

# Survey of Models for Acquiring the Optical Properties of Translucent Materials

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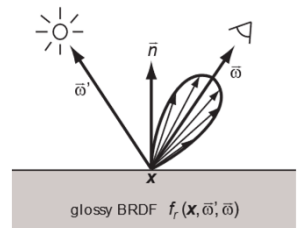
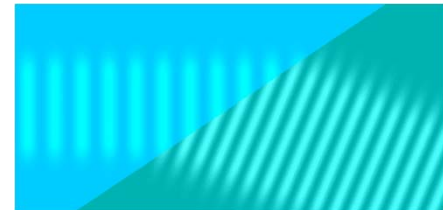
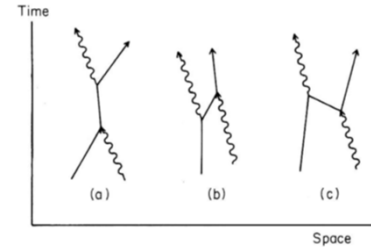
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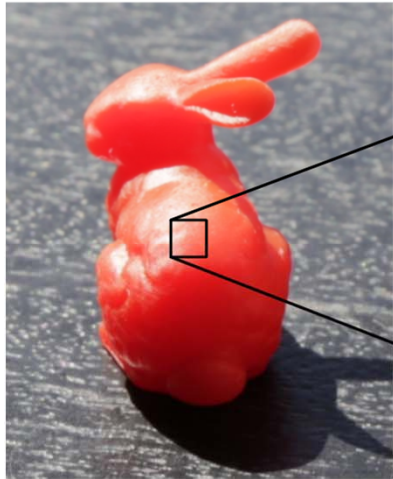
# Optical properties

- Parameters that determine how light interacts with a material.
- Quantum and wave theories:
  - Quantum scale: photon-electron interactions in atomic systems.
  - Nanoscopic scale: charge and current densities in atomic systems.
  - Microscopic scale: polarisation and magnetisation vectors.
  - Macroscopic scale: permittivity, permeability, conductivity.
- Radiative transfer theory:
  - Microscopic scale: complex index of refraction.
  - Mesoscopic scale: surface BSDF, scattering cross section, phase function.
  - Macroscopic scale: scattering properties, BSSRDF, BRDF, BTDF.

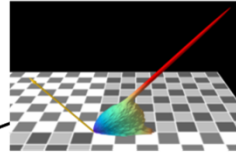


# Multiscale modelling

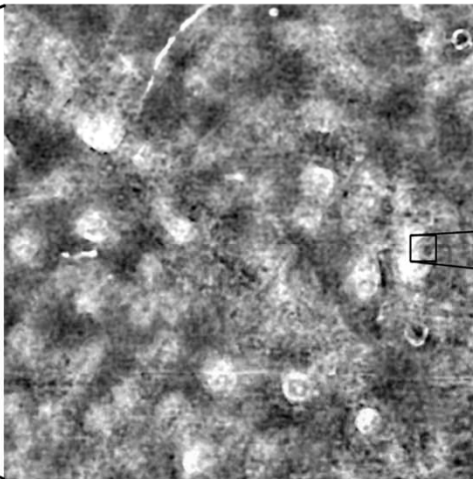
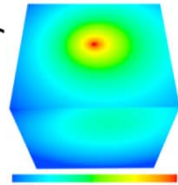
translucent object



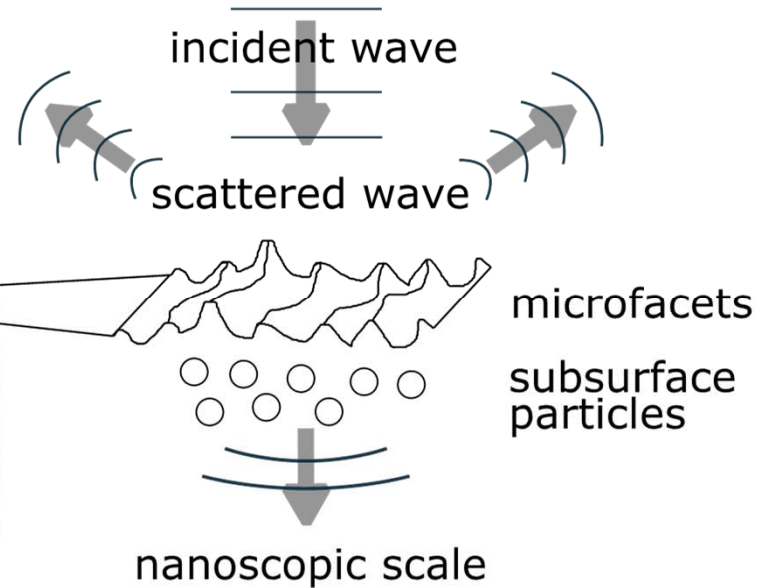
macroscopic scale



BRDF  
or  
BSSRDF



microscopic scale



- With simulation of light propagation, we can compute macroscopic optical properties by considering geometry at different scales.

# Index of refraction (or refractive index)

- Combining permittivity ( $\epsilon$ ), permeability ( $\mu$ ), and conductivity ( $\sigma$ ):

- $$n_{\text{med}} = n' + i n'' = c \sqrt{\mu \left( \epsilon + i \frac{\sigma}{\omega} \right)}$$

- $\omega$  is angular frequency.
- $c$  is the speed of light *in vacuo*.

- Real part  $n' \approx \frac{c}{v}$

- $v$  is the phase velocity of the light wave.

- Imaginary part  $n'' \approx \frac{\sigma_a \lambda}{4\pi}$

- $\sigma_a$  is the absorption coefficient.
- $\lambda$  is the wavelength *in vacuo*.



varying the  
real part  $n'$



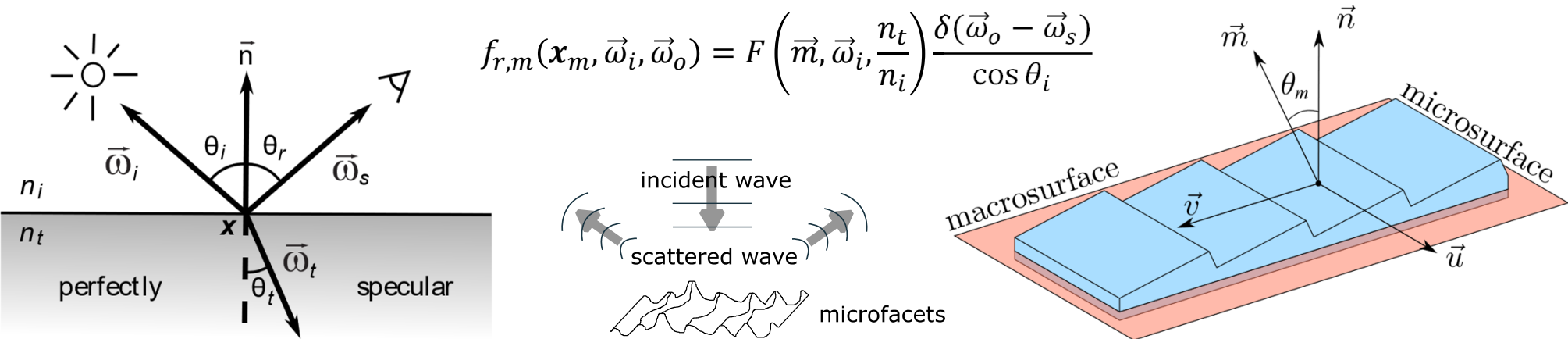
$n''$



Including  
absorption

# Microfacet BSDF

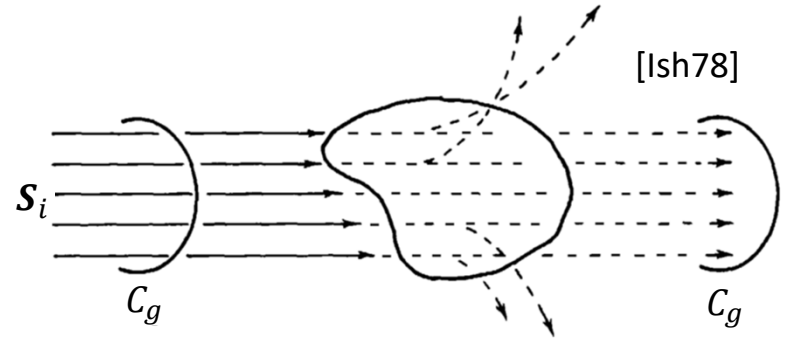
- A surface is **optically smooth** if the surface roughness  $R_q$  is sufficiently small compared with the wavelength  $\lambda$ .
- Rayleigh smooth-surface criterion:  $R_q < \lambda / (8 \cos \theta_i)$ .
- Considering smooth microgeometry we can use  $n_{\text{med}}$  as input for analytic or computational solutions for Maxwell's equations.
- Example: Fresnel reflectance  $F$  for a microfacet BSDF.



# Particle phase function and cross sections

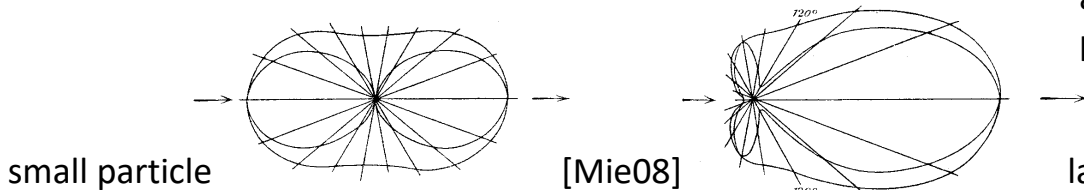
- Particle cross sections

- $C_g$  is the geometric cross section.
- $C_s$  is the scattering cross section.
- $C_a$  is the absorption cross section.
- $C_t = C_s + C_a$  is the extinction cross section.

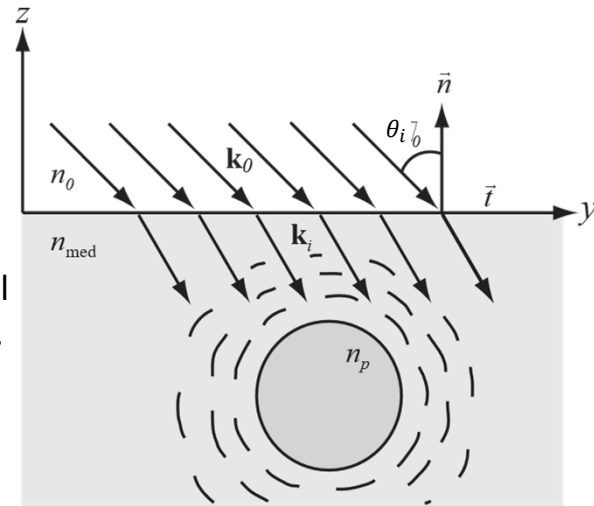


- Particle phase function

- $p_m(\vec{\omega}_i, \vec{\omega}_o)$  is the far field distribution of the scattered light.
- $g = \int_{4\pi} p_m(\vec{\omega}_i, \vec{\omega}_o) (\vec{\omega}_i \cdot \vec{\omega}_o) d\omega$  is the asymmetry parameter in  $[-1,1]$ .



Example: Insert  $x = \frac{2\pi r n_{med}}{\lambda}$  and  $y = \frac{2\pi r n_p}{\lambda}$  in Lorenz-Mie theory to compute  $C_s$ ,  $C_t$ , and  $p$  of a spherical particle of radius  $r$ .



# Scattering properties of a medium

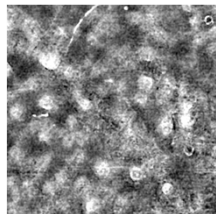
[FCJ07]

- Using a particle size distribution  $N(r)$ :  $\sigma_s = \int_{r_{\min}}^{r_{\max}} C_s(r) N(r) dr$ 
  - $\sigma_s$  is the scattering coefficient.
  - Similarly for  $\sigma_a$  (absorption coefficient) and  $p$  (ensemble phase function).
- Using a microfacet normal distribution  $D(\vec{m})$ :

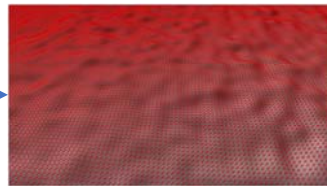
[WMLT07]

$$f_s(\vec{\omega}_i, \vec{\omega}_o, \vec{n}) = \int \left| \frac{\vec{\omega}_i \cdot \vec{m}}{\vec{\omega}_i \cdot \vec{n}} \right| f_m(\vec{\omega}_i, \vec{\omega}_o, \vec{m}) \left| \frac{\vec{\omega}_o \cdot \vec{m}}{\vec{\omega}_o \cdot \vec{n}} \right| G(\vec{\omega}_i, \vec{\omega}_o, \vec{m}) D(\vec{m}) d\omega_m$$

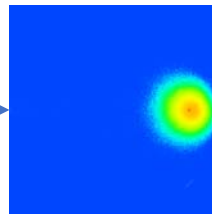
- $G$  is a geometrical attenuation term (shadowing/masking).
- Or we can use explicitly defined microgeometry



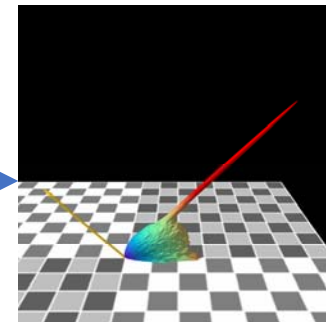
profilometry



triangle mesh



simulation



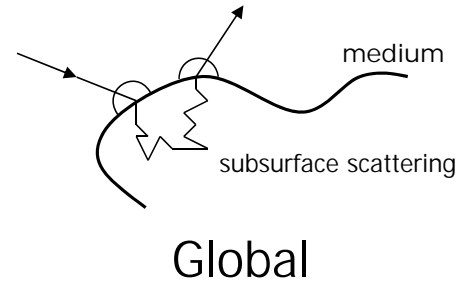
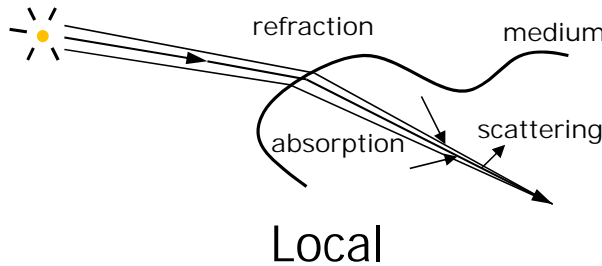
BSDF

# Global scattering function (BSSRDF)

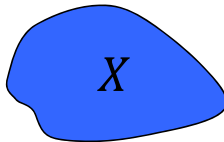
- From local to global formulation using scattering operators [Pre65].

$$L_r = L_i \mathbf{S} = L_i \mathbf{F}_s \sum_{j=0}^{\infty} \mathbf{S}^j \mathbf{F}_s$$

- $\mathbf{F}_s$  is surface scattering
- $\mathbf{S}^0$  is direct transmission
- $\mathbf{S}^j$  for  $j > 0$  is subsurface scattering (with  $j$  scattering events)



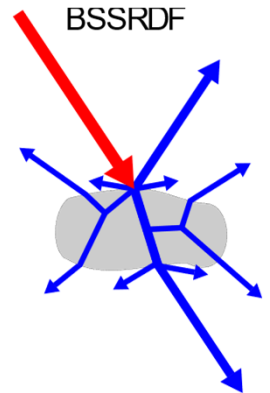
- For



$$\longrightarrow \bullet x, \quad \mathbf{S} \rightarrow \sigma_s p(\vec{\omega}_i, \vec{\omega}_o)$$

- Continuous boundary and interior leads to the BSSRDF:

$$S(X; \mathbf{x}_i, \vec{\omega}_i; \mathbf{x}_o, \vec{\omega}_o) = \lim_{\substack{X_i \rightarrow X_i \\ \Omega_i \rightarrow \vec{\omega}_i}} \frac{L_i \mathbf{S}(\mathbf{x}_o, \vec{\omega}_o)}{L_i(X_i, \Omega_i) A_{i\perp}(X_i) \omega_i(\Omega_i)} = \frac{dL_r(\mathbf{x}_o, \vec{\omega}_o)}{L_i(\mathbf{x}_i, \vec{\omega}_i) dA_{i\perp} d\omega_i} = \frac{dL_r(\mathbf{x}_o, \vec{\omega}_o)}{d\Phi_i(\mathbf{x}_i, \vec{\omega}_i)}$$





# Macroscopic BRDF/BTDF

- Object with homogeneous scattering properties.
- Uniform irradiation of the object over an area  $A_i$  around  $\mathbf{x}_o$ .
  - $A_i$  is large enough to include all  $\mathbf{x}_i$  with subsurface scattering to  $\mathbf{x}_o$ .

- The BRDF is then

$$f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_o) = \int_{A_i} S(X; \mathbf{x}_i, \vec{\omega}_i; \mathbf{x}_o, \vec{\omega}_o) dA_i = \frac{dL_r(\mathbf{x}, \vec{\omega}_o)}{dE(\mathbf{x}, \vec{\omega}_i)}$$

- The equation is the same for the BTDF, but then  $\vec{\omega}_i \cdot \vec{n}_o < 0$ .
  - $\vec{n}_o$  is the surface normal at the point of observation.
- Macroscopic BRDF/BTDF works well for opaque/thin objects.
- Not a good approximation for solid translucent objects.



Lambertian BRDF approximation

# Appearance of translucent materials

- Varying the transport mean free path  $\frac{1}{\sigma'_t} = \frac{1}{\sigma_a + (1-g)\sigma_s}$



$1/\sigma'_t$  increasing  $\rightarrow$

- Varying surface roughness
- Varying lighting environment
- Varying colours (absorption and scattering spectra)

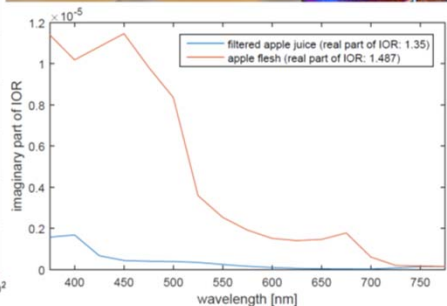
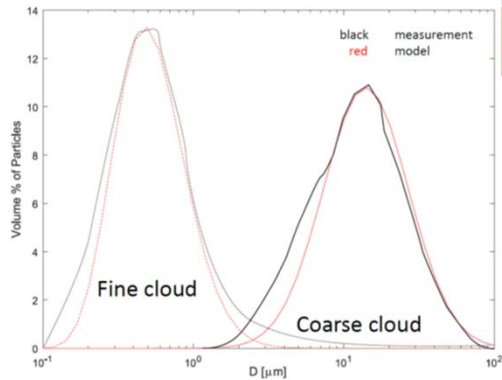
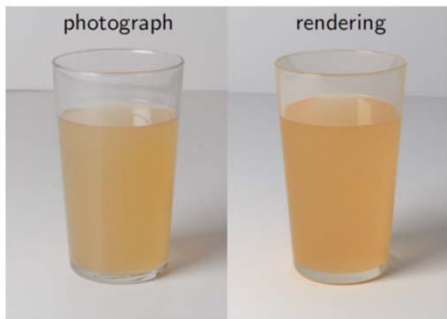
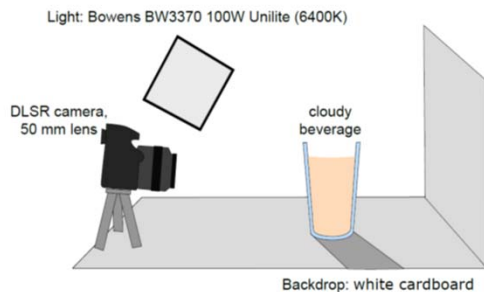


# Influence of particle content

[DFKB16]

- Apple juice example
  - Particle concentration (horizontally).
  - Storage time and handling (vertically).

[FK19]



(peeled and cored)

4 days

9.5 days

9.5 days

27 days



0.0 g/l 0.1 g/l 0.2 g/l 0.5 g/l 1.0 g/l 2.0 g/l

# Discussion

- Translucent objects require optical properties describing both surface and subsurface scattering.
- What is the best appearance specification for translucent objects?
- Influence of surface roughness [LFD\*20]

