Modelling the Directionality of Light Scattered in Translucent Materials

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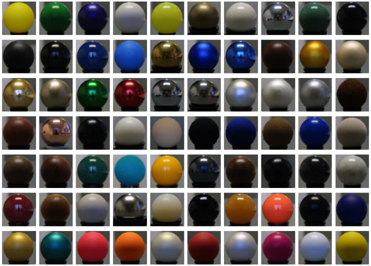
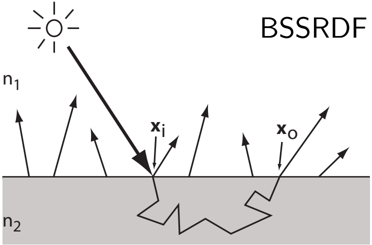
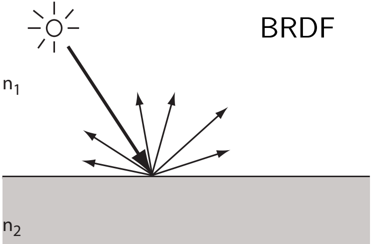
Materials (scattering and absorption of light)

- Optical properties (index of refraction, $n(\lambda) = n'(\lambda) + i\, n''(\lambda)$).
- Reflectance distribution functions, $S(\mathbf{x}_i, \vec{\omega}_i; \mathbf{x}_o, \vec{\omega}_o)$.



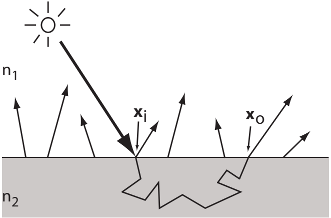




Subsurface scattering

- Behind the rendering equation [Nicodemus et al. 1977]:

$$\frac{\mathrm{d}L_r(\mathbf{x}_o, \vec{\omega}_o)}{\mathrm{d}\Phi_i(\mathbf{x}_i, \vec{\omega}_i)} = S(\mathbf{x}_i, \vec{\omega}_i; \mathbf{x}_o, \vec{\omega}_o)\,.$$



- An element of reflected radiance $\mathrm{d}L_r$ is proportional to an element of incident flux $\mathrm{d}\Phi_i$.
- S (the BSSRDF) is the factor of proportionality.
- Using the definition of radiance $L = \dfrac{\mathrm{d}^2\Phi}{\cos\theta\,\mathrm{d}A\,\mathrm{d}\omega}$, we have

$$L_r(\mathbf{x}_o, \vec{\omega}_o) = \int_A \int_{2\pi} S(\mathbf{x}_i, \vec{\omega}_i; \mathbf{x}_o, \vec{\omega}_o) L_i(\mathbf{x}_i, \vec{\omega}_i) \cos\theta\,\mathrm{d}\omega_i\,\mathrm{d}A\,.$$

References

- Nicodemus, F. E., Richmond, J. C., Hsia, J. J., Ginsberg, I. W., and Limperis, T. Geometrical considerations and nomenclature for reflectance. Tech. rep., National Bureau of Standards (US), 1977.



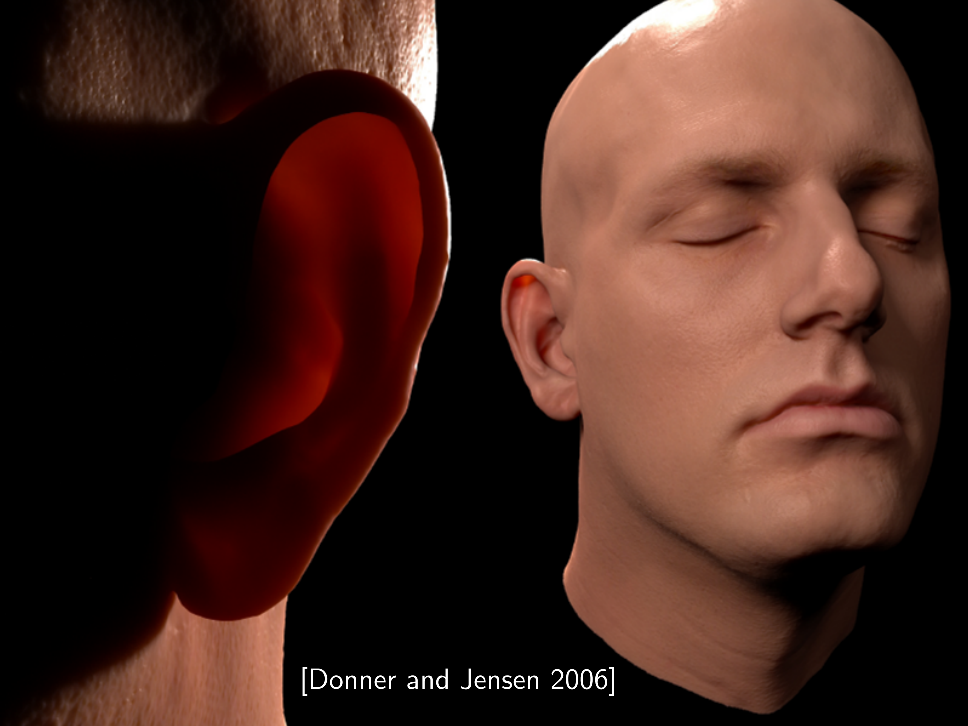




BRDF


BSSRDF

[Jensen et al. 2001]



[Donner and Jensen 2006]

Splitting up the BSSRDF

- Bidirectional Scattering-Surface Reflectance Distribution Function: $S = S(\mathbf{x}_i, \vec{\omega}_i; \mathbf{x}_o, \vec{\omega}_o)$.
- Away from sources and boundaries, we can use diffusion.
- Splitting up the BSSRDF

$$S = T_{12}(S^{(0)} + S^{(1)} + S_d)\,T_{21}\,.$$

  where
  - T_{12} and T_{21} are Fresnel transmittance terms (using $\vec{\omega}_i$, $\vec{\omega}_o$).
  - $S^{(0)}$ is the direct transmission part (using Dirac δ-functions).
  - $S^{(1)}$ is the single scattering part (using all arguments).
  - S_d is the diffusive part (multiple scattering, using $|\mathbf{x}_o - \mathbf{x}_i|$).
- We distribute the single scattering to the other terms using the delta-Eddington approximation:

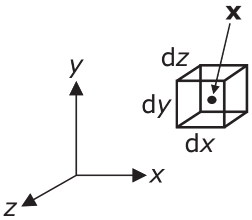
$$S = T_{12}(S_{\delta E} + S_d)\,T_{21} \quad ,$$

  and generalize the model such that $S_d = S_d(\mathbf{x}_i, \vec{\omega}_i; \mathbf{x}_o)$.

Diffusion theory

- Think of multiple scattering as a diffusion process.
- In diffusion theory, we use quantities that describe the light field in an element of volume of the scattering medium.
- Total flux, or fluence, is defined by

$$\phi(\mathbf{x}) = \int_{4\pi} L(\mathbf{x}, \vec{\omega})\, \mathrm{d}\omega \quad .$$



- We find an expression for ϕ by solving the diffusion equation

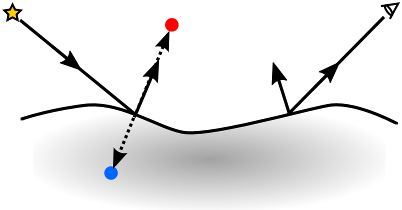
$$(D\nabla^2 - \sigma_a)\phi(\mathbf{x}) = -q(\mathbf{x}) + 3D\,\nabla\cdot\mathbf{Q}(\mathbf{x}) \quad ,$$

where σ_a and D are absorption and diffusion coefficients, while q and $\mathbf{Q}$ are zeroth and first order source terms.

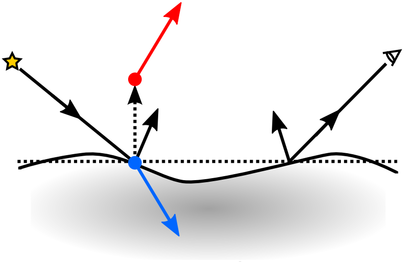
Deriving a BSSRDF

- Assume that emerging light is diffuse due to a large number of scattering events: $S_d(\mathbf{x}_i, \vec{\omega}_i; \mathbf{x}_o, \vec{\omega}_o) = S_d(\mathbf{x}_i, \vec{\omega}_i; \mathbf{x}_o)$.
- Integrating emerging diffuse radiance over outgoing directions, we find
$$S_d = \frac{C_\phi(\eta)\,\phi - C_{\mathbf{E}}(\eta)\,D\,\vec{n}_o\cdot\nabla\phi}{\Phi\,4\pi\,C_\phi(1/\eta)}\ ,$$
where
  - Φ is the flux entering the medium at $\mathbf{x}_i$.
  - $\vec{n}_o$ is the surface normal at the point of emergence $\mathbf{x}_o$.
  - C_ϕ and $C_{\mathbf{E}}$ depend on the relative index of refraction η and are polynomial fits of different hemispherical integrals of the Fresnel transmittance.
- This connects the BSSRDF and the diffusion theory.
- To get an analytical model, we use a special case solution for the diffusion equation (an expression for ϕ).
- Then, "all" we need to do is to find $\nabla\phi$ (do the math) and deal with boundary conditions (build a plausible model).

Point source diffusion or ray source diffusion



standard dipole



our model

- Point source diffusion [Bothe 1941; 1942]

$$\phi(r) = \frac{\Phi}{4\pi D} \frac{e^{-\sigma_{\text{tr}} r}}{r} \quad ,$$

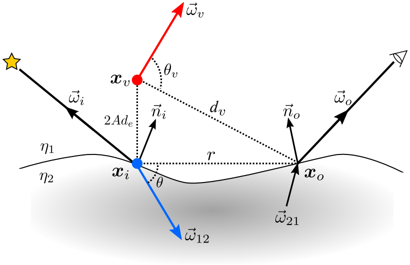
where $r = |\mathbf{x}_o - \mathbf{x}_i|$ and $\sigma_{\text{tr}} = \sqrt{\sigma_a / D}$ is the effective transport coefficient.

- Ray source diffusion [Menon et al. 2005a; 2005b]

$$\phi(r, \theta) = \frac{\Phi}{4\pi D} \frac{e^{-\sigma_{\text{tr}} r}}{r} \left(1 + 3D \frac{1 + \sigma_{\text{tr}} r}{r} \cos\theta\right) ,$$

where θ is the angle between the refracted ray and $\mathbf{x}_o - \mathbf{x}_i$.

Our BSSRDF when disregarding the boundary

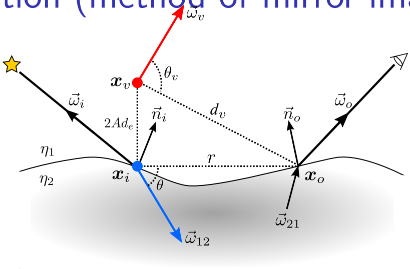


- Using $\mathbf{x} = \mathbf{x}_o - \mathbf{x}_i$, $r = |\mathbf{x}|$, $\cos\theta = \mathbf{x} \cdot \vec{\omega}_{12}/r$, we take the gradient of $\phi(r, \theta)$ (the expression for ray source diffusion) and insert to find

$$S_d'(\mathbf{x}, \vec{\omega}_{12}, r) = \frac{1}{4C_\phi(1/\eta)} \frac{1}{4\pi^2} \frac{e^{-\sigma_{\text{tr}} r}}{r^3} \Bigg[C_\phi(\eta) \left(\frac{r^2}{D} + 3(1 + \sigma_{\text{tr}} r)\, \mathbf{x} \cdot \vec{\omega}_{12} \right) - C_{\mathbf{E}}(\eta) \left(3D(1 + \sigma_{\text{tr}} r)\, \vec{\omega}_{12} \cdot \vec{n}_o - \left((1 + \sigma_{\text{tr}} r) + 3D \frac{3(1+\sigma_{\text{tr}} r) + (\sigma_{\text{tr}} r)^2}{r^2} \mathbf{x} \cdot \vec{\omega}_{12} \right) \mathbf{x} \cdot \vec{n}_o \right) \Bigg],$$

which would be the BSSRDF if we neglect the boundary.

Dipole configuration (method of mirror images)



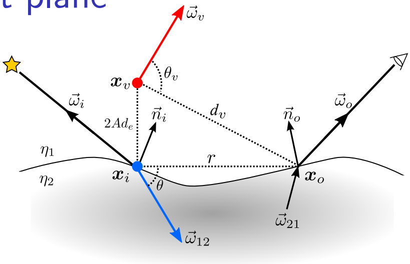
- We place the “real” ray source at the boundary and reflect it in an extrapolated boundary to place the “virtual” ray source.
- Distance to the extrapolated boundary [Davison 1958]:

$$d_e = 2.131\, D/\sqrt{1 - 3D\sigma_a} \ .$$

- In case of a refractive boundary ($\eta_1 \neq \eta_2$), the distance is

$$Ad_e \quad \text{with} \quad A = \frac{1 - C_{\mathbf{E}}(\eta)}{2C_\phi(\eta)} \ .$$

Modified tangent plane



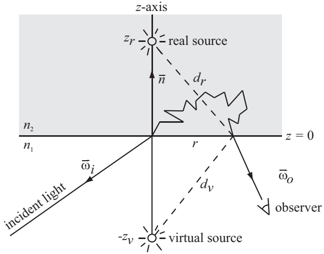
- The dipole assumes a semi-infinite medium.
- We assume that the boundary contains the vector $\mathbf{x}_o - \mathbf{x}_i$ and that it is perpendicular to the plane spanned by $\vec{n}_i$ and $\mathbf{x}_o - \mathbf{x}_i$.
- The normal of the assumed boundary plane is then

$$\vec{n}_i^* = \frac{\mathbf{x}_o - \mathbf{x}_i}{|\mathbf{x}_o - \mathbf{x}_i|} \times \frac{\vec{n}_i \times (\mathbf{x}_o - \mathbf{x}_i)}{|\vec{n}_i \times (\mathbf{x}_o - \mathbf{x}_i)|}, \quad \text{or } \vec{n}_i^* = \vec{n}_i \text{ if } \mathbf{x}_o = \mathbf{x}_i .$$

  and the virtual source is given by

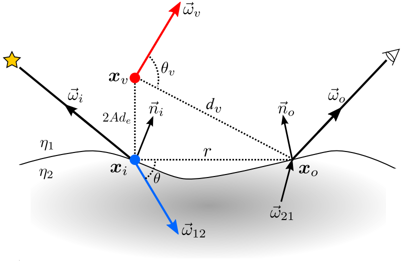
$$\mathbf{x}_v = \mathbf{x}_i + 2Ad_e\vec{n}_i^* , \;\; d_v = |\mathbf{x}_v - \mathbf{x}_i| , \;\; \vec{\omega}_v = \vec{\omega}_{12} - 2(\vec{\omega}_{12} \cdot \vec{n}_i^*)\vec{n}_i^* .$$

Distance to the real source (handling the singularity)



standard dipole

$$d_r = \sqrt{r^2 + z_r^2} \ .$$



our model

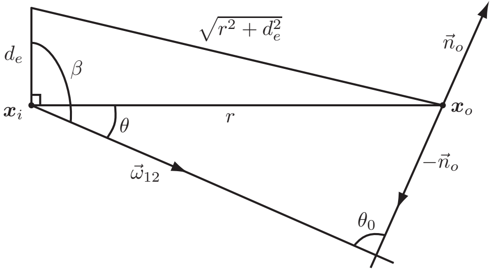
$$d_r = r \ ?$$

- Emergent radiance is an integral over z of a Hankel transform of a Green function which is Fourier transformed in x and y.
- Approximate analytic evaluation is possible if r is corrected to

$$R^2 = r^2 + (z' + d_e)^2 \ .$$

- The resulting model for $z' = 0$ corresponds to the standard dipole where $z' = z_r$ and d_e is replaced by the virtual source.

Distance to the real source (handling the singularity)



- Since we neither have normal incidence nor $\mathbf{x}_o$ in the tangent plane, we modify the distance correction:

$$R^2 = r^2 + z'^2 + d_e^2 - 2z' d_e \cos\beta \ .$$

- It is possible to reformulate the integral over z to an integral along the refracted ray.
- We can approximate this integral by choosing an offset D^* along the refracted ray. Then $z' = D^* |\cos\theta_0|$.

Our BSSRDF when considering boundary conditions

- Our final distance to the real source becomes

$$d_r^2 = \begin{cases} r^2 + D\mu_0(D\mu_0 - 2d_e \cos\beta) & \text{for } \mu_0 > 0 \text{ (frontlit)} \\ r^2 + 1/(3\sigma_t)^2 & \text{otherwise (backlit)}, \end{cases}$$

with $\mu_0 = \cos\theta_0 = -\vec{n}_o \cdot \vec{\omega}_{12}$ and

$$\cos\beta = -\sin\theta \frac{r}{\sqrt{r^2 + d_e^2}} = -\sqrt{\frac{r^2 - (\mathbf{x} \cdot \omega_{12})^2}{r^2 + d_e^2}} \ .$$

- The diffusive part of our BSSRDF is then

$$S_d(\mathbf{x}_i, \vec{\omega}_i; \mathbf{x}_o) = S'_d(\mathbf{x}_o - \mathbf{x}_i, \vec{\omega}_{12}, d_r) - S'_d(\mathbf{x}_o - \mathbf{x}_v, \vec{\omega}_v, d_v) \ ,$$

while the full BSSRDF is as before:

$$S = T_{12}(S_{\delta E} + S_d)\, T_{21} \ .$$

Previous Models

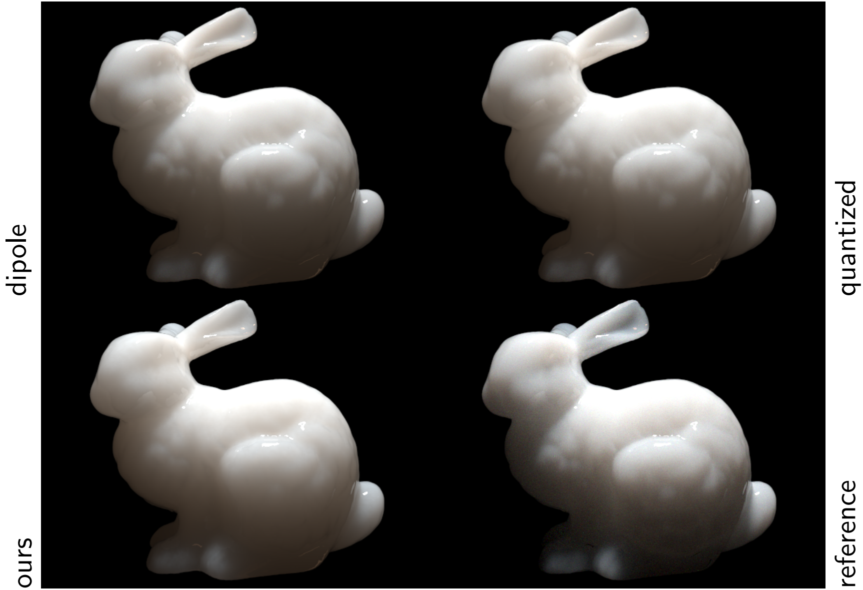
- Previous models are based on the point source solution of the diffusion equation and have the problems listed below.

1. Ignore incoming light direction:
   - Standard dipole [Jensen et al. 2001].
   - Multipole [Donner and Jensen 2005].
   - Quantized diffusion [d'Eon and Irving 2011].
2. Require precomputation:
   - Precomputed BSSRDF [Donner et al. 2009, Yan et al. 2012].
3. Rely on numerical integration:
   - Photon diffusion [Donner and Jensen 2007, Habel et al. 2013].

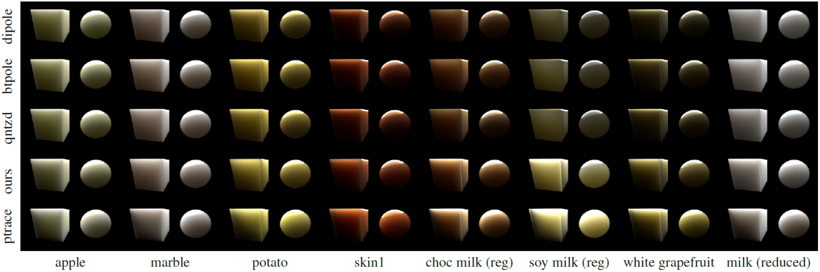
- Using ray source diffusion, we can get rid of those problems.

Results (Grapefruit Bunnies)

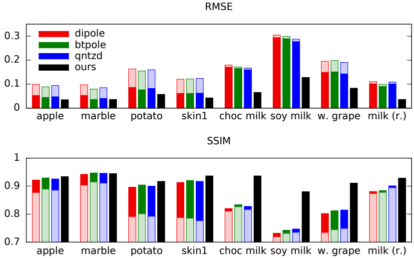
Results (marble Bunnies)



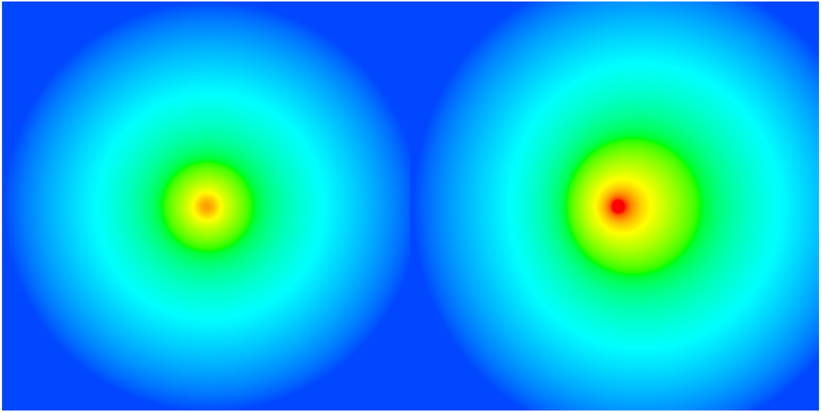
Results (Simple Scene)



- Path traced single scattering was added to the existing models but not to ours.
- Faded bars show quality measurements when single scattering is not added.
- The four leftmost materials scatter light isotropically.



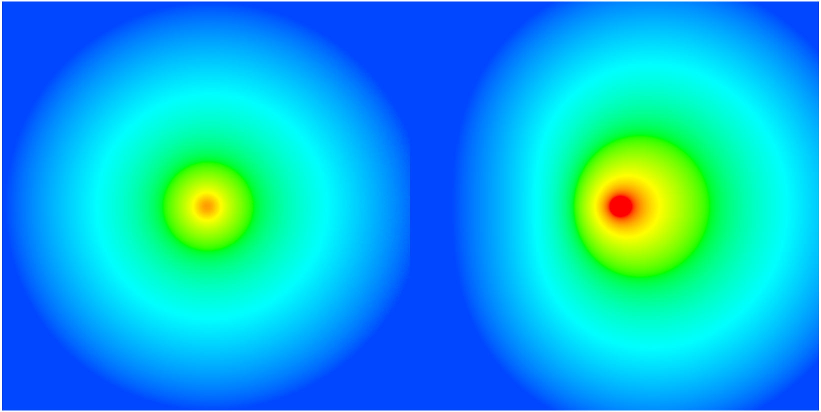
Results (2D plots, 30° Oblique Incidence)



quantized ours

- Our model is significantly different
  - when the angle of incidence changes
  - when the direction toward the point of emergence changes.

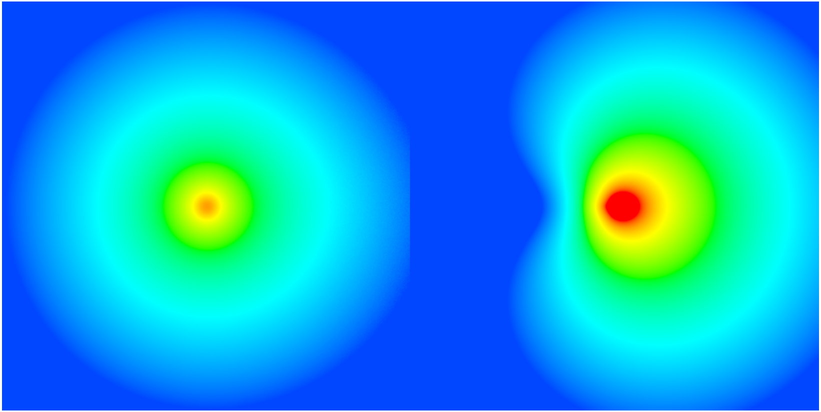
Results (2D plots, 45° Oblique Incidence)



quantized ours

- Our model is significantly different
  - when the angle of incidence changes
  - when the direction toward the point of emergence changes.

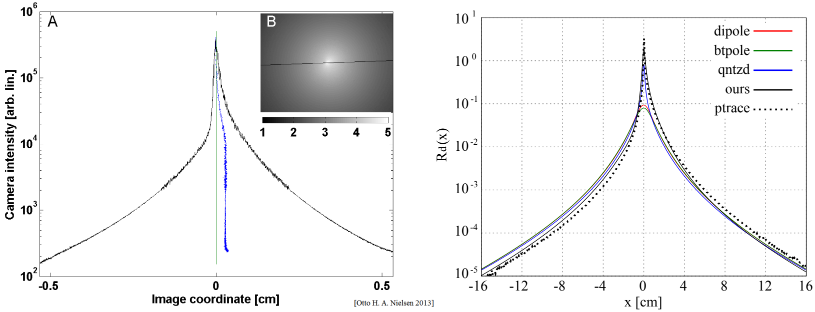
Results (2D plots, 60° Oblique Incidence)



quantized ours

- Our model is significantly different
  - when the angle of incidence changes
  - when the direction toward the point of emergence changes.

Results (Diffuse Reflectance Curves)



- Our model comes closer than the existing analytical models to measured and simulated diffuse reflectance curves.

Results (Image Based Lighting)

quantized ours

The 3Shape Buddha! (scanned with a TRIOS Scanner)

matte milk-coloured　　　　mini milk

Conclusion

- First BSSRDF which...
  - Considers the direction of the incident light.
  - Requires no precomputation.
  - Provides a fully analytical solution.
- Much more accurate than previous models.
- Incorporates single scattering in the analytical model.

- Future work:
  - Consider the direction of the emergent light.
  - Real-time approximations.
  - Directional multipole and quadpole extensions.
  - Directional photon diffusion.
  - Anisotropic media (skewed dipole).