NoMath: Nominalistic Logic for Computer Mathematics

NoMath: Nominalistic Logic for Computer Mathematics

Type: $\tau \ ::= \ \sigma \mid \tau \mapsto \tau \mid \iota$

NoMath: Nominalistic Logic for Computer Mathematics

Type: $\tau ::= \sigma \mid \tau \mapsto \tau \mid \iota \qquad \mathsf{v} ::= \tau \mid \ell \mathsf{v}$

NoMath: Nominalistic Logic for Computer Mathematics

Type: $\tau ::= \sigma \mid \tau \mapsto \tau \mid \iota \qquad v ::= \tau \mid \ell v \qquad t ::= tt \mid \lambda v.t \mid v \mid \circ \mid \vartriangle\tau \mid \lessgtr\tau$

NoMath: Nominalistic Logic for Computer Mathematics

Type: $\tau ::= \sigma \mid \tau \mapsto \tau \mid \iota$ $\quad v ::= \tau \mid \ell v$ $\quad t ::= tt \mid \lambda v.t \mid v \mid \circ \mid \vartriangle\tau \mid \lessgtr\tau$

$ft : \tau$ if $f : \tau_0\tau$ and $t : \tau_1$ where τ_0 is τ_1

NoMath: Nominalistic Logic for Computer Mathematics

Type: $\tau ::= \sigma \mid \tau \mapsto \tau \mid \iota \qquad v ::= \tau \mid \ell v \qquad t ::= tt \mid \lambda v.t \mid v \mid \circ \mid \vartriangle\tau \mid \lesssim\tau$

$ft : \tau$ if $f : \tau_0\tau$ and $t : \tau_1$ where τ_0 is τ_1 or else τ_0 is ι and free $i : \iota$ in t

NoMath: Nominalistic Logic for Computer Mathematics

Type: $\tau ::= \sigma \mid \tau \mapsto \tau \mid \iota \qquad v ::= \tau \mid \ell v \qquad t ::= tt \mid \lambda v.t \mid v \mid \circ \mid \vartriangle\tau \mid \lessgtr\tau$

$ft : \tau$ if $f : \tau_0\tau$ and $t : \tau_1$ where τ_0 is τ_1 or else τ_0 is ι and free $i : \iota$ in t

$\lambda v.t : \tau_0\tau$ if $v : \tau_0$ and $t : \tau$

NoMath: Nominalistic Logic for Computer Mathematics

Type: $\tau ::= \sigma \mid \tau \mapsto \tau \mid \iota \qquad v ::= \tau \mid \ell v \qquad t ::= tt \mid \lambda v.t \mid v \mid \circ \mid \vartriangle\tau \mid \lessgtr\tau$

$ft : \tau$ if $f : \tau_0\tau$ and $t : \tau_1$ where τ_0 is τ_1 or else τ_0 is ι and free $i : \iota$ in t

$\lambda v.t : \tau_0\tau$ if $v : \tau_0$ and $t : \tau \qquad v : \tau$

NoMath: Nominalistic Logic for Computer Mathematics

Type: $\tau ::= \sigma \mid \tau \mapsto \tau \mid \iota \qquad v ::= \tau \mid \ell v \qquad t ::= tt \mid \lambda v.t \mid v \mid \circ \mid \vartriangle\tau \mid \lesssim\tau$

$ft : \tau$ if $f : \tau_0\tau$ and $t : \tau_1$ where τ_0 is τ_1 or else τ_0 is ι and free $i : \iota$ in t

$\lambda v.t : \tau_0\tau$ if $v : \tau_0$ and $t : \tau \qquad v : \tau \qquad \circ : \sigma\sigma\sigma$

NoMath: Nominalistic Logic for Computer Mathematics

Type: $\tau ::= \sigma \mid \tau \mapsto \tau \mid \iota \qquad v ::= \tau \mid \ell v \qquad t ::= tt \mid \lambda v.t \mid v \mid \circ \mid \vartriangle\tau \mid \lesssim\tau$

$ft : \tau$ if $f : \tau_0\tau$ and $t : \tau_1$ where τ_0 is τ_1 or else τ_0 is ι and free $i : \iota$ in t

$\lambda v.t : \tau_0\tau$ if $v : \tau_0$ and $t : \tau \qquad v : \tau \qquad \circ : \sigma\sigma\sigma \qquad \vartriangle : (\tau\sigma)\sigma$

NoMath: Nominalistic Logic for Computer Mathematics

Type: $\tau \ ::= \ \sigma \mid \tau \mapsto \tau \mid \iota \qquad v \ ::= \ \tau \mid \ell v \qquad t \ ::= \ tt \mid \lambda v.t \mid v \mid \circ \mid \vartriangle\tau \mid \lessgtr\tau$

$ft : \tau$ if $f : \tau_0\tau$ and $t : \tau_1$ where τ_0 is τ_1 or else τ_0 is ι and free $i : \iota$ in t

$\lambda v.t : \tau_0\tau$ if $v : \tau_0$ and $t : \tau \qquad v : \tau \qquad \circ : \sigma\sigma\sigma \qquad \vartriangle : (\tau\sigma)\sigma \qquad \lessgtr : (\tau\sigma)\tau$

NoMath: Nominalistic Logic for Computer Mathematics

Type: $\tau \ ::= \ \sigma \mid \tau \mapsto \tau \mid \iota \qquad v \ ::= \ \tau \mid \ell v \qquad t \ ::= \ tt \mid \lambda v.t \mid v \mid \circ \mid \vartriangle\tau \mid \lessgtr\tau$

$ft : \tau$ if $f : \tau_0\tau$ and $t : \tau_1$ where τ_0 is τ_1 or else τ_0 is ι and free $i : \iota$ in t

$\lambda v.t : \tau_0\tau$ if $v : \tau_0$ and $t : \tau \qquad v : \tau \qquad \circ : \sigma\sigma\sigma \qquad \vartriangle : (\tau\sigma)\sigma \qquad \lessgtr : (\tau\sigma)\tau$

$\neg p \ := \ \circ pp$

NoMath: Nominalistic Logic for Computer Mathematics

Type: $\tau ::= \sigma \mid \tau \mapsto \tau \mid \iota \qquad v ::= \tau \mid \ell v \qquad t ::= tt \mid \lambda v.t \mid v \mid \circ \mid \vartriangle\tau \mid \lessgtr\tau$

$ft : \tau$ if $f : \tau_0\tau$ and $t : \tau_1$ where τ_0 is τ_1 or else τ_0 is ι and free $i : \iota$ in t

$\lambda v.t : \tau_0\tau$ if $v : \tau_0$ and $t : \tau \qquad v : \tau \qquad \circ : \sigma\sigma\sigma \qquad \vartriangle : (\tau\sigma)\sigma \qquad \lessgtr : (\tau\sigma)\tau$

$\neg p \coloneqq \circ pp \qquad p \vee q \coloneqq \neg\circ pq$

NoMath: Nominalistic Logic for Computer Mathematics

Type: $\tau ::= \sigma \mid \tau \mapsto \tau \mid \iota \qquad v ::= \tau \mid \ell v \qquad t ::= tt \mid \lambda v.t \mid v \mid \circ \mid \vartriangle\tau \mid \lessgtr\tau$

$ft : \tau$ if $f : \tau_0\tau$ and $t : \tau_1$ where τ_0 is τ_1 or else τ_0 is ι and free $i : \iota$ in t

$\lambda v.t : \tau_0\tau$ if $v : \tau_0$ and $t : \tau \qquad v : \tau \qquad \circ : \sigma\sigma\sigma \qquad \vartriangle : (\tau\sigma)\sigma \qquad \lessgtr : (\tau\sigma)\tau$

$\neg p \coloneqq \circ pp \qquad p \vee q \coloneqq \neg\circ pq \qquad p \leftarrow q \coloneqq p \vee \neg q$

NoMath: Nominalistic Logic for Computer Mathematics

Type: $\tau ::= \sigma \mid \tau \mapsto \tau \mid \iota \qquad v ::= \tau \mid \ell v \qquad t ::= tt \mid \lambda v.t \mid v \mid \circ \mid \vartriangle\tau \mid \lessgtr\tau$

$ft : \tau$ if $f : \tau_0\tau$ and $t : \tau_1$ where τ_0 is τ_1 or else τ_0 is ι and free $i : \iota$ in t

$\lambda v.t : \tau_0\tau$ if $v : \tau_0$ and $t : \tau \qquad v : \tau \qquad \circ : \sigma\sigma\sigma \qquad \vartriangle : (\tau\sigma)\sigma \qquad \lessgtr : (\tau\sigma)\tau$

$$\neg p := \circ pp \qquad p \vee q := \neg\circ pq \qquad p \leftarrow q := p \vee \neg q \qquad p \rightarrow q := \neg p \vee q$$

NoMath: Nominalistic Logic for Computer Mathematics

Type: $\tau ::= \sigma \mid \tau \mapsto \tau \mid \iota \qquad v ::= \tau \mid \ell v \qquad t ::= tt \mid \lambda v.t \mid v \mid \circ \mid \vartriangle\tau \mid \lessgtr\tau$

$ft : \tau$ if $f : \tau_0\tau$ and $t : \tau_1$ where τ_0 is τ_1 or else τ_0 is ι and free $i : \iota$ in t

$\lambda v.t : \tau_0\tau$ if $v : \tau_0$ and $t : \tau \qquad v : \tau \qquad \circ : \sigma\sigma\sigma \qquad \vartriangle : (\tau\sigma)\sigma \qquad \lessgtr : (\tau\sigma)\tau$

$$\neg p \coloneqq \circ pp \qquad p \vee q \coloneqq \neg\circ pq \qquad p \leftarrow q \coloneqq p \vee \neg q \qquad p \rightarrow q \coloneqq \neg p \vee q$$

$$\forall v.p \coloneqq \vartriangle\lambda v.p$$

NoMath: Nominalistic Logic for Computer Mathematics

Type: $\tau ::= \sigma \mid \tau \mapsto \tau \mid \iota \qquad v ::= \tau \mid \ell v \qquad t ::= tt \mid \lambda v.t \mid v \mid \circ \mid \vartriangle\tau \mid \lesssim\tau$

$ft : \tau$ if $f : \tau_0\tau$ and $t : \tau_1$ where τ_0 is τ_1 or else τ_0 is ι and free $i : \iota$ in t

$\lambda v.t : \tau_0\tau$ if $v : \tau_0$ and $t : \tau \qquad v : \tau \qquad \circ : \sigma\sigma\sigma \qquad \vartriangle : (\tau\sigma)\sigma \qquad \lesssim : (\tau\sigma)\tau$

$$\neg p \coloneqq \circ pp \qquad p \vee q \coloneqq \neg\circ pq \qquad p \leftarrow q \coloneqq p \vee \neg q \qquad p \rightarrow q \coloneqq \neg p \vee q$$

$$\forall v.p \coloneqq \vartriangle\lambda v.p \qquad \bot \coloneqq \forall v.v$$

NoMath: Nominalistic Logic for Computer Mathematics

Type: $\tau ::= \sigma \mid \tau \mapsto \tau \mid \iota \qquad v ::= \tau \mid \ell v \qquad t ::= tt \mid \lambda v.t \mid v \mid \circ \mid \vartriangle\tau \mid \lesssim\tau$

$ft : \tau$ if $f : \tau_0\tau$ and $t : \tau_1$ where τ_0 is τ_1 or else τ_0 is ι and free $i : \iota$ in t

$\lambda v.t : \tau_0\tau$ if $v : \tau_0$ and $t : \tau \qquad v : \tau \qquad \circ : \sigma\sigma\sigma \qquad \vartriangle : (\tau\sigma)\sigma \qquad \lesssim : (\tau\sigma)\tau$

$$\neg p \coloneqq \circ pp \quad p \vee q \coloneqq \neg\circ pq \quad p \leftarrow q \coloneqq p \vee \neg q \quad p \rightarrow q \coloneqq \neg p \vee q$$

$$\forall v.p \coloneqq \vartriangle\lambda v.p \quad \bot \coloneqq \forall v.v \quad \top \coloneqq \neg\bot$$

NoMath: Nominalistic Logic for Computer Mathematics

Type: $\tau ::= \sigma \mid \tau \mapsto \tau \mid \iota$ $\quad v ::= \tau \mid \ell v$ $\quad t ::= tt \mid \lambda v.t \mid v \mid \circ \mid \vartriangle\tau \mid \lesssim\tau$

$ft : \tau$ if $f : \tau_0\tau$ and $t : \tau_1$ where τ_0 is τ_1 or else τ_0 is ι and free $i : \iota$ in t

$\lambda v.t : \tau_0\tau$ if $v : \tau_0$ and $t : \tau$ $\quad v : \tau$ $\quad \circ : \sigma\sigma\sigma$ $\quad \vartriangle : (\tau\sigma)\sigma$ $\quad \lesssim : (\tau\sigma)\tau$

$$\neg p := \circ pp \quad p \vee q := \neg\circ pq \quad p \leftarrow q := p \vee \neg q \quad p \rightarrow q := \neg p \vee q$$

$$\forall v.p := \vartriangle\lambda v.p \quad \bot := \forall v.v \quad \top := \neg\bot \quad \exists v.p := \neg\forall v.\neg p$$

NoMath: Nominalistic Logic for Computer Mathematics

Type: $\tau ::= \sigma \mid \tau \mapsto \tau \mid \iota \qquad v ::= \tau \mid \ell v \qquad t ::= tt \mid \lambda v.t \mid v \mid \circ \mid \vartriangle\tau \mid \lessgtr\tau$

$ft : \tau$ if $f : \tau_0\tau$ and $t : \tau_1$ where τ_0 is τ_1 or else τ_0 is ι and free $i : \iota$ in t

$\lambda v.t : \tau_0\tau$ if $v : \tau_0$ and $t : \tau \qquad v : \tau \qquad \circ : \sigma\sigma\sigma \qquad \vartriangle : (\tau\sigma)\sigma \qquad \lessgtr : (\tau\sigma)\tau$

$$\neg p \;\coloneqq\; \circ pp \qquad p \vee q \;\coloneqq\; \neg\circ pq \qquad p \leftarrow q \;\coloneqq\; p \vee \neg q \qquad p \rightarrow q \;\coloneqq\; \neg p \vee q$$

$$\forall v.p \;\coloneqq\; \vartriangle\lambda v.p \qquad \bot \;\coloneqq\; \forall v.v \qquad \top \;\coloneqq\; \neg\bot \qquad \exists v.p \;\coloneqq\; \neg\forall v.\neg p$$

$$p \wedge q \;\coloneqq\; \neg(\neg p \vee \neg q)$$

NoMath: Nominalistic Logic for Computer Mathematics

Type: $\tau ::= \sigma \mid \tau \mapsto \tau \mid \iota \qquad v ::= \tau \mid \ell v \qquad t ::= tt \mid \lambda v.t \mid v \mid \circ \mid \vartriangle\tau \mid \lessgtr\tau$

$ft : \tau$ if $f : \tau_0\tau$ and $t : \tau_1$ where τ_0 is τ_1 or else τ_0 is ι and free $i : \iota$ in t

$\lambda v.t : \tau_0\tau$ if $v : \tau_0$ and $t : \tau \qquad v : \tau \qquad \circ : \sigma\sigma\sigma \qquad \vartriangle : (\tau\sigma)\sigma \qquad \lessgtr : (\tau\sigma)\tau$

$$\neg p \coloneqq \circ pp \qquad p \vee q \coloneqq \neg\circ pq \qquad p \leftarrow q \coloneqq p \vee \neg q \qquad p \rightarrow q \coloneqq \neg p \vee q$$

$$\forall v.p \coloneqq \vartriangle\lambda v.p \qquad \bot \coloneqq \forall v.v \qquad \top \coloneqq \neg\bot \qquad \exists v.p \coloneqq \neg\forall v.\neg p$$

$$p \wedge q \coloneqq \neg(\neg p \vee \neg q) \qquad p \leftrightarrow q \coloneqq (p \leftarrow q) \wedge (p \rightarrow q)$$

NoMath: Nominalistic Logic for Computer Mathematics

Type: $\tau ::= \sigma \mid \tau \mapsto \tau \mid \iota \qquad v ::= \tau \mid \ell v \qquad t ::= tt \mid \lambda v.t \mid v \mid \circ \mid \vartriangle\tau \mid \lessapprox\tau$

$ft : \tau$ if $f : \tau_0\tau$ and $t : \tau_1$ where τ_0 is τ_1 or else τ_0 is ι and free $i : \iota$ in t

$\lambda v.t : \tau_0\tau$ if $v : \tau_0$ and $t : \tau \qquad v : \tau \qquad \circ : \sigma\sigma\sigma \qquad \vartriangle : (\tau\sigma)\sigma \qquad \lessapprox : (\tau\sigma)\tau$

$$\neg p \coloneqq \circ pp \quad p \vee q \coloneqq \neg\circ pq \quad p \leftarrow q \coloneqq p \vee \neg q \quad p \rightarrow q \coloneqq \neg p \vee q$$

$$\forall v.p \coloneqq \vartriangle\lambda v.p \quad \bot \coloneqq \forall v.v \quad \top \coloneqq \neg\bot \quad \exists v.p \coloneqq \neg\forall v.\neg p$$

$$p \wedge q \coloneqq \neg(\neg p \vee \neg q) \quad p \leftrightarrow q \coloneqq (p \leftarrow q) \wedge (p \rightarrow q) \quad \varnothing \coloneqq \lambda i.\bot$$

NoMath: Nominalistic Logic for Computer Mathematics

Type: $\tau \ ::= \ \sigma \mid \tau \mapsto \tau \mid \iota \qquad v \ ::= \ \tau \mid \ell v \qquad t \ ::= \ tt \mid \lambda v.t \mid v \mid \circ \mid \vartriangle\tau \mid \lessgtr\tau$

$ft : \tau$ if $f : \tau_0\tau$ and $t : \tau_1$ where τ_0 is τ_1 or else τ_0 is ι and free $i : \iota$ in t

$\lambda v.t : \tau_0\tau$ if $v : \tau_0$ and $t : \tau \qquad v : \tau \qquad \circ : \sigma\sigma\sigma \qquad \vartriangle : (\tau\sigma)\sigma \qquad \lessgtr : (\tau\sigma)\tau$

$$\neg p \ := \ \circ pp \qquad p \vee q \ := \ \neg\circ pq \qquad p \leftarrow q \ := \ p \vee \neg q \qquad p \rightarrow q \ := \ \neg p \vee q$$

$$\forall v.p \ := \ \vartriangle\lambda v.p \qquad \bot \ := \ \forall v.v \qquad \top \ := \ \neg\bot \qquad \exists v.p \ := \ \neg\forall v.\neg p$$

$$p \wedge q \ := \ \neg(\neg p \vee \neg q) \qquad p \leftrightarrow q \ := \ (p \leftarrow q) \wedge (p \rightarrow q) \qquad \varnothing \ := \ \lambda i.\bot$$

$$t = u \ := \ \forall v.vt \leftrightarrow vu$$

NoMath: Nominalistic Logic for Computer Mathematics

Type: $\tau ::= \sigma \mid \tau \mapsto \tau \mid \iota$ $\quad v ::= \tau \mid \ell v$ $\quad t ::= tt \mid \lambda v.t \mid v \mid \circ \mid \vartriangle\tau \mid \lessgtr\tau$

$ft : \tau$ if $f : \tau_0\tau$ and $t : \tau_1$ where τ_0 is τ_1 or else τ_0 is ι and free $i : \iota$ in t

$\lambda v.t : \tau_0\tau$ if $v : \tau_0$ and $t : \tau$ $\quad v : \tau$ $\quad \circ : \sigma\sigma\sigma$ $\quad \vartriangle : (\tau\sigma)\sigma$ $\quad \lessgtr : (\tau\sigma)\tau$

$$\neg p := \circ pp \quad p \vee q := \neg\circ pq \quad p \leftarrow q := p \vee \neg q \quad p \rightarrow q := \neg p \vee q$$

$$\forall v.p := \vartriangle\lambda v.p \quad \bot := \forall v.v \quad \top := \neg\bot \quad \exists v.p := \neg\forall v.\neg p$$

$$p \wedge q := \neg(\neg p \vee \neg q) \quad p \leftrightarrow q := (p \leftarrow q) \wedge (p \rightarrow q) \quad \varnothing := \lambda i.\bot$$

$$t = u := \forall v.vt \leftrightarrow vu \quad t \doteq u := (\lambda ij.i = j)tu$$

NoMath: Nominalistic Logic for Computer Mathematics

Type: $\tau ::= \sigma \mid \tau \mapsto \tau \mid \iota \qquad v ::= \tau \mid \ell v \qquad t ::= tt \mid \lambda v.t \mid v \mid \circ \mid \vartriangle\tau \mid \lessgtr\tau$

$ft : \tau$ if $f : \tau_0\tau$ and $t : \tau_1$ where τ_0 is τ_1 or else τ_0 is ι and free $i : \iota$ in t

$\lambda v.t : \tau_0\tau$ if $v : \tau_0$ and $t : \tau \qquad v : \tau \qquad \circ : \sigma\sigma\sigma \qquad \vartriangle : (\tau\sigma)\sigma \qquad \lessgtr : (\tau\sigma)\tau$

$$\neg p \coloneqq \circ pp \qquad p \vee q \coloneqq \neg\circ pq \qquad p \leftarrow q \coloneqq p \vee \neg q \qquad p \rightarrow q \coloneqq \neg p \vee q$$

$$\forall v.p \coloneqq \vartriangle\lambda v.p \qquad \bot \coloneqq \forall v.v \qquad \top \coloneqq \neg\bot \qquad \exists v.p \coloneqq \neg\forall v.\neg p$$

$$p \wedge q \coloneqq \neg(\neg p \vee \neg q) \qquad p \leftrightarrow q \coloneqq (p \leftarrow q) \wedge (p \rightarrow q) \qquad \varnothing \coloneqq \lambda i.\bot$$

$$t = u \coloneqq \forall v.vt \leftrightarrow vu \qquad t \doteq u \coloneqq (\lambda ij.i = j)tu \qquad t' \coloneqq (\lambda ij.i = j)t$$

NoMath: Nominalistic Logic for Computer Mathematics

Type: $\tau \ ::= \ \sigma \mid \tau \mapsto \tau \mid \iota \qquad v \ ::= \ \tau \mid \ell v \qquad t \ ::= \ tt \mid \lambda v.t \mid v \mid \circ \mid \vartriangle\!\tau \mid \lesssim\!\tau$

$ft : \tau$ if $f : \tau_0\tau$ and $t : \tau_1$ where τ_0 is τ_1 or else τ_0 is ι and free $i : \iota$ in t

$\lambda v.t : \tau_0\tau$ if $v : \tau_0$ and $t : \tau \qquad v : \tau \qquad \circ : \sigma\sigma\sigma \qquad \vartriangle : (\tau\sigma)\sigma \qquad \lesssim : (\tau\sigma)\tau$

$$\neg p \ := \ \circ pp \qquad p \vee q \ := \ \neg\circ pq \qquad p \leftarrow q \ := \ p \vee \neg q \qquad p \rightarrow q \ := \ \neg p \vee q$$

$$\forall v.p \ := \ \vartriangle\lambda v.p \qquad \bot \ := \ \forall v.v \qquad \top \ := \ \neg\bot \qquad \exists v.p \ := \ \neg\forall v.\neg p$$

$$p \wedge q \ := \ \neg(\neg p \vee \neg q) \qquad p \leftrightarrow q \ := \ (p \leftarrow q) \wedge (p \rightarrow q) \qquad \varnothing \ := \ \lambda i.\bot$$

$$t = u \ := \ \forall v.vt \leftrightarrow vu \qquad t \doteq u \ := \ (\lambda ij.i = j)tu \qquad t' \ := \ (\lambda ij.i = j)t$$

A sequent calculus: Theorem if p has a proof
consisting of steps justified by the rules

NoMath: Nominalistic Logic for Computer Mathematics

Type: $\tau ::= \sigma \mid \tau \mapsto \tau \mid \iota \qquad v ::= \tau \mid \ell v \qquad t ::= tt \mid \lambda v.t \mid v \mid \circ \mid \vartriangle\tau \mid \lesssim\tau$

$ft : \tau$ if $f : \tau_0\tau$ and $t : \tau_1$ where τ_0 is τ_1 or else τ_0 is ι and free $i : \iota$ in t

$\lambda v.t : \tau_0\tau$ if $v : \tau_0$ and $t : \tau \qquad v : \tau \qquad \circ : \sigma\sigma\sigma \qquad \vartriangle : (\tau\sigma)\sigma \qquad \lesssim : (\tau\sigma)\tau$

$$\neg p := \circ pp \quad p \vee q := \neg\circ pq \quad p \leftarrow q := p \vee \neg q \quad p \rightarrow q := \neg p \vee q$$

$$\forall v.p := \vartriangle\lambda v.p \quad \bot := \forall v.v \quad \top := \neg\bot \quad \exists v.p := \neg\forall v.\neg p$$

$$p \wedge q := \neg(\neg p \vee \neg q) \quad p \leftrightarrow q := (p \leftarrow q) \wedge (p \rightarrow q) \quad \varnothing := \lambda i.\bot$$

$$t = u := \forall v.vt \leftrightarrow vu \quad t \doteq u := (\lambda ij.i = j)tu \quad t' := (\lambda ij.i = j)t$$

A sequent calculus: Theorem if p has a proof consisting of steps justified by the rules

Each step is a sequence of alternatives:

$$\varphi_1, \ldots, \varphi_n$$

NoMath: Nominalistic Logic for Computer Mathematics

Type: $\tau ::= \sigma \mid \tau \mapsto \tau \mid \iota \qquad v ::= \tau \mid \ell v \qquad t ::= tt \mid \lambda v.t \mid v \mid \circ \mid \vartriangle\tau \mid \lessgtr\tau$

$ft : \tau$ if $f : \tau_0\tau$ and $t : \tau_1$ where τ_0 is τ_1 or else τ_0 is ι and free $i : \iota$ in t

$\lambda v.t : \tau_0\tau$ if $v : \tau_0$ and $t : \tau \qquad v : \tau \qquad \circ : \sigma\sigma\sigma \qquad \vartriangle : (\tau\sigma)\sigma \qquad \lessgtr : (\tau\sigma)\tau$

$$\neg p \;\coloneqq\; \circ pp \qquad p \vee q \;\coloneqq\; \neg\circ pq \qquad p \leftarrow q \;\coloneqq\; p \vee \neg q \qquad p \rightarrow q \;\coloneqq\; \neg p \vee q$$

$$\forall v.p \;\coloneqq\; \vartriangle\lambda v.p \qquad \bot \;\coloneqq\; \forall v.v \qquad \top \;\coloneqq\; \neg\bot \qquad \exists v.p \;\coloneqq\; \neg\forall v.\neg p$$

$$p \wedge q \;\coloneqq\; \neg(\neg p \vee \neg q) \qquad p \leftrightarrow q \;\coloneqq\; (p \leftarrow q) \wedge (p \rightarrow q) \qquad \varnothing \;\coloneqq\; \lambda i.\bot$$

$$t = u \;\coloneqq\; \forall v.vt \leftrightarrow vu \qquad t \doteq u \;\coloneqq\; (\lambda ij.i = j)tu \qquad t' \;\coloneqq\; (\lambda ij.i = j)t$$

A sequent calculus: Theorem if p has a proof consisting of steps justified by the rules

Each step is a sequence of alternatives:

$$\varphi_1, \ldots, \varphi_n$$

Each alternative φ is either p or denial $\overline{p}$

NoMath: Nominalistic Logic for Computer Mathematics

Type: $\tau \ ::= \ \sigma \mid \tau \mapsto \tau \mid \iota \qquad v \ ::= \ \tau \mid \ell v \qquad t \ ::= \ tt \mid \lambda v.t \mid v \mid \circ \mid \vartriangle\tau \mid \lesssim\tau$

$ft : \tau$ if $f : \tau_0\tau$ and $t : \tau_1$ where τ_0 is τ_1 or else τ_0 is ι and free $i : \iota$ in t

$\lambda v.t : \tau_0\tau$ if $v : \tau_0$ and $t : \tau \qquad v : \tau \qquad \circ : \sigma\sigma\sigma \qquad \vartriangle : (\tau\sigma)\sigma \qquad \lesssim : (\tau\sigma)\tau$

$$\neg p \ := \ \circ pp \qquad p \vee q \ := \ \neg\circ pq \qquad p \leftarrow q \ := \ p \vee \neg q \qquad p \rightarrow q \ := \ \neg p \vee q$$

$$\forall v.p \ := \ \vartriangle\lambda v.p \qquad \bot \ := \ \forall v.v \qquad \top \ := \ \neg\bot \qquad \exists v.p \ := \ \neg\forall v.\neg p$$

$$p \wedge q \ := \ \neg(\neg p \vee \neg q) \qquad p \leftrightarrow q \ := \ (p \leftarrow q) \wedge (p \rightarrow q) \qquad \varnothing \ := \ \lambda i.\bot$$

$$t = u \ := \ \forall v.vt \leftrightarrow vu \qquad t \doteq u \ := \ (\lambda ij.i = j)tu \qquad t' \ := \ (\lambda ij.i = j)t$$

A sequent calculus: Theorem if p has a proof consisting of steps justified by the rules

Each step is a sequence of alternatives:

$$\varphi_1, \ldots, \varphi_n$$

Each alternative φ is either p or denial $\overline{p}$

NoMath: Nominalistic Logic for Computer Mathematics

Type: $\tau ::= \sigma \mid \tau \mapsto \tau \mid \iota \qquad v ::= \tau \mid \ell v \qquad t ::= tt \mid \lambda v.t \mid v \mid \circ \mid \vartriangle\tau \mid \lessapprox\tau$

$ft : \tau$ if $f : \tau_0\tau$ and $t : \tau_1$ where τ_0 is τ_1 or else τ_0 is ι and free $i : \iota$ in t

$\lambda v.t : \tau_0\tau$ if $v : \tau_0$ and $t : \tau \qquad v : \tau \qquad \circ : \sigma\sigma\sigma \qquad \vartriangle : (\tau\sigma)\sigma \qquad \lessapprox : (\tau\sigma)\tau$

$$\neg p \coloneqq \circ pp \quad p \vee q \coloneqq \neg\circ pq \quad p \leftarrow q \coloneqq p \vee \neg q \quad p \rightarrow q \coloneqq \neg p \vee q$$

$$\forall v.p \coloneqq \vartriangle\lambda v.p \quad \bot \coloneqq \forall v.v \quad \top \coloneqq \neg\bot \quad \exists v.p \coloneqq \neg\forall v.\neg p$$

$$p \wedge q \coloneqq \neg(\neg p \vee \neg q) \quad p \leftrightarrow q \coloneqq (p \leftarrow q) \wedge (p \rightarrow q) \quad \varnothing \coloneqq \lambda i.\bot$$

$$t = u \coloneqq \forall v.vt \leftrightarrow vu \quad t \doteq u \coloneqq (\lambda ij.i=j)tu \quad t' \coloneqq (\lambda ij.i=j)t$$

A sequent calculus: Theorem if p has a proof consisting of steps justified by the rules

Each step is a sequence of alternatives:

$$\varphi_1, \ldots, \varphi_n$$

Each alternative φ is either p or denial $\overline{p}$

$\varphi, \psi \,/\, \psi, \varphi$

NoMath: Nominalistic Logic for Computer Mathematics

Type: $\tau ::= \sigma \mid \tau \mapsto \tau \mid \iota$ $\quad v ::= \tau \mid \ell v$ $\quad t ::= tt \mid \lambda v.t \mid v \mid \circ \mid \vartriangle\tau \mid \lessgtr\tau$

$ft : \tau$ if $f : \tau_0\tau$ and $t : \tau_1$ where τ_0 is τ_1 or else τ_0 is ι and free $i : \iota$ in t

$\lambda v.t : \tau_0\tau$ if $v : \tau_0$ and $t : \tau$ $\quad v : \tau$ $\quad \circ : \sigma\sigma\sigma$ $\quad \vartriangle : (\tau\sigma)\sigma$ $\quad \lessgtr : (\tau\sigma)\tau$

$$\neg p \coloneqq \circ pp \quad p \vee q \coloneqq \neg\circ pq \quad p \leftarrow q \coloneqq p \vee \neg q \quad p \rightarrow q \coloneqq \neg p \vee q$$

$$\forall v.p \coloneqq \vartriangle\lambda v.p \quad \bot \coloneqq \forall v.v \quad \top \coloneqq \neg\bot \quad \exists v.p \coloneqq \neg\forall v.\neg p$$

$$p \wedge q \coloneqq \neg(\neg p \vee \neg q) \quad p \leftrightarrow q \coloneqq (p \leftarrow q) \wedge (p \rightarrow q) \quad \varnothing \coloneqq \lambda i.\bot$$

$$t = u \coloneqq \forall v.vt \leftrightarrow vu \quad t \doteq u \coloneqq (\lambda ij.i = j)tu \quad t' \coloneqq (\lambda ij.i = j)t$$

A sequent calculus: Theorem if p has a proof consisting of steps justified by the rules

Each step is a sequence of alternatives:

$$\varphi_1, \ldots, \varphi_n$$

Each alternative φ is either p or denial $\overline{p}$

$\varphi, \psi \,/\, \psi, \varphi \quad \varphi, \varphi \,/\, \varphi$

NoMath: Nominalistic Logic for Computer Mathematics

Type: $\tau ::= \sigma \mid \tau \mapsto \tau \mid \iota \qquad v ::= \tau \mid \ell v \qquad t ::= tt \mid \lambda v.t \mid v \mid \circ \mid \vartriangle\tau \mid \lessgtr\tau$

$ft : \tau$ if $f : \tau_0\tau$ and $t : \tau_1$ where τ_0 is τ_1 or else τ_0 is ι and free $i : \iota$ in t

$\lambda v.t : \tau_0\tau$ if $v : \tau_0$ and $t : \tau \qquad v : \tau \qquad \circ : \sigma\sigma\sigma \qquad \vartriangle : (\tau\sigma)\sigma \qquad \lessgtr : (\tau\sigma)\tau$

$$\neg p := \circ pp \quad p \vee q := \neg\circ pq \quad p \leftarrow q := p \vee \neg q \quad p \rightarrow q := \neg p \vee q$$

$$\forall v.p := \vartriangle\lambda v.p \quad \bot := \forall v.v \quad \top := \neg\bot \quad \exists v.p := \neg\forall v.\neg p$$

$$p \wedge q := \neg(\neg p \vee \neg q) \quad p \leftrightarrow q := (p \leftarrow q) \wedge (p \rightarrow q) \quad \varnothing := \lambda i.\bot$$

$$t = u := \forall v.vt \leftrightarrow vu \quad t \doteq u := (\lambda ij.i = j)tu \quad t' := (\lambda ij.i = j)t$$

A sequent calculus: Theorem if p has a proof consisting of steps justified by the rules

Each step is a sequence of alternatives:

$$\varphi_1, \ldots, \varphi_n$$

Each alternative φ is either p or denial $\overline{p}$

$\varphi, \psi \,/\, \psi, \varphi \qquad \varphi, \varphi \,/\, \varphi$

$/\!/\, \varphi$

NoMath: Nominalistic Logic for Computer Mathematics

Type: $\tau ::= \sigma \mid \tau \mapsto \tau \mid \iota \qquad v ::= \tau \mid \ell v \qquad t ::= tt \mid \lambda v.t \mid v \mid \circ \mid \vartriangle\tau \mid \lessapprox\tau$

$ft : \tau$ if $f : \tau_0\tau$ and $t : \tau_1$ where τ_0 is τ_1 or else τ_0 is ι and free $i : \iota$ in t

$\lambda v.t : \tau_0\tau$ if $v : \tau_0$ and $t : \tau \qquad v : \tau \qquad \circ : \sigma\sigma\sigma \qquad \vartriangle : (\tau\sigma)\sigma \qquad \lessapprox : (\tau\sigma)\tau$

$$\neg p \;:=\; \circ pp \qquad p \vee q \;:=\; \neg\circ pq \qquad p \leftarrow q \;:=\; p \vee \neg q \qquad p \rightarrow q \;:=\; \neg p \vee q$$

$$\forall v.p \;:=\; \vartriangle\lambda v.p \qquad \bot \;:=\; \forall v.v \qquad \top \;:=\; \neg\bot \qquad \exists v.p \;:=\; \neg\forall v.\neg p$$

$$p \wedge q \;:=\; \neg(\neg p \vee \neg q) \qquad p \leftrightarrow q \;:=\; (p \leftarrow q) \wedge (p \rightarrow q) \qquad \varnothing \;:=\; \lambda i.\bot$$

$$t = u \;:=\; \forall v.vt \leftrightarrow vu \qquad t \doteq u \;:=\; (\lambda ij.i = j)tu \qquad t' \;:=\; (\lambda ij.i = j)t$$

A sequent calculus: Theorem if p has a proof consisting of steps justified by the rules

Each step is a sequence of alternatives:

$$\varphi_1, \ldots, \varphi_n$$

Each alternative φ is either p or denial $\overline{p}$

$$\varphi, \psi \,/\, \psi, \varphi \qquad \varphi, \varphi \,/\, \varphi$$

$$/\!/\, \varphi \qquad \varphi, \widetilde{\varphi}$$

NoMath: Nominalistic Logic for Computer Mathematics

Type: $\tau ::= \sigma \mid \tau \mapsto \tau \mid \iota \qquad v ::= \tau \mid \ell v \qquad t ::= tt \mid \lambda v.t \mid v \mid \circ \mid \vartriangle\tau \mid \lessgtr\tau$

$ft : \tau$ if $f : \tau_0\tau$ and $t : \tau_1$ where τ_0 is τ_1 or else τ_0 is ι and free $i : \iota$ in t

$\lambda v.t : \tau_0\tau$ if $v : \tau_0$ and $t : \tau \qquad v : \tau \qquad \circ : \sigma\sigma\sigma \qquad \vartriangle : (\tau\sigma)\sigma \qquad \lessgtr : (\tau\sigma)\tau$

$$\neg p \coloneqq \circ pp \quad p \vee q \coloneqq \neg\circ pq \quad p \leftarrow q \coloneqq p \vee \neg q \quad p \rightarrow q \coloneqq \neg p \vee q$$

$$\forall v.p \coloneqq \vartriangle\lambda v.p \quad \bot \coloneqq \forall v.v \quad \top \coloneqq \neg\bot \quad \exists v.p \coloneqq \neg\forall v.\neg p$$

$$p \wedge q \coloneqq \neg(\neg p \vee \neg q) \quad p \leftrightarrow q \coloneqq (p \leftarrow q) \wedge (p \rightarrow q) \quad \varnothing \coloneqq \lambda i.\bot$$

$$t = u \coloneqq \forall v.vt \leftrightarrow vu \quad t \doteq u \coloneqq (\lambda ij.i = j)tu \quad t' \coloneqq (\lambda ij.i = j)t$$

A sequent calculus: Theorem if p has a proof consisting of steps justified by the rules

Each step is a sequence of alternatives:

$$\varphi_1, \ldots, \varphi_n$$

Each alternative φ is either p or denial $\overline{p}$

$\varphi, \psi \,/\, \psi, \varphi \qquad \varphi, \varphi \,/\, \varphi$

$/\!/\, \varphi \qquad \varphi, \widetilde{\varphi}$

$p, q, \circ pq$

NoMath: Nominalistic Logic for Computer Mathematics

Type: $\tau ::= \sigma \mid \tau \mapsto \tau \mid \iota \qquad v ::= \tau \mid \ell v \qquad t ::= tt \mid \lambda v.t \mid v \mid \circ \mid \vartriangle\tau \mid \lessgtr\tau$

$ft : \tau$ if $f : \tau_0\tau$ and $t : \tau_1$ where τ_0 is τ_1 or else τ_0 is ι and free $i : \iota$ in t

$\lambda v.t : \tau_0\tau$ if $v : \tau_0$ and $t : \tau \qquad v : \tau \qquad \circ : \sigma\sigma\sigma \qquad \vartriangle : (\tau\sigma)\sigma \qquad \lessgtr : (\tau\sigma)\tau$

$$\neg p \;:=\; \circ pp \qquad p \vee q \;:=\; \neg\circ pq \qquad p \leftarrow q \;:=\; p \vee \neg q \qquad p \rightarrow q \;:=\; \neg p \vee q$$

$$\forall v.p \;:=\; \vartriangle\lambda v.p \qquad \bot \;:=\; \forall v.v \qquad \top \;:=\; \neg\bot \qquad \exists v.p \;:=\; \neg\forall v.\neg p$$

$$p \wedge q \;:=\; \neg(\neg p \vee \neg q) \qquad p \leftrightarrow q \;:=\; (p \leftarrow q) \wedge (p \rightarrow q) \qquad \varnothing \;:=\; \lambda i.\bot$$

$$t = u \;:=\; \forall v.vt \leftrightarrow vu \qquad t \doteq u \;:=\; (\lambda ij.i = j)tu \qquad t' \;:=\; (\lambda ij.i = j)t$$

A sequent calculus: Theorem if p has a proof consisting of steps justified by the rules

Each step is a sequence of alternatives:

$$\varphi_1, \ldots, \varphi_n$$

Each alternative φ is either p or denial $\overline{p}$

$$\varphi, \psi \,/\, \psi, \varphi \qquad \varphi, \varphi \,/\, \varphi$$

$$/\!/\, \varphi \qquad \varphi, \widetilde{\varphi}$$

$$p, q, \circ pq \qquad p, q \,/\, \overline{\circ pq}$$

NoMath: Nominalistic Logic for Computer Mathematics

Type: $\tau ::= \sigma \mid \tau \mapsto \tau \mid \iota \qquad v ::= \tau \mid \ell v \qquad t ::= tt \mid \lambda v.t \mid v \mid \circ \mid \vartriangle\tau \mid \lessgtr\tau$

$ft : \tau$ if $f : \tau_0\tau$ and $t : \tau_1$ where τ_0 is τ_1 or else τ_0 is ι and free $i : \iota$ in t

$\lambda v.t : \tau_0\tau$ if $v : \tau_0$ and $t : \tau \qquad v : \tau \qquad \circ : \sigma\sigma\sigma \qquad \vartriangle : (\tau\sigma)\sigma \qquad \lessgtr : (\tau\sigma)\tau$

$$\neg p := \circ pp \qquad p \vee q := \neg\circ pq \qquad p \leftarrow q := p \vee \neg q \qquad p \rightarrow q := \neg p \vee q$$

$$\forall v.p := \vartriangle\lambda v.p \qquad \bot := \forall v.v \qquad \top := \neg\bot \qquad \exists v.p := \neg\forall v.\neg p$$

$$p \wedge q := \neg(\neg p \vee \neg q) \qquad p \leftrightarrow q := (p \leftarrow q) \wedge (p \rightarrow q) \qquad \varnothing := \lambda i.\bot$$

$$t = u := \forall v.vt \leftrightarrow vu \qquad t \doteq u := (\lambda ij.i=j)tu \qquad t' := (\lambda ij.i=j)t$$

A sequent calculus: Theorem if p has a proof consisting of steps justified by the rules

Each step is a sequence of alternatives:

$$\varphi_1, \ldots, \varphi_n$$

Each alternative φ is either p or denial $\overline{p}$

$\varphi, \psi \,/\, \psi, \varphi \qquad \varphi, \varphi \,/\, \varphi$

$/\!/\, \varphi \qquad \varphi, \widetilde{\varphi}$

$p, q, \circ pq \qquad p, q \,/\, \overline{\circ pq}$

$pv \,/\, \vartriangle p \ [v]$

NoMath: Nominalistic Logic for Computer Mathematics

Type: $\tau ::= \sigma \mid \tau \mapsto \tau \mid \iota \qquad v ::= \tau \mid \ell v \qquad t ::= tt \mid \lambda v.t \mid v \mid \circ \mid \vartriangle\tau \mid \lessgtr\tau$

$ft : \tau$ if $f : \tau_0\tau$ and $t : \tau_1$ where τ_0 is τ_1 or else τ_0 is ι and free $i : \iota$ in t

$\lambda v.t : \tau_0\tau$ if $v : \tau_0$ and $t : \tau \qquad v : \tau \qquad \circ : \sigma\sigma\sigma \qquad \vartriangle : (\tau\sigma)\sigma \qquad \lessgtr : (\tau\sigma)\tau$

$$\neg p \;:=\; \circ pp \qquad p \vee q \;:=\; \neg\circ pq \qquad p \leftarrow q \;:=\; p \vee \neg q \qquad p \rightarrow q \;:=\; \neg p \vee q$$

$$\forall v.p \;:=\; \vartriangle\lambda v.p \qquad \bot \;:=\; \forall v.v \qquad \top \;:=\; \neg\bot \qquad \exists v.p \;:=\; \neg\forall v.\neg p$$

$$p \wedge q \;:=\; \neg(\neg p \vee \neg q) \qquad p \leftrightarrow q \;:=\; (p \leftarrow q) \wedge (p \rightarrow q) \qquad \varnothing \;:=\; \lambda i.\bot$$

$$t = u \;:=\; \forall v.vt \leftrightarrow vu \qquad t \doteq u \;:=\; (\lambda ij.i = j)tu \qquad t' \;:=\; (\lambda ij.i = j)t$$

A sequent calculus: Theorem if p has a proof consisting of steps justified by the rules

Each step is a sequence of alternatives:

$$\varphi_1, \ldots, \varphi_n$$

Each alternative φ is either p or denial $\overline{p}$

$$\varphi, \psi \,/\, \psi, \varphi \qquad \varphi, \varphi \,/\, \varphi$$

$$/\!/\, \varphi \qquad \varphi, \widetilde{\varphi}$$

$$p, q, \circ pq \qquad p, q \,/\, \overline{\circ pq}$$

$$pv \,/\, \vartriangle p \; [v] \qquad \vartriangle p \,/\, pt$$

NoMath: Nominalistic Logic for Computer Mathematics

Type: $\tau ::= \sigma \mid \tau \mapsto \tau \mid \iota \qquad v ::= \tau \mid \ell v \qquad t ::= tt \mid \lambda v.t \mid v \mid \circ \mid \vartriangle\tau \mid \lessgtr\tau$

$ft : \tau$ if $f : \tau_0\tau$ and $t : \tau_1$ where τ_0 is τ_1 or else τ_0 is ι and free $i : \iota$ in t

$\lambda v.t : \tau_0\tau$ if $v : \tau_0$ and $t : \tau \qquad v : \tau \qquad \circ : \sigma\sigma\sigma \qquad \vartriangle : (\tau\sigma)\sigma \qquad \lessgtr : (\tau\sigma)\tau$

$$\neg p \coloneqq \circ pp \qquad p \vee q \coloneqq \neg\circ pq \qquad p \leftarrow q \coloneqq p \vee \neg q \qquad p \rightarrow q \coloneqq \neg p \vee q$$

$$\forall v.p \coloneqq \vartriangle\lambda v.p \qquad \bot \coloneqq \forall v.v \qquad \top \coloneqq \neg\bot \qquad \exists v.p \coloneqq \neg\forall v.\neg p$$

$$p \wedge q \coloneqq \neg(\neg p \vee \neg q) \qquad p \leftrightarrow q \coloneqq (p \leftarrow q) \wedge (p \rightarrow q) \qquad \varnothing \coloneqq \lambda i.\bot$$

$$t = u \coloneqq \forall v.vt \leftrightarrow vu \qquad t \doteq u \coloneqq (\lambda ij.i = j)tu \qquad t' \coloneqq (\lambda ij.i = j)t$$

A sequent calculus: Theorem if p has a proof consisting of steps justified by the rules

Each step is a sequence of alternatives:

$$\varphi_1, \ldots, \varphi_n$$

Each alternative φ is either p or denial $\overline{p}$

$\varphi, \psi \,/\, \psi, \varphi \qquad \varphi, \varphi \,/\, \varphi$

$/\!/ \, \varphi \qquad \varphi, \widetilde{\varphi}$

$p, q, \circ pq \qquad p, q \,/\, \overline{\circ pq}$

$pv \,/\, \vartriangle p \; [v] \qquad \vartriangle p \,/\, pt$

$t = u \;\leftrightarrow\; t \doteq u$

NoMath: Nominalistic Logic for Computer Mathematics

Type: $\tau ::= \sigma \mid \tau \mapsto \tau \mid \iota \qquad v ::= \tau \mid \ell v \qquad t ::= tt \mid \lambda v.t \mid v \mid \circ \mid \vartriangle\tau \mid \lessapprox\tau$

$ft : \tau$ if $f : \tau_0\tau$ and $t : \tau_1$ where τ_0 is τ_1 or else τ_0 is ι and free $i : \iota$ in t

$\lambda v.t : \tau_0\tau$ if $v : \tau_0$ and $t : \tau \qquad v : \tau \qquad \circ : \sigma\sigma\sigma \qquad \vartriangle : (\tau\sigma)\sigma \qquad \lessapprox : (\tau\sigma)\tau$

$$\neg p \ := \ \circ pp \qquad p \vee q \ := \ \neg\circ pq \qquad p \leftarrow q \ := \ p \vee \neg q \qquad p \rightarrow q \ := \ \neg p \vee q$$

$$\forall v.p \ := \ \vartriangle\lambda v.p \qquad \bot \ := \ \forall v.v \qquad \top \ := \ \neg\bot \qquad \exists v.p \ := \ \neg\forall v.\neg p$$

$$p \wedge q \ := \ \neg(\neg p \vee \neg q) \qquad p \leftrightarrow q \ := \ (p \leftarrow q) \wedge (p \rightarrow q) \qquad \varnothing \ := \ \lambda i.\bot$$

$$t = u \ := \ \forall v.vt \leftrightarrow vu \qquad t \doteq u \ := \ (\lambda ij.i = j)tu \qquad t' \ := \ (\lambda ij.i = j)t$$

A sequent calculus: Theorem if p has a proof consisting of steps justified by the rules

Each step is a sequence of alternatives:

$$\varphi_1, \ldots, \varphi_n$$

Each alternative φ is either p or denial $\overline{p}$

$$\varphi, \psi \,/\, \psi, \varphi \qquad \varphi, \varphi \,/\, \varphi$$

$$/\!/\, \varphi \qquad \varphi, \widetilde{\varphi}$$

$$p, q, \circ pq \qquad p, q \,/\, \overline{\circ pq}$$

$$pv \,/\, \vartriangle p \; [v] \qquad \vartriangle p \,/\, pt$$

$$t = u \ \leftrightarrow \ t \doteq u$$

$$wv \rightarrow w(\lessapprox w)$$

NoMath: Nominalistic Logic for Computer Mathematics

Type: $\tau ::= \sigma \mid \tau \mapsto \tau \mid \iota \qquad v ::= \tau \mid \ell v \qquad t ::= tt \mid \lambda v.t \mid v \mid \circ \mid \vartriangle\tau \mid \lessgtr\tau$

$ft : \tau$ if $f : \tau_0\tau$ and $t : \tau_1$ where τ_0 is τ_1 or else τ_0 is ι and free $i : \iota$ in t

$\lambda v.t : \tau_0\tau$ if $v : \tau_0$ and $t : \tau \qquad v : \tau \qquad \circ : \sigma\sigma\sigma \qquad \vartriangle : (\tau\sigma)\sigma \qquad \lessgtr : (\tau\sigma)\tau$

$$\neg p \;:=\; \circ pp \qquad p \vee q \;:=\; \neg\circ pq \qquad p \leftarrow q \;:=\; p \vee \neg q \qquad p \rightarrow q \;:=\; \neg p \vee q$$

$$\forall v.p \;:=\; \vartriangle\lambda v.p \qquad \bot \;:=\; \forall v.v \qquad \top \;:=\; \neg\bot \qquad \exists v.p \;:=\; \neg\forall v.\neg p$$

$$p \wedge q \;:=\; \neg(\neg p \vee \neg q) \qquad p \leftrightarrow q \;:=\; (p \leftarrow q) \wedge (p \rightarrow q) \qquad \varnothing \;:=\; \lambda i.\bot$$

$$t = u \;:=\; \forall v.vt \leftrightarrow vu \qquad t \doteq u \;:=\; (\lambda ij.i = j)tu \qquad t' \;:=\; (\lambda ij.i = j)t$$

A sequent calculus: Theorem if p has a proof consisting of steps justified by the rules

Each step is a sequence of alternatives:

$$\varphi_1, \ldots, \varphi_n$$

Each alternative φ is either p or denial $\overline{p}$

$$\varphi, \psi \,/\, \psi, \varphi \qquad \varphi, \varphi \,/\, \varphi$$

$$/\!/\, \varphi \qquad \varphi, \widetilde{\varphi}$$

$$p, q, \circ pq \qquad p, q \,/\, \overline{\circ pq}$$

$$pv \,/\, \vartriangle p \; [v] \qquad \vartriangle p \,/\, pt$$

$$t = u \;\leftrightarrow\; t \doteq u$$

$$wv \longrightarrow w(\lessgtr w)$$