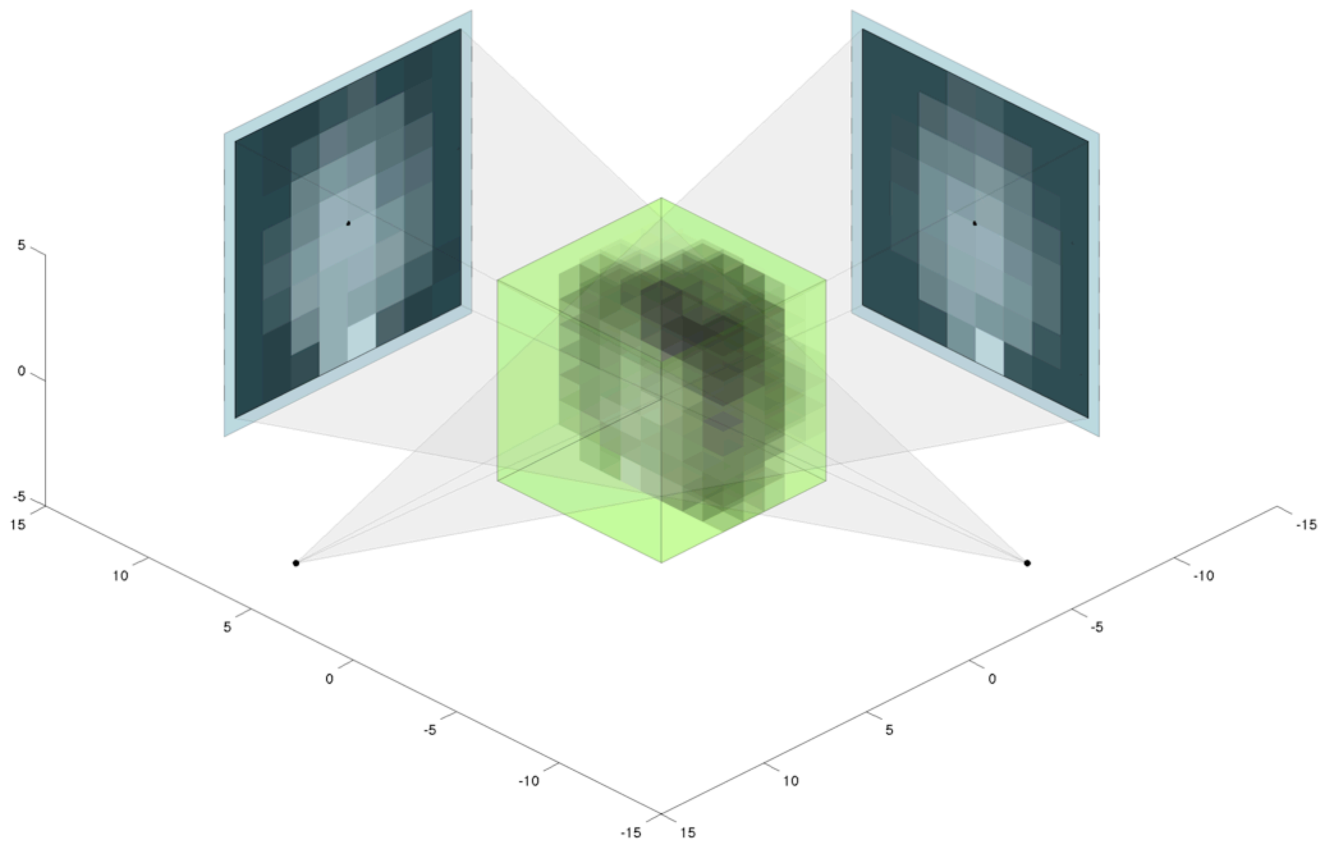
Block method (one per projection)

- Correction ($y = (b - A_1 x) ./\mathrm{diag}(A_1 A_1^T)$)



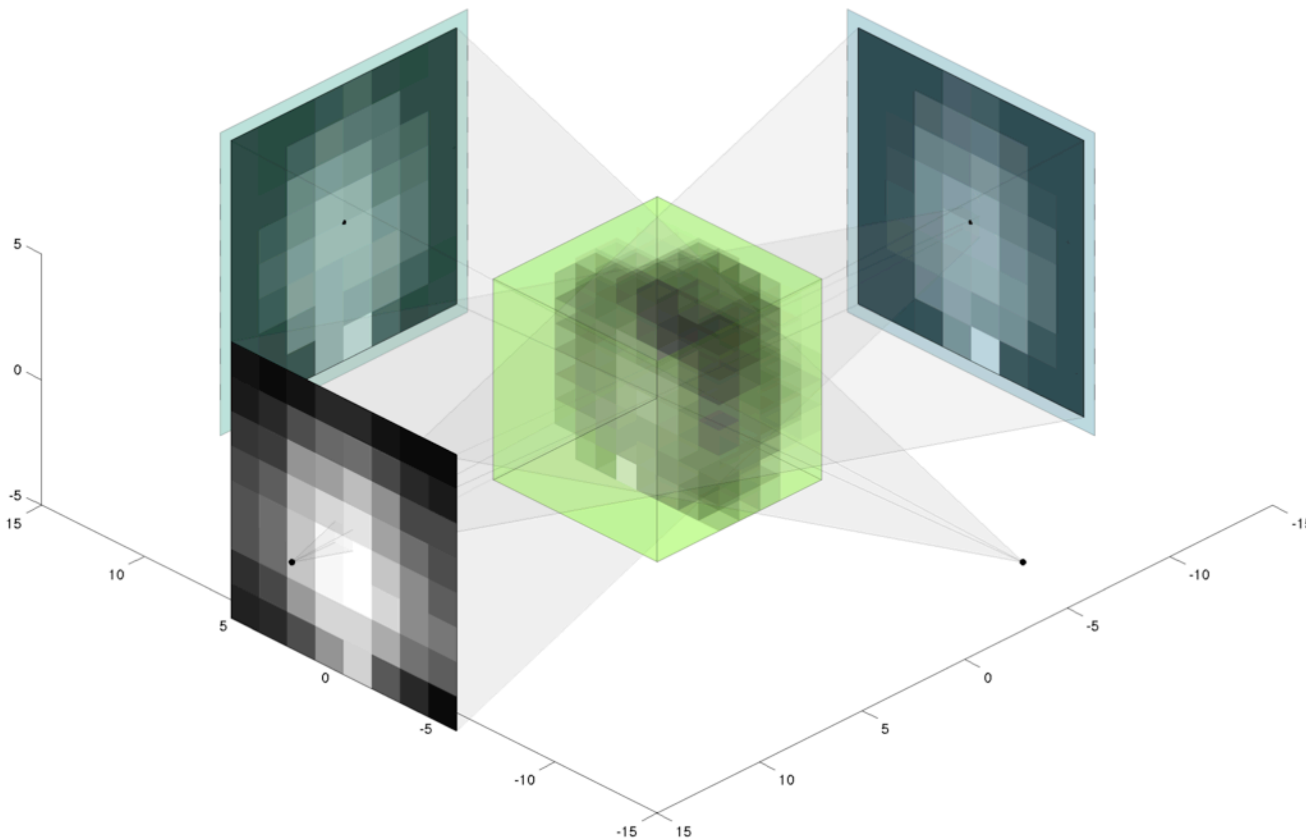
Block method (one per projection)

- Back projection ($x := x + \lambda A_1^T y$)



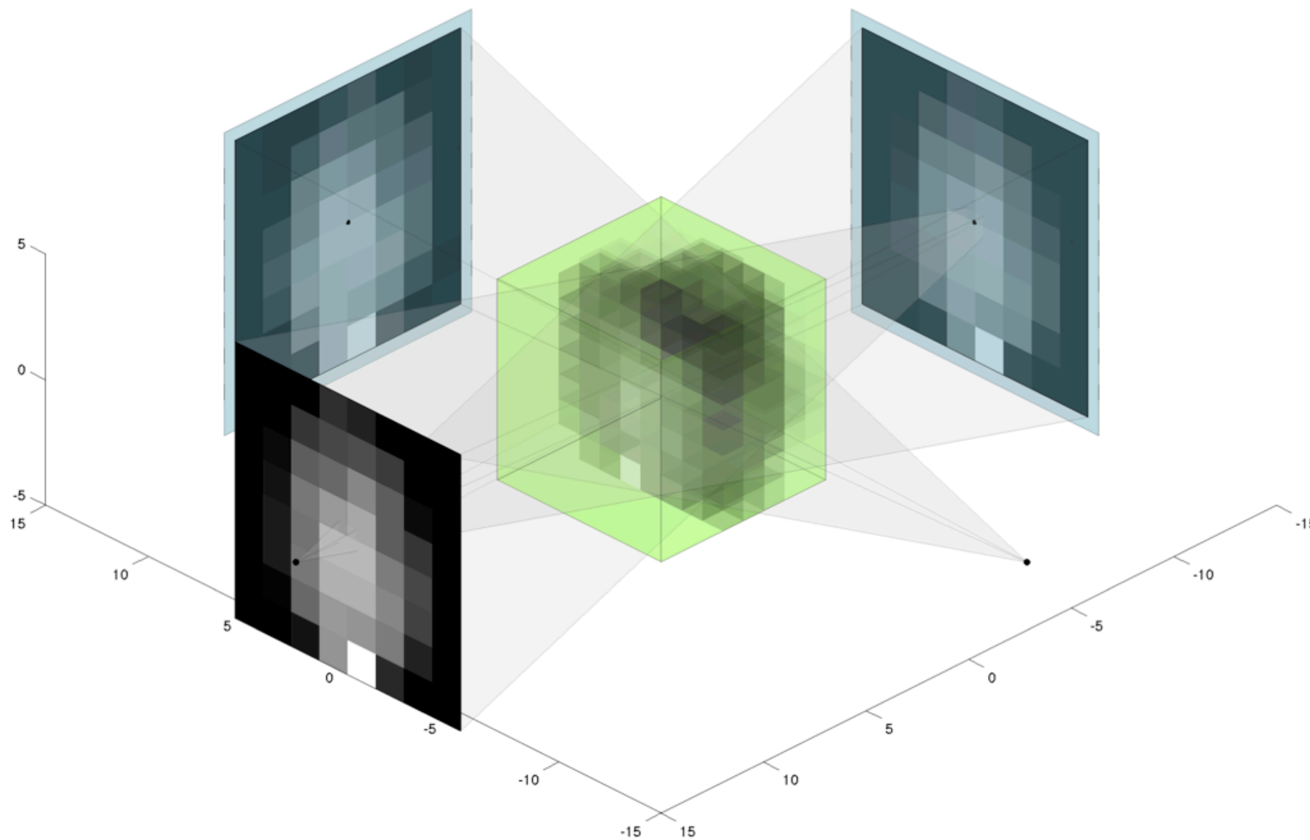
Block method (one per projection)

- Forward projection (A_2x)



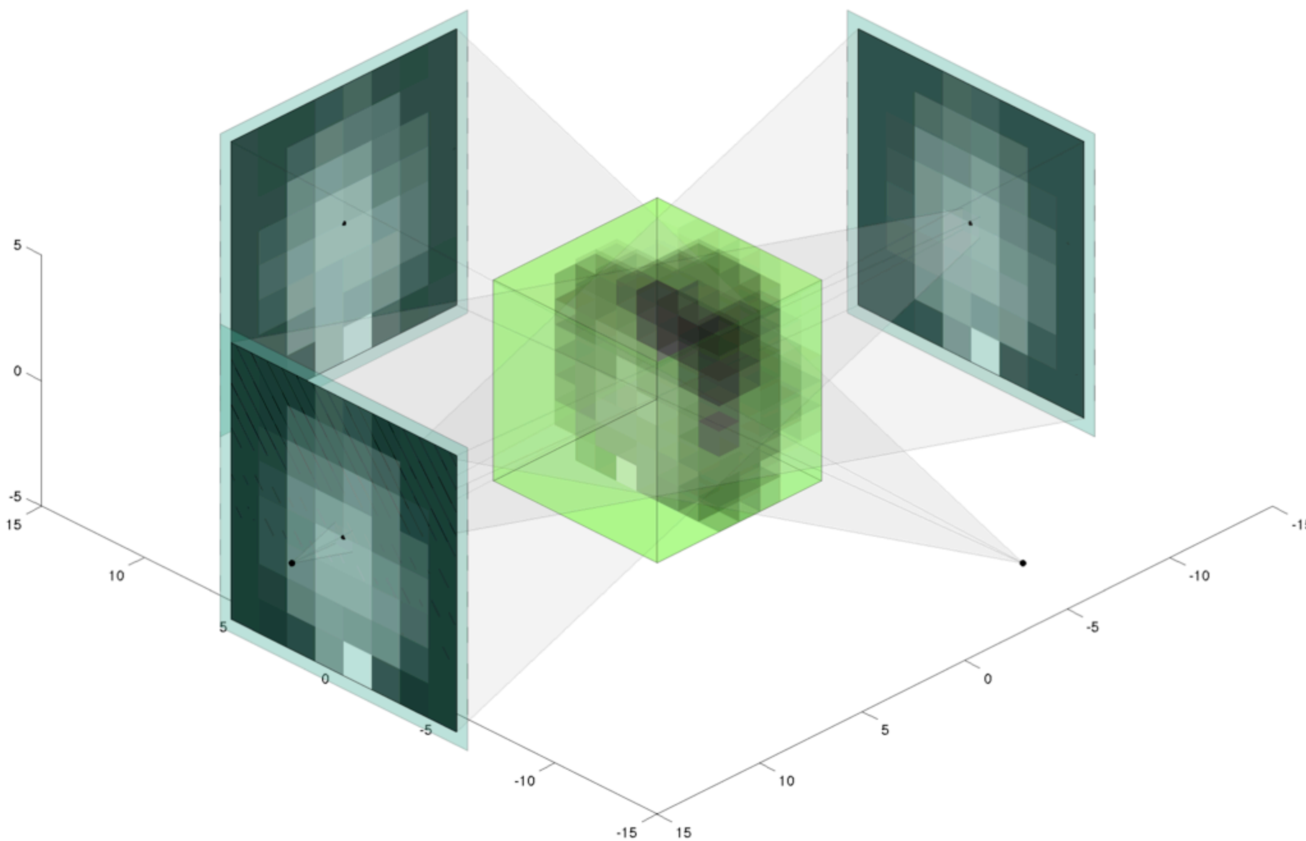
Block method (one per projection)

- Correction ($y = (b\text{-}A_2x)\ ./\mathrm{diag}(A_2A_2^T)$)



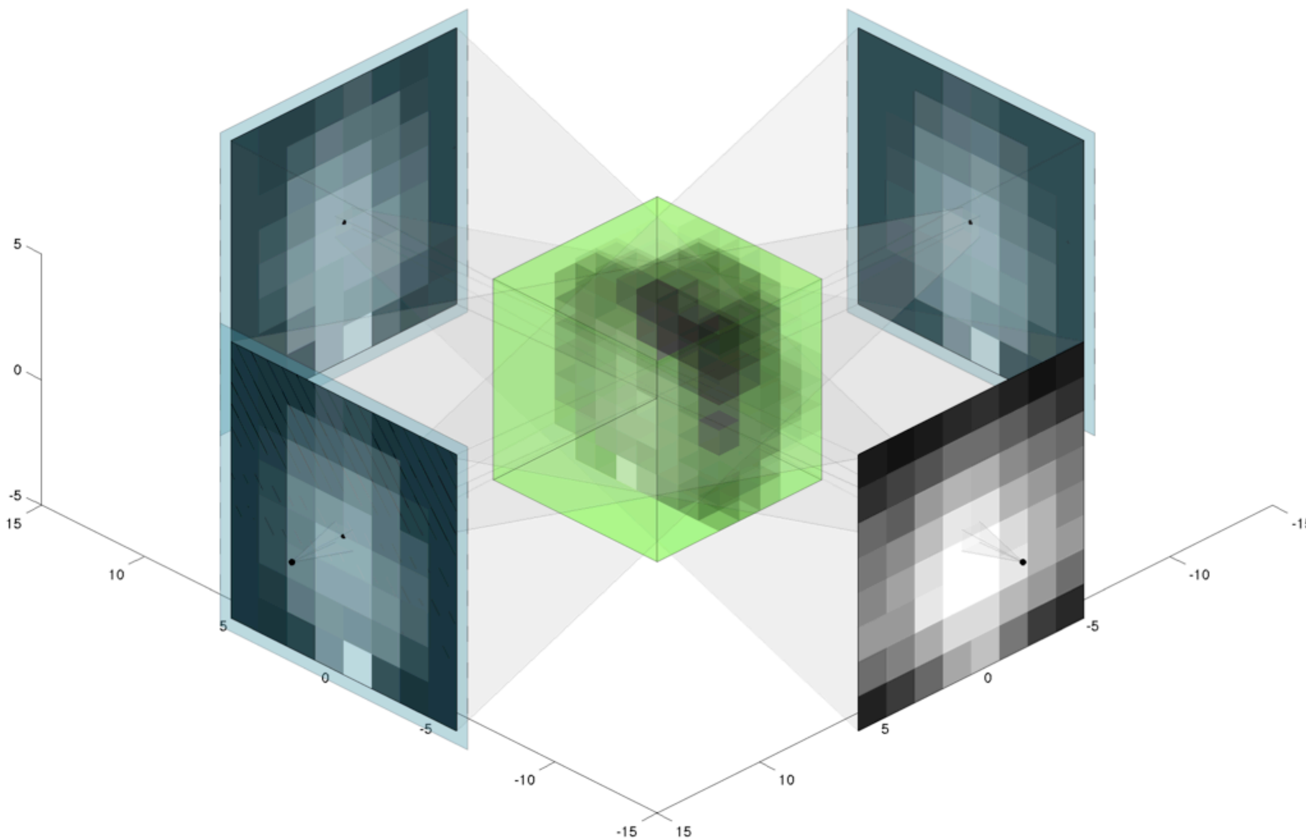
Block method (one per projection)

- Back projection ($x := x + \lambda A_2{}^T y$)



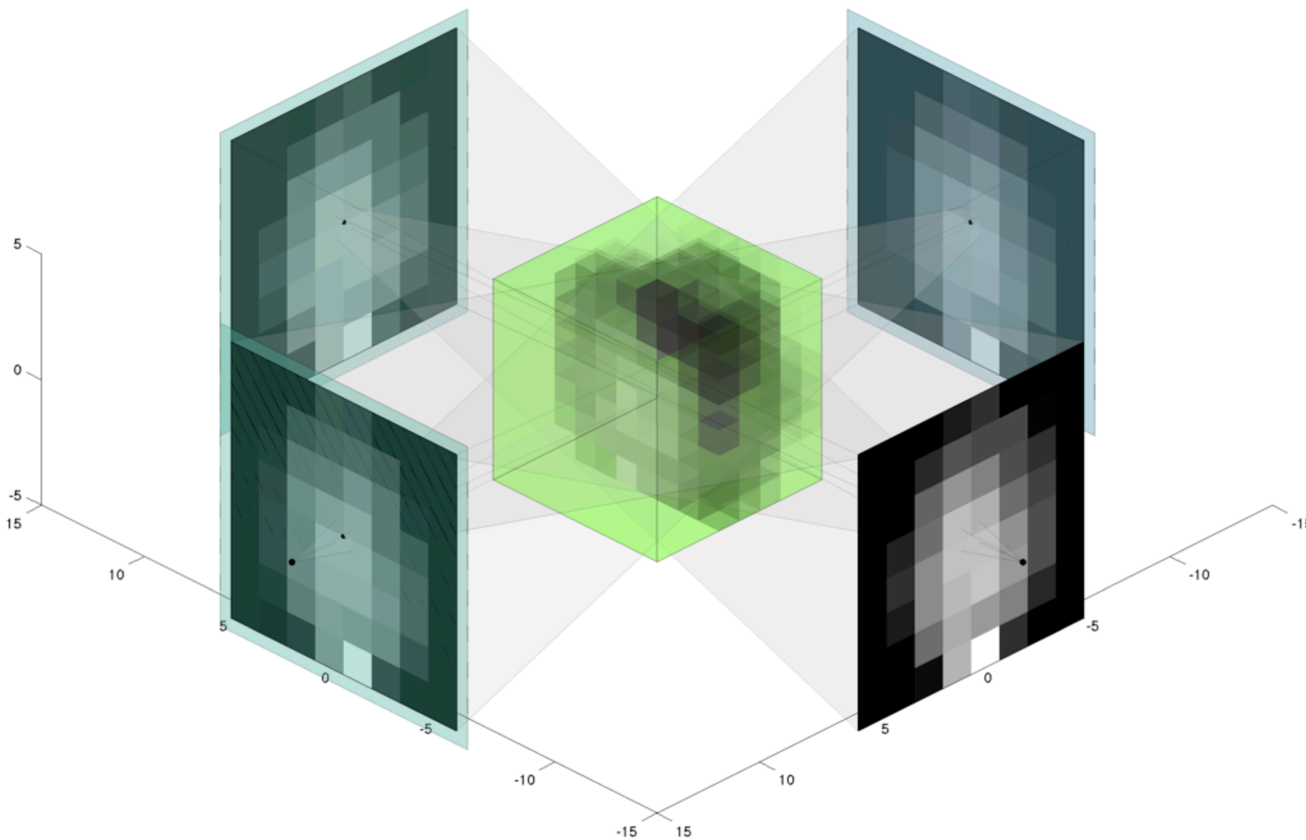
Block method (one per projection)

- Forward projection (A_3x)



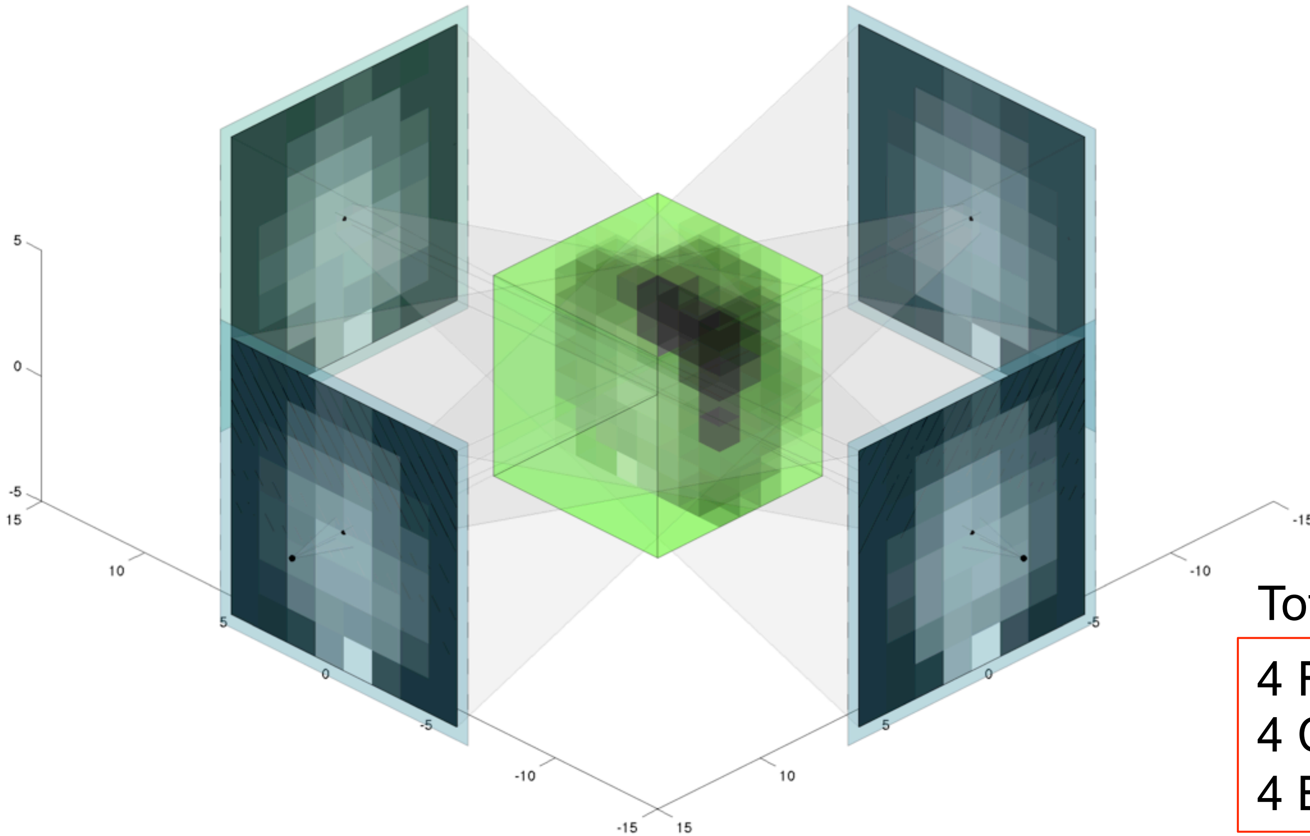
Block method (one per projection)

- Correction $(y = (b - A_3 x)\ ./\mathrm{diag}(A_3 A_3^T))$



Block method (one per projection)

- Back projection ($x := x + \lambda A_3{}^T y$)



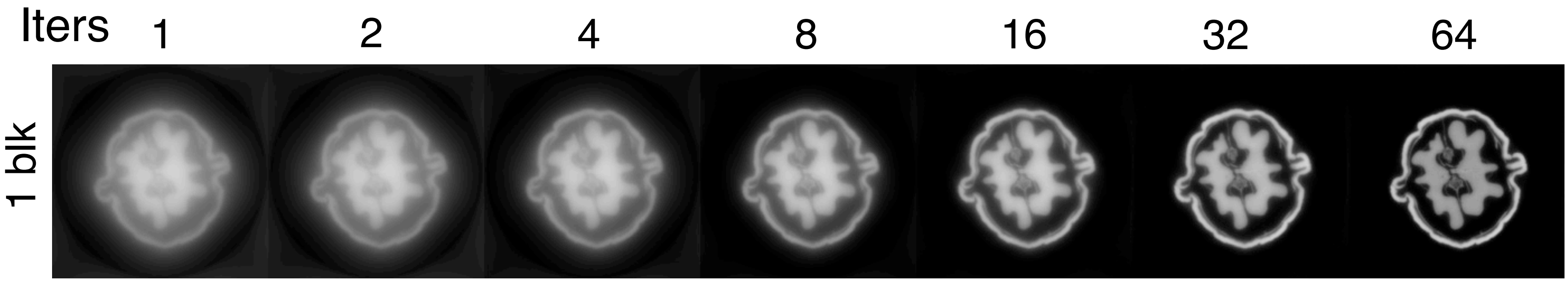
Total:

4 FP
4 Corrections
4 BP

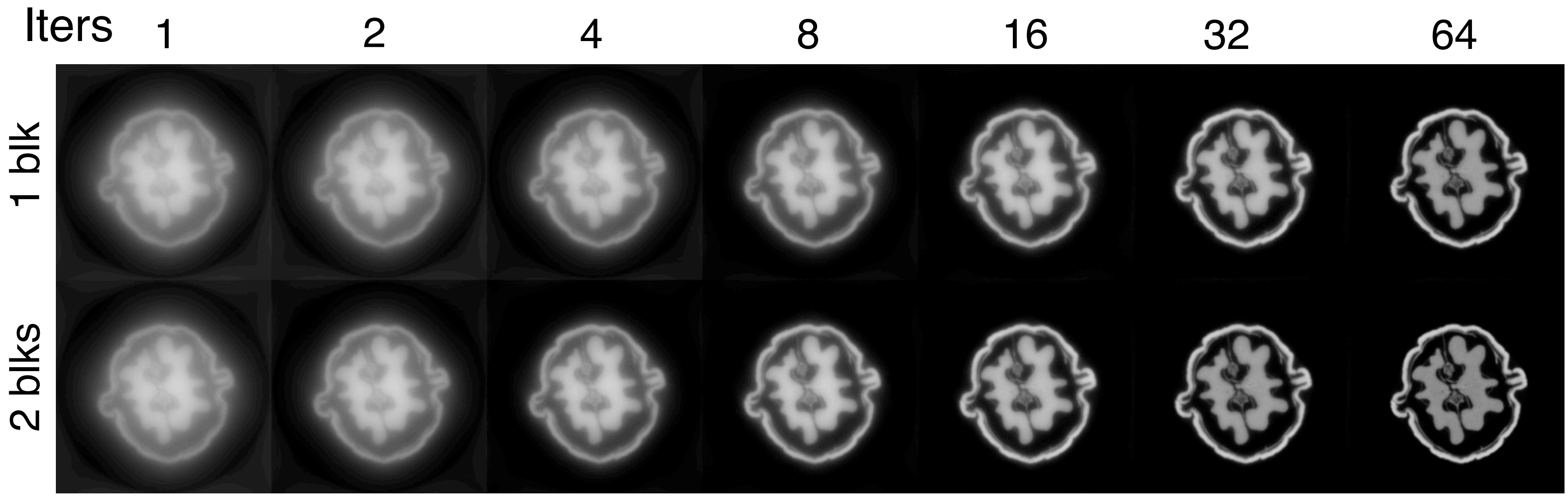
Block method using ASTRA

```
...
for iter = 1:num_iter
    for blk = 1:num_blks
        % Create the algorithm object
        alg_id = astra_mex_algorithm('create', cfg{blk});
        % Run 1 SIRT iteration with this block
        astra_mex_algorithm('iterate', alg_id, 1);
        % Delete algorithm object to conserve memory
        astra_mex_algorithm('delete', alg_id);
    end
end
% Get the result
rec = astra_mex_data3d('get', rec_id);
```

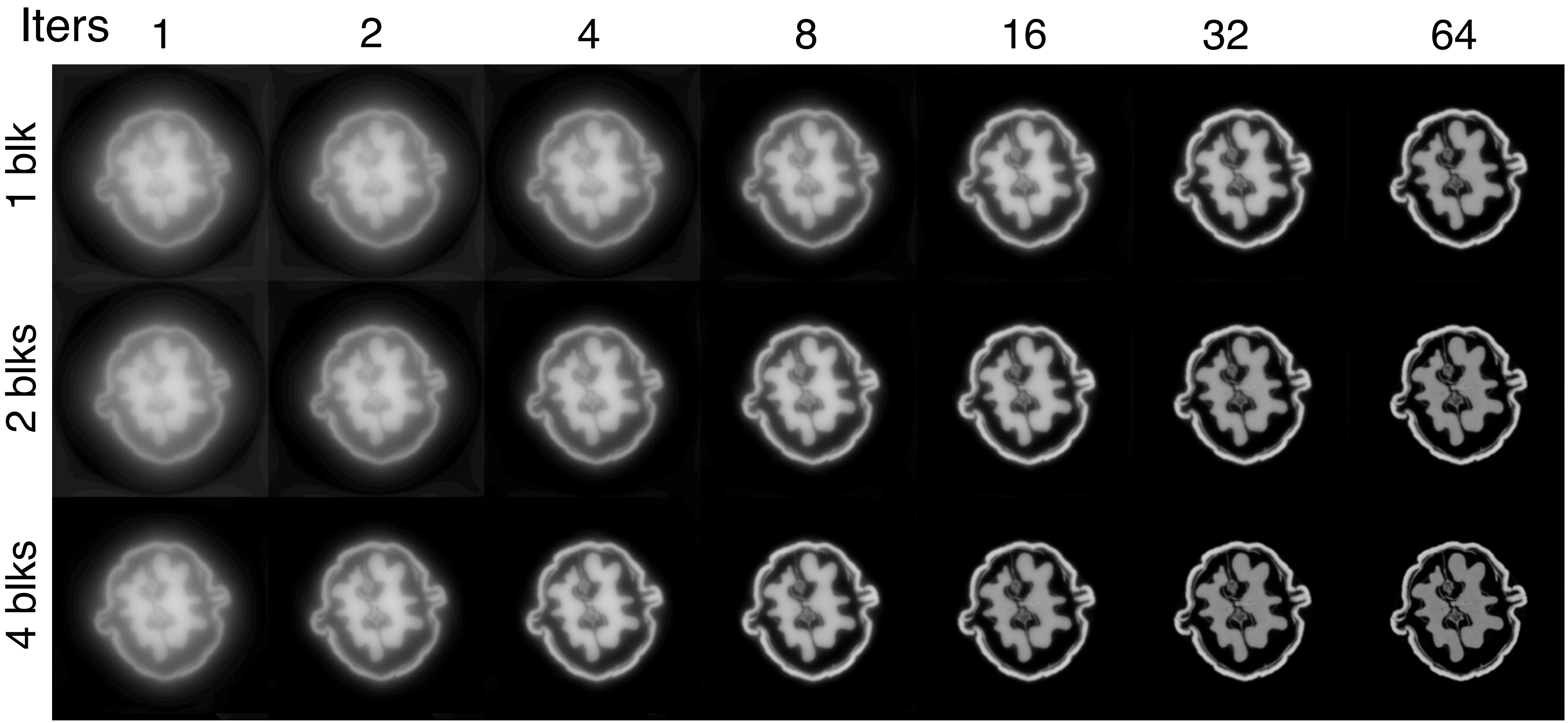
Block method reconstruction



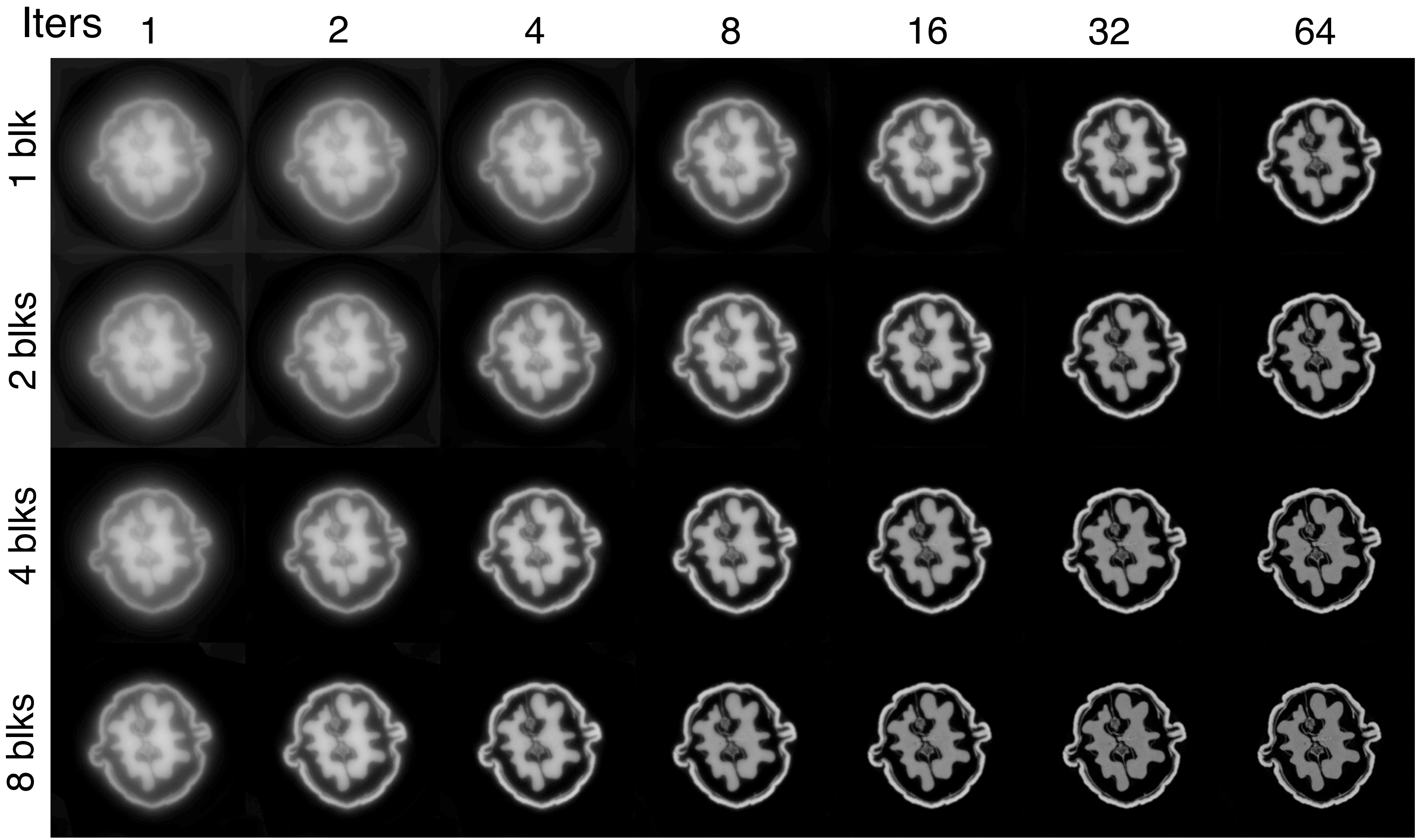
Block method reconstruction



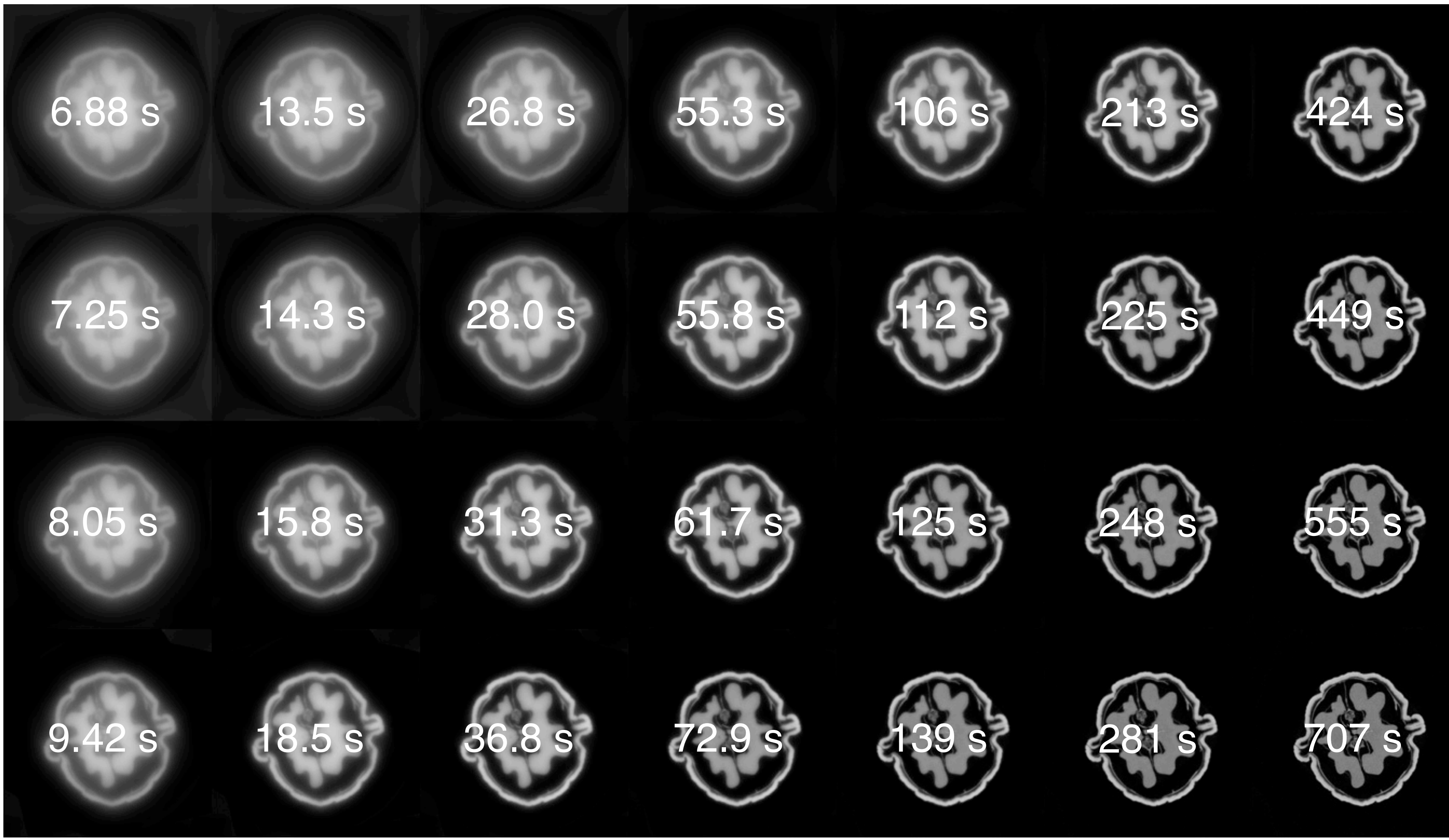
Block method reconstruction



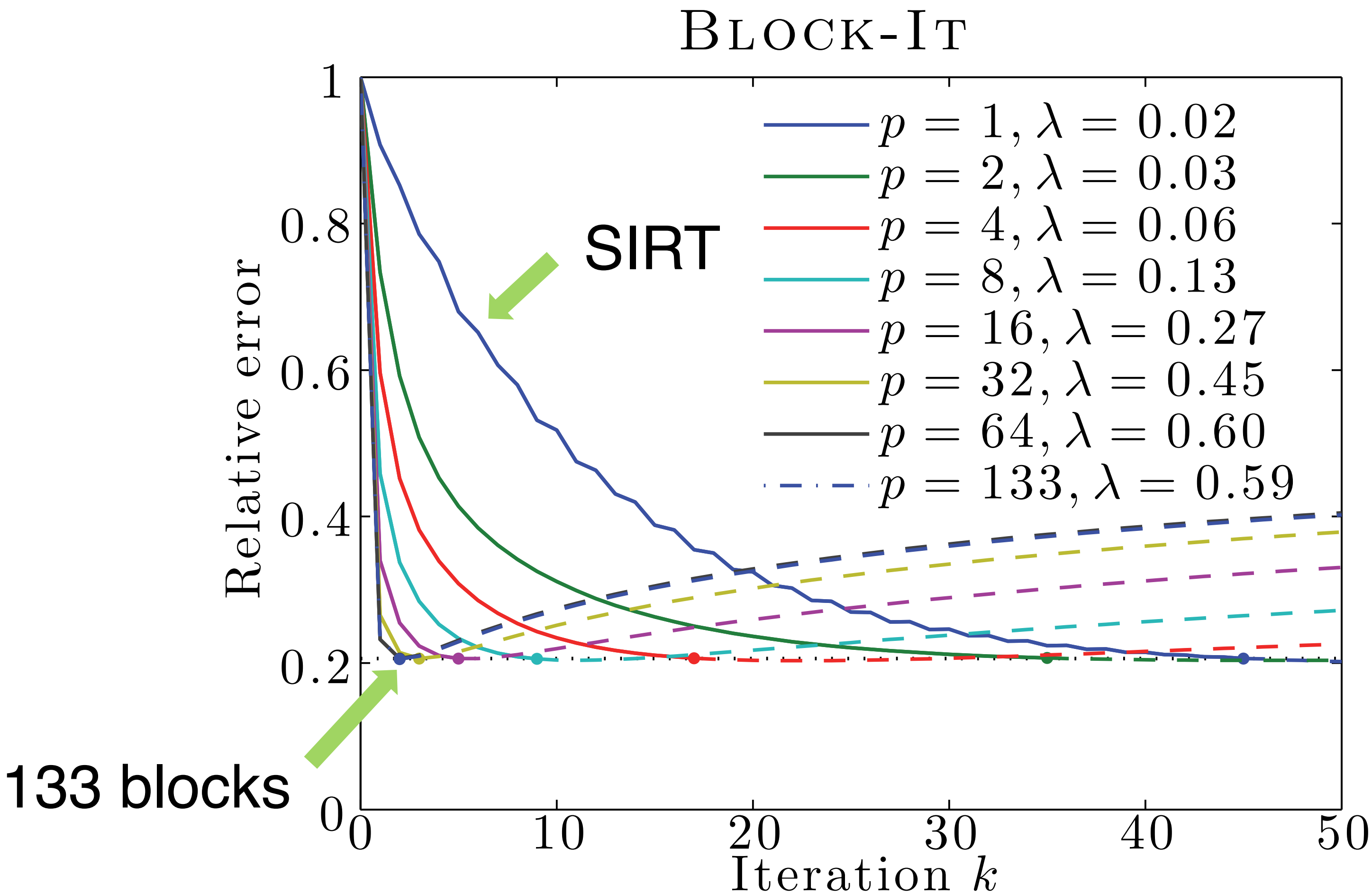
Block method reconstruction



Block method reconstruction



Block methods converge faster



Fully optimized block code

1 iter | 2 iters | 3 iters

400 blks

GPU	GTX 680	K20	K40	K80(x1)	TITAN X
Runtime (s)	47.0	25.1	20.7	22.8	10.5

- <1 min is ok! - but what about larger problems?

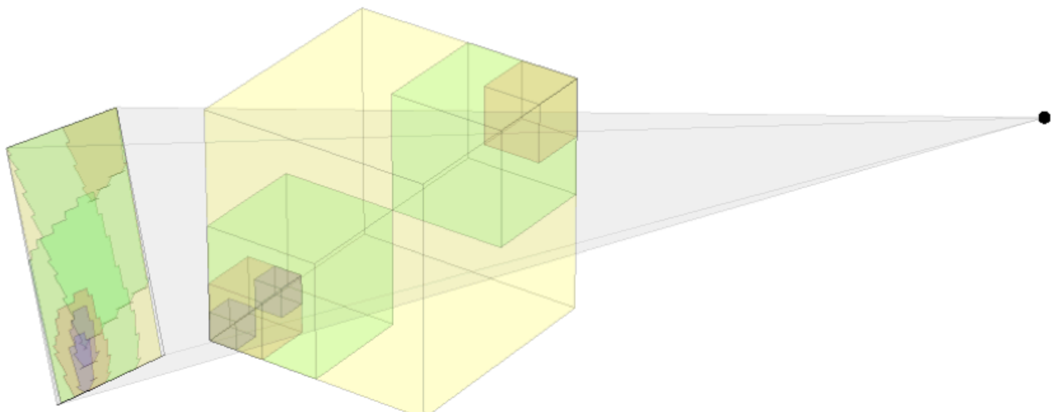
Distributed / Multi-GPU computing

Domain decomposition

```
Domains:                                             Detector:       Angles
                           512
            +-------------------------------+                    +       +-----
           /                               /|                   /|        \ 400
          /                               / |                  / |         \
         /                               /  |                 /  |          \
    512 /               3               /   +            512 /   |
       /                               /   /|               /    |
      /                               / 3 / |              /     |
     /                               /   /  |             /      |
    +-------------------------------+   /   + 512        +       | 512
    |                               |  /   /|            |       |
128 |               3               | / 2 / |            |       |
    |                               |/   /  |            |       |
    +-------------------------------+   /   +            |       |
    |                               |  /   /|            |       |
128 |               2               | / 1 / |            |       |
    |                               |/   /  |            |       |
    +-------------------------------+   /   +        512 |       +
    |                               |  /   /             |      /
128 |               1               | / 0 /              |     /
    |                               |/   /               |    /
    +-------------------------------+   / 512            |   / 512
    |                               |  /                 |  /
128 |               0               | /                  | /
    |                               |/                   |/
    +-------------------------------+                    +
                   512
```

Communication

- Setting up for distributed/multi-GPU computing
  - Top level domain decomposition of the solution x
  - One domain for each MPI thread / GPU



- We communicate the FP once per block
  - Note that only part of the FP is required from others

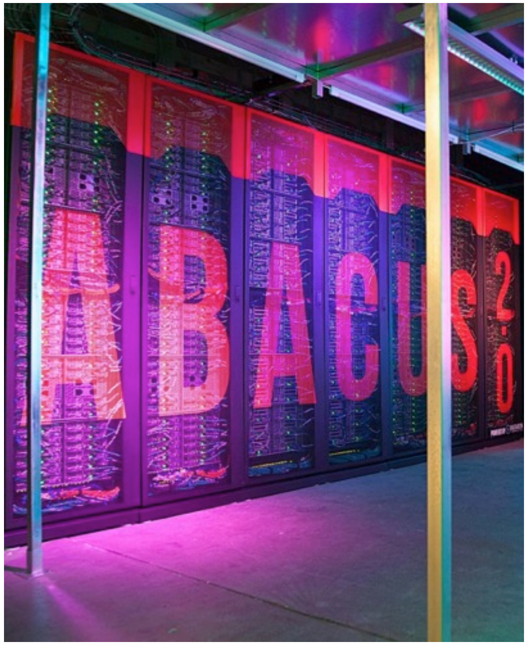
Multi-GPU results (K80)

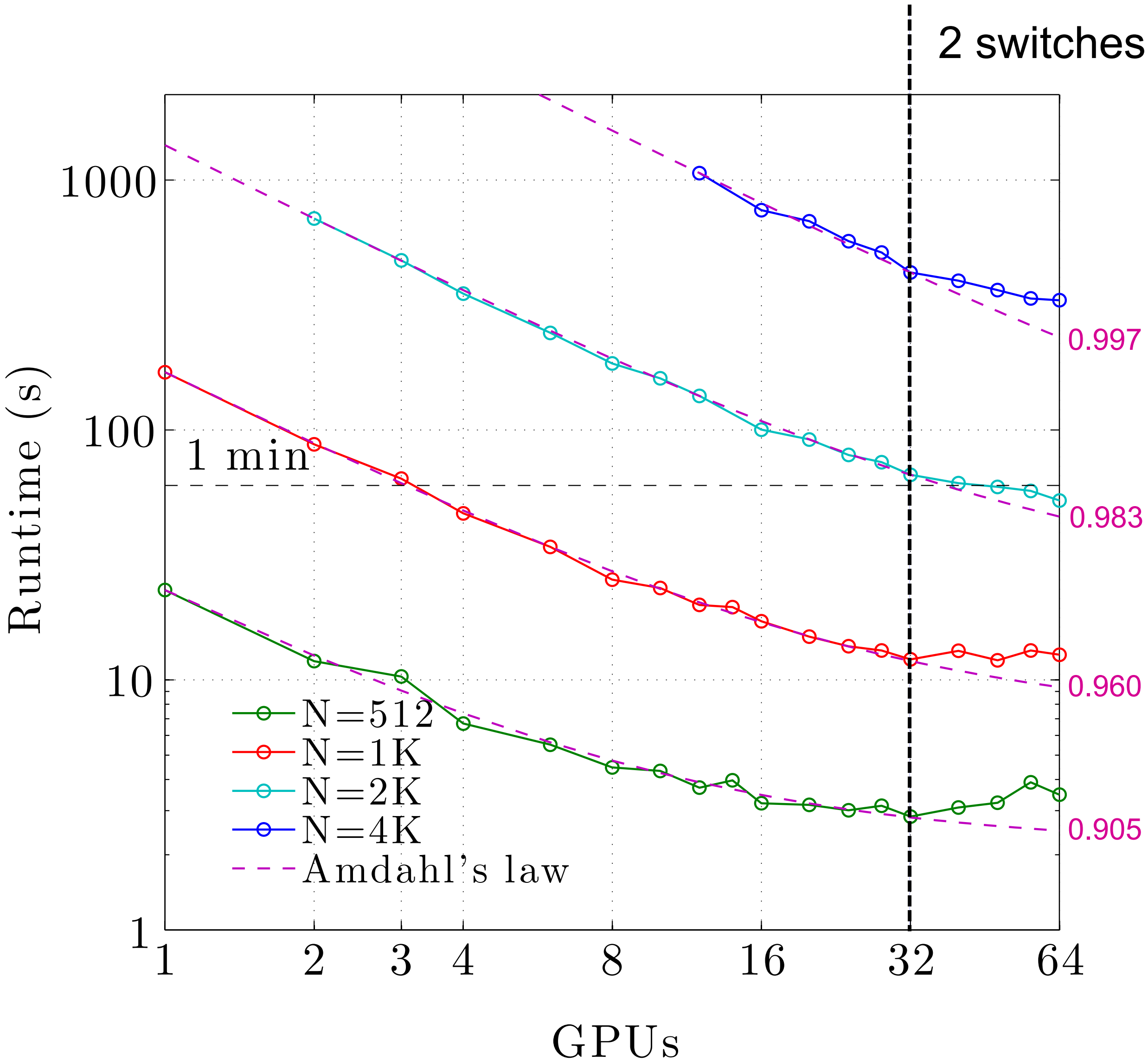
N	256	512	1024	2048
1 GPU	3.33	22.8	181	-
2 GPU	2.31	12.6	95.6	798
4 GPU	2.08	7.68	51.1	398
8 GPU	2.00	4.65	26.4	211

Walnut case - full reconstruction time (2 iters) without disk I/O in seconds.

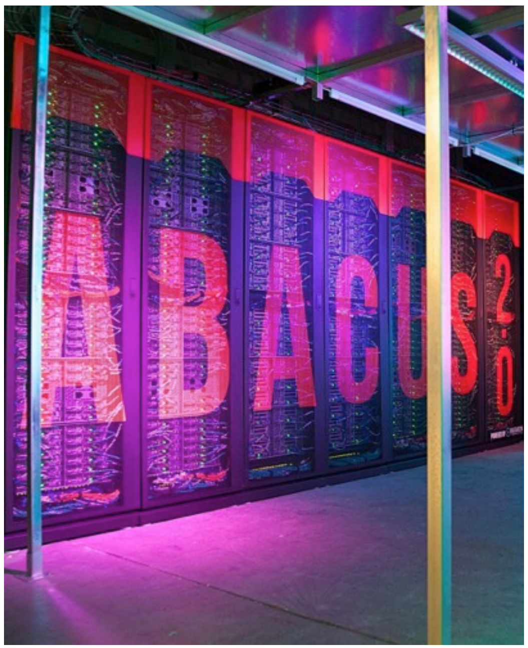
Solution size: N^3 / Projections: $400 \times N^2$

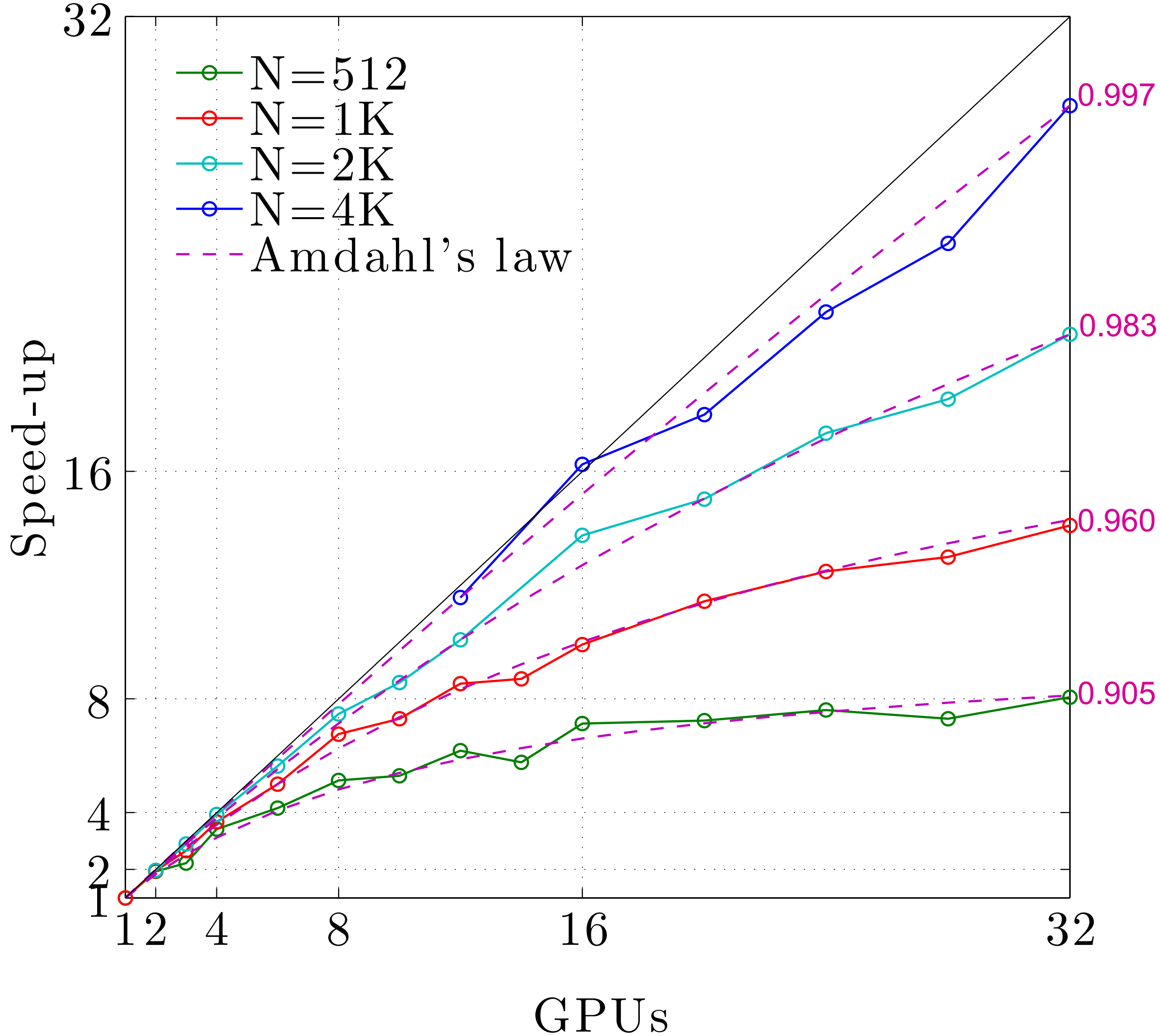
GPU cluster results





GPU cluster results





CPU cluster results (Xeon 2680)

N	256	512	1024	2048
20 cores	5.54	38.7	316	2380
40 cores	3.39	21.8	171	1260
80 cores	1.94	13.3	97,3	700
160 cores	1.36	9.55	60.1	413
320 cores	-	8.01	46.5	314

Walnut case - full reconstruction time (2 iters) without disk I/O in seconds.

Solution size: N^3 / Projections: $400xN^2$

Summary

- Block methods are convenient for large-scale reconstructions where runtimes are an issue
- ASTRA allows you to do block methods with little effort and good performance for limited blocks
- Block methods are well suited for GPUs & CPUs
  - We have an implementation with one projection per block that performs close to SIRT runtimes
- Multi-GPU approach is also efficient by domain decomposition and communication of the FP
- GPU-cluster performance scales but operates at the limit of current communication bandwidths

Summary

Thank you for your attention!

Block method

- Formulated in terms of matrices (as yesterday)

Basic Block-Iteration algorithm

$x^{(0)}$ = initial vector
for $k = 0, 1, 2, \ldots$
$\quad \boldsymbol{x}^{(k,0)} = \boldsymbol{x}^{(k)}$
$\quad$ for $l = 0, 1, 2, \ldots, \mathrm{blks}-1$
$$\boldsymbol{x}^{(k,l+1)} = P_{\mathcal{C}}\left(\boldsymbol{x}^{(k,l)} + \lambda_k \boldsymbol{D}^{-1} \boldsymbol{A}_l^T \boldsymbol{M}_l^{-1} (\boldsymbol{b}_l - \boldsymbol{A}_l \boldsymbol{x}^{(k,l)})\right)$$
$\quad$ end
$\quad \boldsymbol{x}^{(k+1)} = \boldsymbol{x}^{(k,\mathrm{blks})}$
end