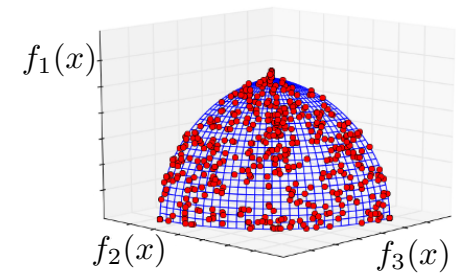
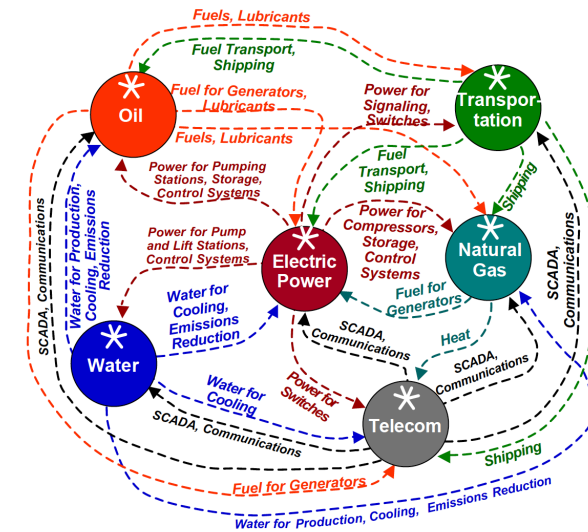


# A Framework for Multi-Stakeholder Decision-Making and Conflict Resolution



**Victor M. Zavala**

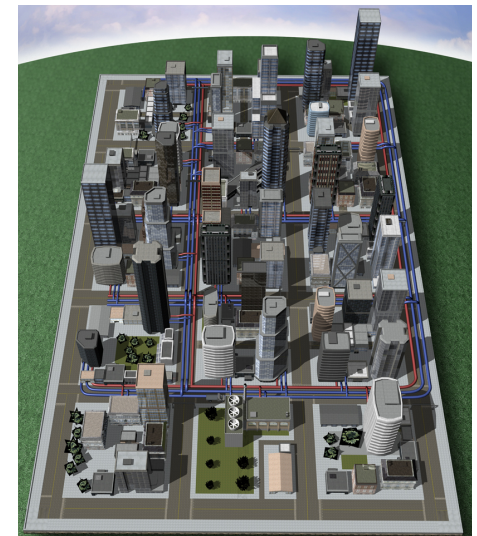
Department of Chemical and Biological Engineering  
University of Wisconsin-Madison

<http://zavalab.engr.wisc.edu>

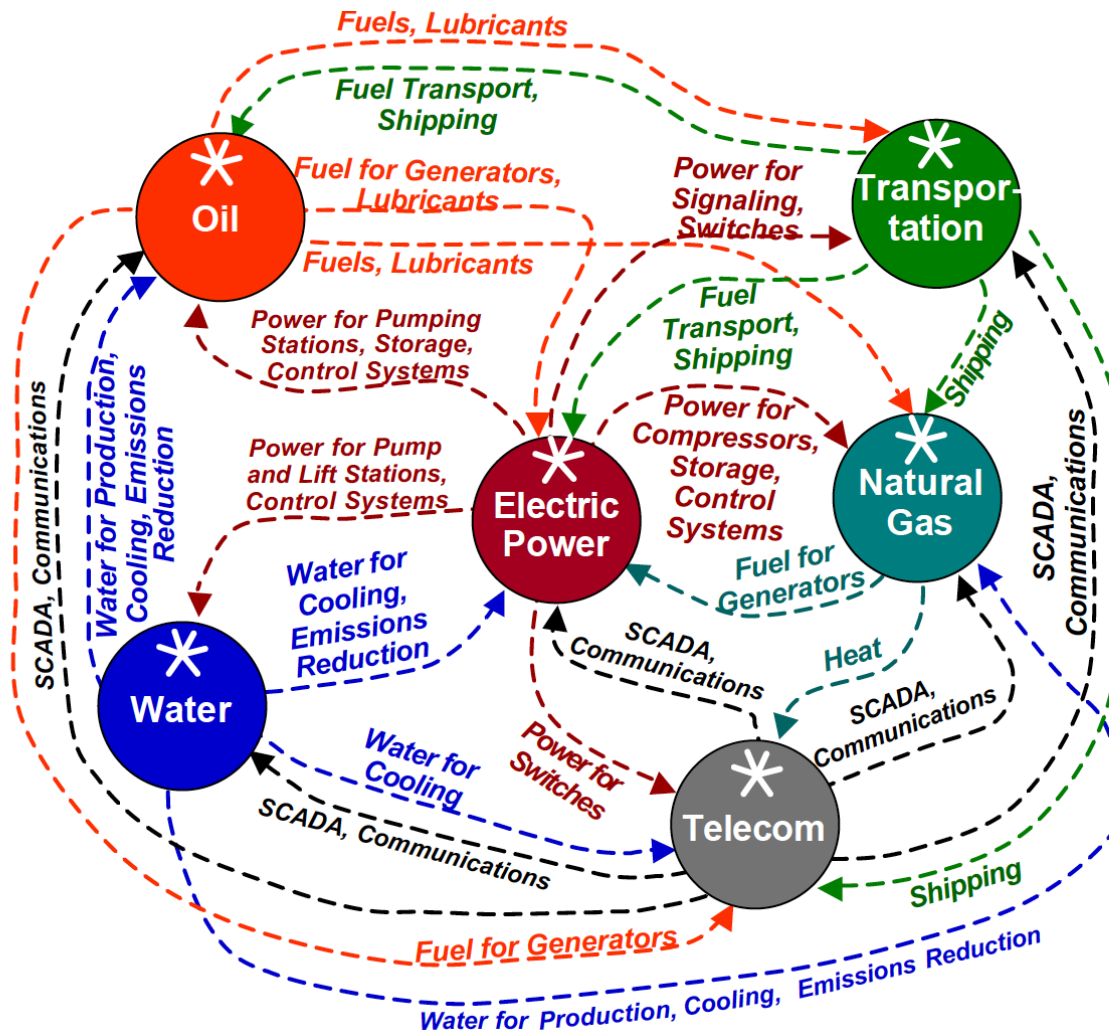
With: **A. Dowling (UWisc)**

**L.F. Fuentes J.M. Ponce (UMichoacan)**

**G. Ruiz-Mercado (EPA)**



# Interdependent Infrastructures





# Multi-Stakeholder/MultiObjective Optimization

## MultiObjective Optimization

$$\min_x \{f_1(x), f_2(x), \dots, f_N(x)\}$$

$$\text{s.t.} \quad g(x) \leq 0$$

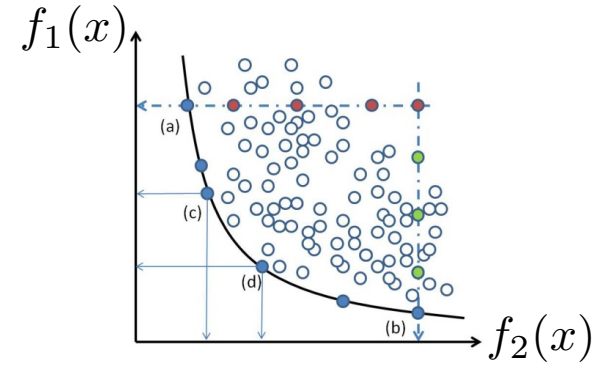
## Weighted Form

$$\min_x w_1 f_1(x) + w_2 f_2(x) + \dots + w_N f_N(x)$$

$$\text{s.t.} \quad g(x) \leq 0$$

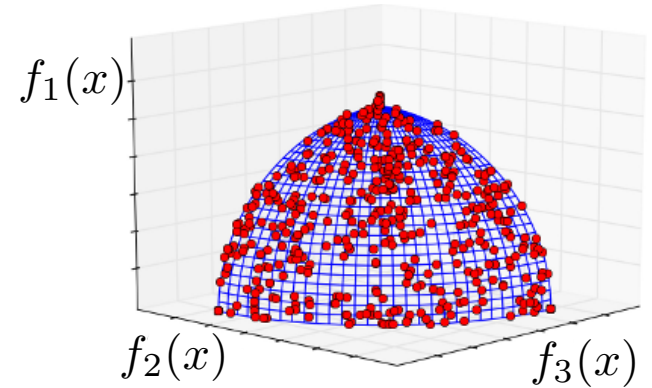
$$\min_x \mathbf{w}^T \mathbf{f}(x)$$

$$\text{s.t.} \quad g(x) \leq 0$$



## Technical Issues:

- Multiple Decision-Makers → Ambiguity, Disagreement
- Multiple Objectives → Dimensionality



High Cost  
Low Environmental Impact

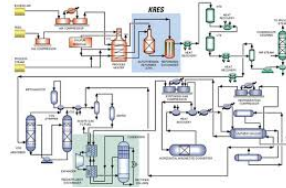
Low Cost  
High Environmental Impact

Power Plant I

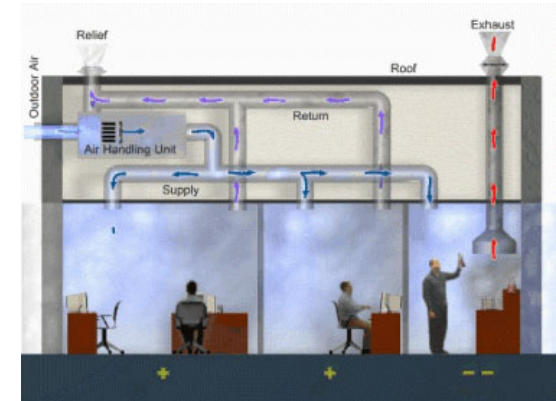
Power Plant II



Stakeholders



Stakeholders




Stakeholders

# Metrics Involved in Evaluation of Lightbulbs

**Incandescent Lamp**

60 Watt  
900 Lumens  
1,000 lifetime hours

~ 22 Incandescent lamps



**Compact Fluorescent Lamp**

15 Watt  
900 Lumens  
8,500 lifetime hours


~ 3 CFLs



**LED Lamp**

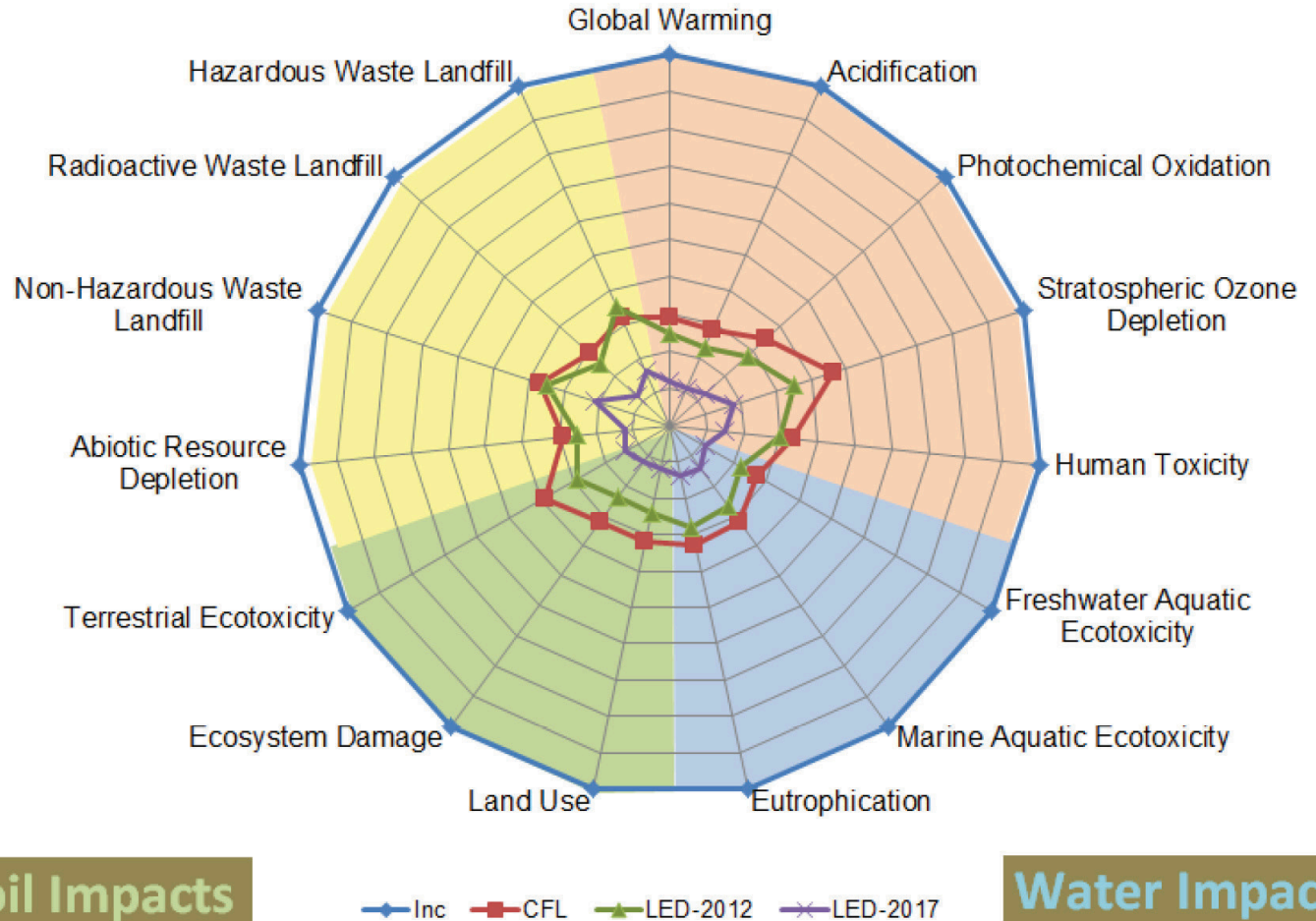
12.5 Watt  
800 Lumens  
25,000 lifetime hours

~ 1 LED lamp



## Resource Impacts

## Air Impacts



## Soil Impacts

## Water Impacts

# Multiobjective Optimization

## MOO

$$\min_{x \in \mathcal{X}} (f_1(x), f_2(x), \dots, f_n(x))$$

## Utopia Point

$$\underline{f}_i := \min_{x \in \mathcal{X}} f_i(x), \quad i \in \mathcal{O} := \{1..n\}$$

$$\underline{x}_i := \operatorname{argmin}_{x \in \mathcal{X}} f_i(x), \quad i \in \mathcal{O}$$

## Nadir Point

$$\bar{f}_i := \max\{f_i(\underline{x}_1), f_i(\underline{x}_2), \dots, f_i(\underline{x}_n)\}, \quad i \in \mathcal{O}$$

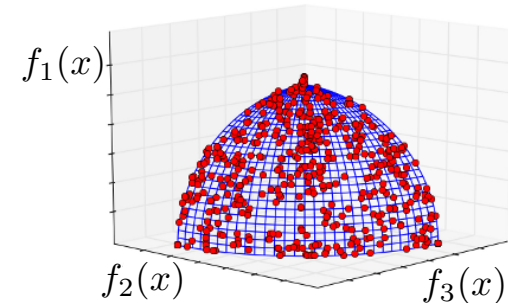
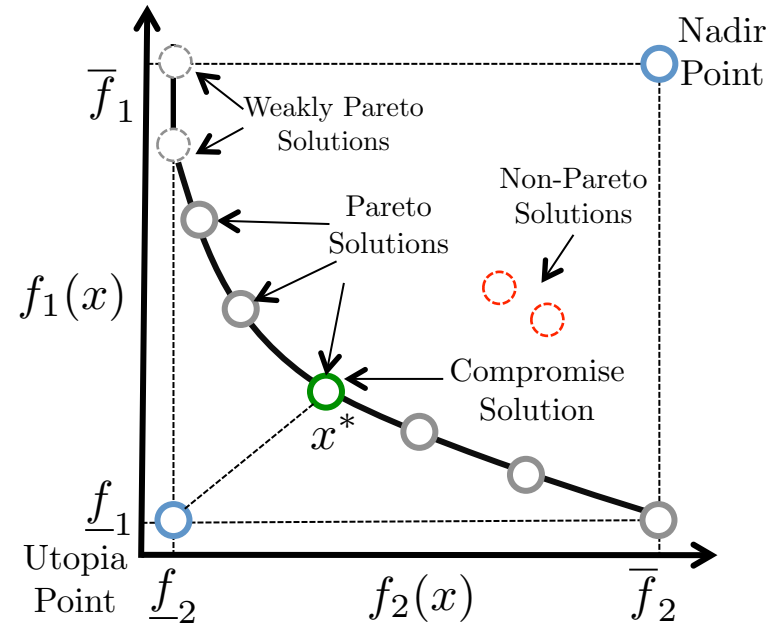
## Re-Scaling

$$\hat{f}_i(x) \leftarrow \frac{f_i(x) - \underline{f}_i}{\bar{f}_i - \underline{f}_i}, \quad i \in \mathcal{O}$$

## Compromise Solution

$$x^* = \min_{x \in \mathcal{X}} \|\mathbf{f}(x)\|_p$$

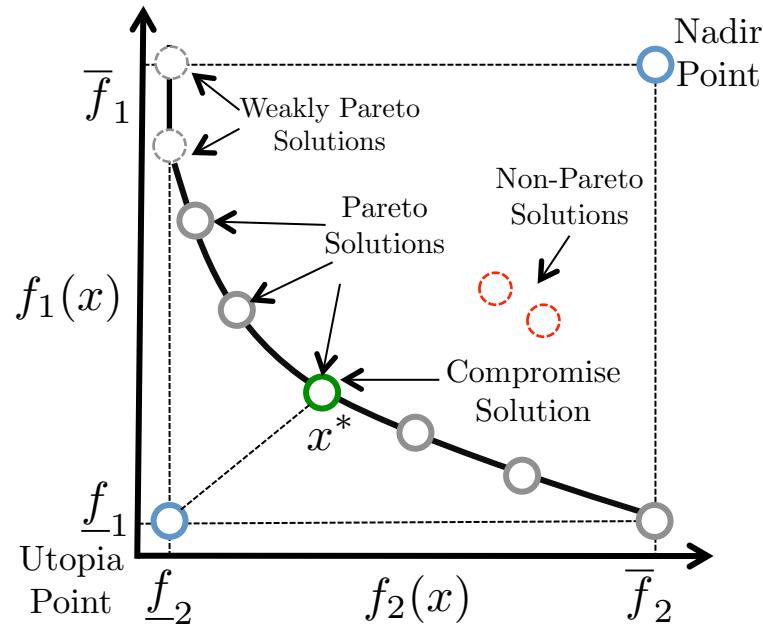
$$\mathbf{f}(x) := (f_1(x), f_2(x), \dots, f_n(x))$$



## Issues:

- **Ambiguity:** Meaning of Compromise?
- **Dimensionality:** Construct Pareto Set?

# Multiobjective Optimization



**Definition: (Weak Pareto Optimality)** A decision  $x^*$  with  $f_i(x^*)$ ,  $i \in \mathcal{O}$  is a *weakly Pareto optimal* solution of MOO if there does not exist an alternative solution  $\bar{x}$  with objectives  $f_i(\bar{x})$ ,  $i \in \mathcal{O}$  satisfying  $f_i(\bar{x}) < f_i(x^*)$  for all  $i \in \mathcal{O}$ .

**Definition: (Pareto Optimality)** A decision  $x^*$  with  $f_i(x^*)$ ,  $i \in \mathcal{O}$  is a *Pareto optimal* solution of MOO if there does not exist an alternative solution  $\bar{x}$  with objectives  $f_i(\bar{x})$ ,  $i \in \mathcal{O}$  satisfying  $f_i(\bar{x}) \leq f_i(x^*)$  for all  $i \in \mathcal{O}$  and at least one index  $i$  satisfying  $f_i(\bar{x}) < f_i(x^*)$ .



# Multistakeholder Optimization

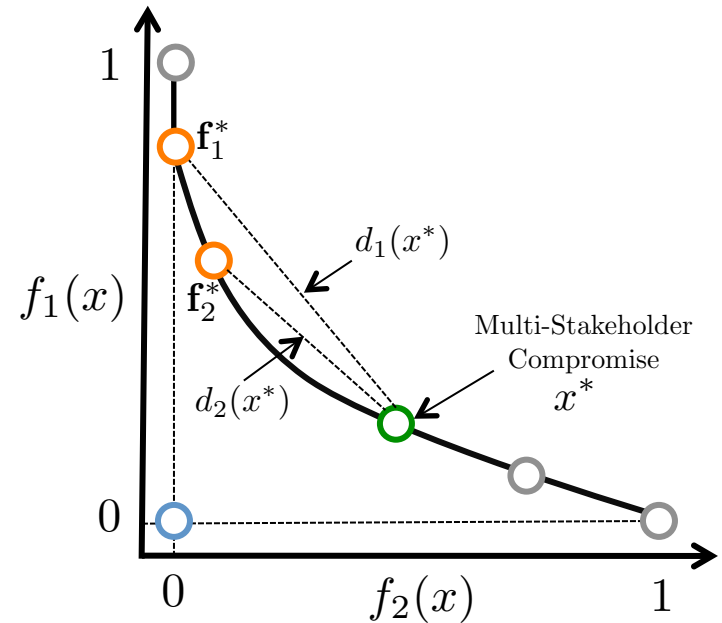
## Ideal Stakeholder Solution

$$x_j^* := \operatorname{argmin}_{x \in \mathcal{X}} \mathbf{w}_j^T \mathbf{f}(x), \quad j \in \mathcal{S} := \{1..m\}$$

$\mathbf{w}_j$  : Stakeholder Priority Vector

## Stakeholder Dissatisfaction Function

$$\begin{aligned} d_j(x) &:= \mathbf{w}_j^T (\mathbf{f}(x) - \mathbf{f}_j^*) \\ &= \mathbf{w}_j^T \mathbf{f}(x) - \mathbf{w}_j^T \mathbf{f}_j^* \end{aligned} \quad \mathbf{f}_j^* := \mathbf{f}(x_j^*)$$



## Average Dissatisfaction *Dyer, 1992*

$$\min_{x \in \mathcal{X}} \frac{1}{m} \sum_{j \in \mathcal{S}} d_j(x)$$

## Worst-Case Dissatisfaction *Mehrotra, 2012*

$$\min_{x \in \mathcal{X}} \max_{j \in \mathcal{S}} \{d_j(x)\}$$

## Conditional Value-at-Risk *This Work*

$$\min_{x \in \mathcal{X}} \operatorname{CVaR}_\alpha (d(x))$$

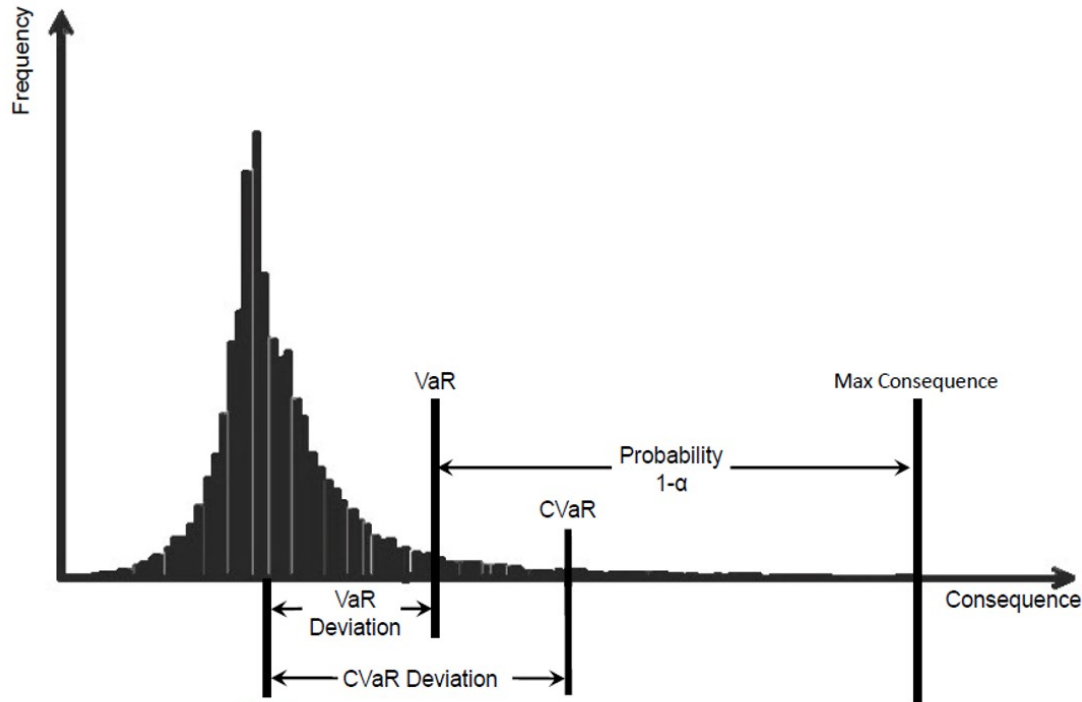
**Key Observation:** Interpret Opinions as Samples from Population

# Conditional Value At Risk (CVaR) Rockafellar & Uryasev, 2000

$$\begin{aligned} \text{CVaR}_\alpha[d(x)] \\ = \min_{\nu} \frac{1}{m} \sum_{j=1}^m \left[ \nu + \frac{1}{1-\alpha} [d_j(x) - \nu]_+ \right] \end{aligned}$$

**Key Property:**

$$\begin{aligned} \lim_{\alpha \rightarrow 0} \text{CVaR}_\alpha[d(x)] &= \frac{1}{m} \sum_{j \in \mathcal{S}} d_j(x) \\ \lim_{\alpha \rightarrow 1} \text{CVaR}_\alpha[d(x)] &= \max_{j \in \mathcal{S}} \{d_j(x)\} \end{aligned}$$



**Question:** Are CVaR Solutions Pareto Optimal?

# Geometric Interpretation

## Disagreement Vector

$$d_j(x) := \mathbf{w}_j^T (\mathbf{f}_j(x) - \mathbf{f}_j^*), j \in \mathcal{S}$$

$$\mathbf{d}(x) := [d_1(x), d_2(x), \dots, d_m(x)]$$

**Definition: (Scaled  $L_p$  norm).** Consider a fixed decision  $x \in \mathfrak{R}^{n_x}$  and the dissatisfaction vector  $\mathbf{d}(x) \in \mathfrak{R}^m$ . The scaled  $L_p$  norm (denoted as  $L_p^m$ ) of  $\mathbf{d}(x)$  is defined as,

$$\|\mathbf{d}(x)\|_p^m := \left( \frac{1}{m} \sum_{j=1}^m |d_j(x)|^p \right)^{\frac{1}{p}}, p \geq 1.$$

The scaled  $L_p$  norm has the following extreme cases,

$$\|\mathbf{d}(x)\|_1^m = \frac{1}{m} \sum_{j=1}^m |d_j(x)|$$

$$\|\mathbf{d}(x)\|_\infty^m = \max_j |d_j(x)|.$$

# Geometric Interpretation

$$\min_{x \in \mathcal{X}} \frac{1}{m} \sum_{j \in \mathcal{S}} d_j(x)$$

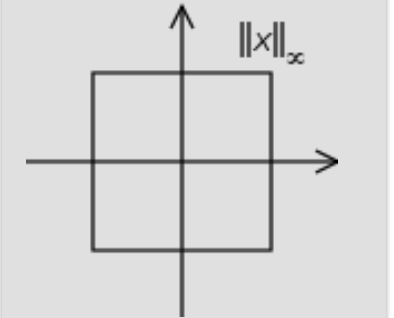
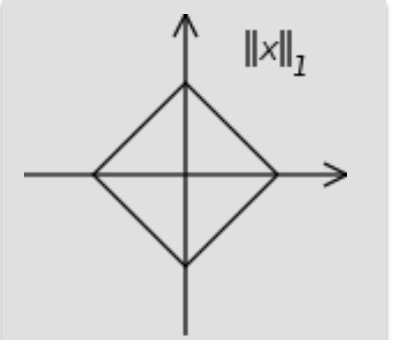
$$\min_{x \in \mathcal{X}} \max_{j \in \mathcal{S}} \{d_j(x)\}$$

$$\min_{x \in \mathcal{X}} \text{CVaR}_\alpha(d(x))$$

$$\min_{x \in \mathcal{X}} \|\mathbf{d}(x)\|_1^m$$

$$\min_{x \in \mathcal{X}} \|\mathbf{d}(x)\|_\infty^m$$

?



?



**Definition: (Scaled CVaR norm).** Consider the vector  $\mathbf{d}(x) \in \mathbb{R}^m$  and assume (without loss of generality) that  $d_1(x) \leq d_2(x) \leq \dots \leq d_m(x)$  holds. Define also the scalars  $\alpha_j := \frac{j}{m}$ ,  $j = 0, \dots, m-1$ . The *scaled CVaR norm* of vector  $\mathbf{d}(x)$  with parameter  $\alpha_j$  is defined as,

$$\ll \mathbf{d}(x) \gg_{\alpha_j}^m := \frac{1}{m-j} \sum_{i=j+1}^m d_i(x).$$

## Norm Conditions:

**Homogeneity:**

$$\rho(\lambda \mathbf{x}) = \lambda \rho(\mathbf{x})$$

**Subadditivity:**

$$\rho(\mathbf{x}_1, \mathbf{x}_2) \leq \rho(\mathbf{x}_1) + \rho(\mathbf{x}_2)$$

**Normalized:**

$$\rho(0) = 0$$

**CVaR Norm Properties:** For fixed  $x$  consider the discrete random variable  $d(x)$  with outcomes  $d_1(x), d_2(x), \dots, d_m(x)$ , probabilities  $p_j = \frac{1}{m}$ ,  $j \in \mathcal{S}$ , and the corresponding vector  $\mathbf{d}(x)$ .

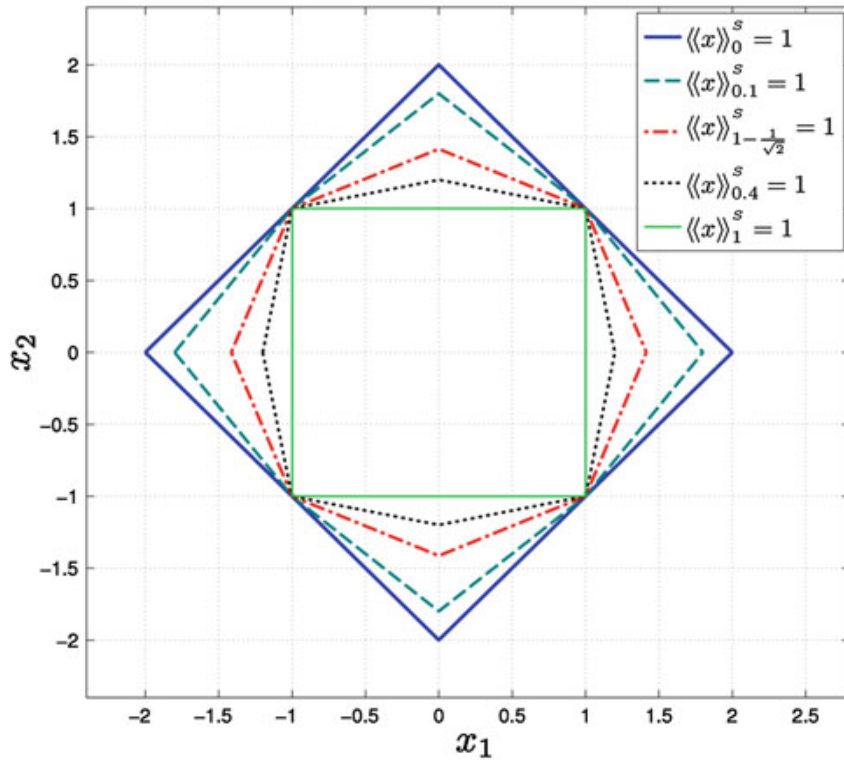
- i)  $\ll \cdot \gg_{\alpha}^m$  is a Norm for  $\alpha \in [0, 1]$
- ii)  $\ll \mathbf{d}(x) \gg_{\alpha}^m = CVaR_{\alpha}(d(x))$  for  $\alpha \in [0, 1]$ .
- iii)  $\ll \mathbf{d}(x) \gg_0^m = \|\mathbf{d}(x)\|_1^m$
- iv)  $\ll \mathbf{d}(x) \gg_{\alpha}^m = \|\mathbf{d}(x)\|_{\infty}^m$  for  $\frac{m-1}{m} \leq \alpha \leq 1$ .
- v) For  $\alpha$  such that  $\alpha_j < \alpha < \alpha_{j+1}$ ,  $j = 0, \dots, m-2$ :

$$\ll \mathbf{d}(x) \gg_{\alpha}^m = \mu \ll \mathbf{d}(x) \gg_{\alpha_j}^m + (1 - \mu) \ll \mathbf{d}(x) \gg_{\alpha_{j+1}}^m$$

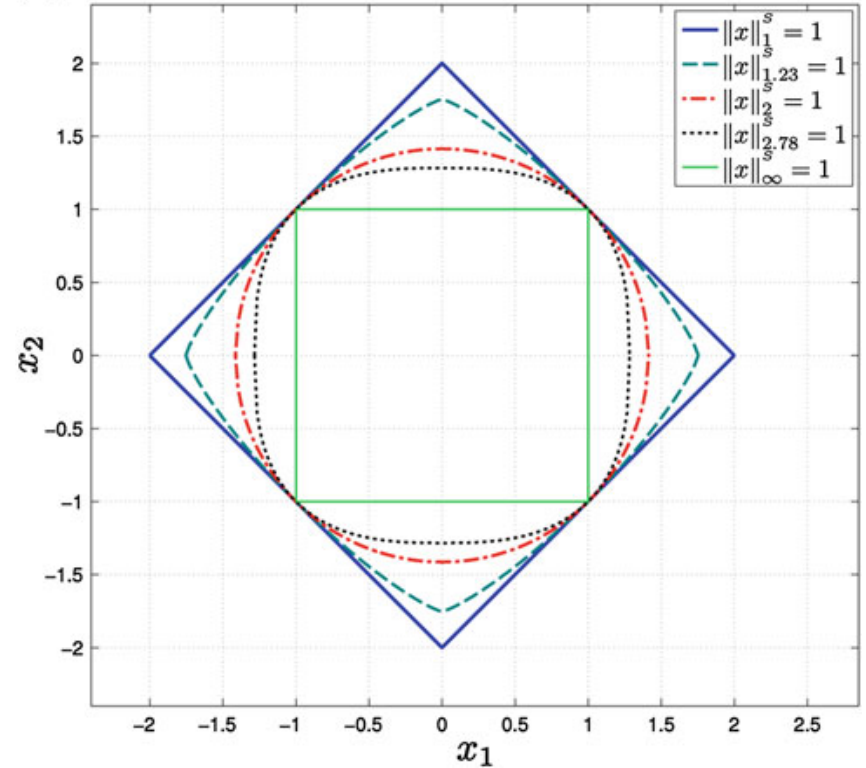
$$\text{with } \mu := \frac{(\alpha_{j+1} - \alpha)(1 - \alpha_j)}{(\alpha_{j+1} - \alpha_j)(1 - \alpha)}.$$

- vi)  $\ll \mathbf{d}(x) \gg_{\alpha}^m$  is a nondecreasing function of  $\alpha \in [0, 1]$ .

## CVaR Norm



## $L_p^S$ Norm



**CVaR Norm Combinatorial But Can be Computed Using Continuous Formulation**

$$\min_{x \in \mathcal{X}} \text{CVaR}_\alpha [d(x)] \iff \min_{x \in \mathcal{X}} \langle\langle \mathbf{d}(x) \rangle\rangle_\alpha^m \iff \min_{(x,y) \in \mathcal{X} \times \mathbb{R}} y + \frac{1}{(1-\alpha)m} \sum_{j=1}^m (d_j(x) - y)_+$$

# Pareto Optimality of CVaR Solutions

## MOO

$$\min_{x \in \mathcal{X}} (f_1(x), f_2(x), \dots, f_n(x))$$

## CVaR Problem

$$\min_{x \in \mathcal{X}} \ll \mathbf{d}(x) \gg_{\alpha}^m \iff \min_{(x,y) \in \mathcal{X} \times \mathcal{R}} y + \frac{1}{(1-\alpha)m} \sum_{j=1}^m (d_j(x) - y)_+$$

**Lemma:** Consider decisions  $\bar{x}, x^*$  with corresponding  $d_j(\bar{x}), d_j(x^*)$ . We have:

$$d_j(\bar{x}) < d_j(x^*), j \in \mathcal{S} \implies \ll \mathbf{d}(\bar{x}) \gg_{\alpha}^m < \ll \mathbf{d}(x^*) \gg_{\alpha}^m, \alpha \in [0, 1].$$

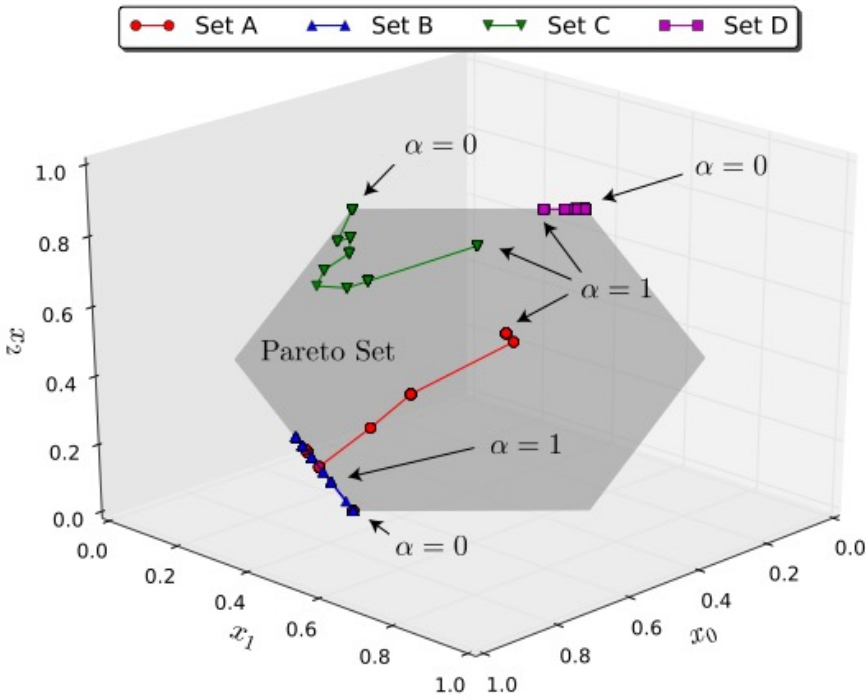
**Theorem:** Let  $x^*$  be a solution of the CVaR problem. We have:

1. If  $\mathbf{w}_j^{(i)} \geq 0, j \in \mathcal{S}, i \in \mathcal{O}$  then  $x^*$  is weak Pareto for MOO  $\forall \alpha \in [0, 1]$ .
2. If  $\mathbf{w}_j^{(i)} > 0, j \in \mathcal{S}, i \in \mathcal{O}$  then  $x^*$  is Pareto for MOO  $\forall \alpha \in [0, 1]$ .

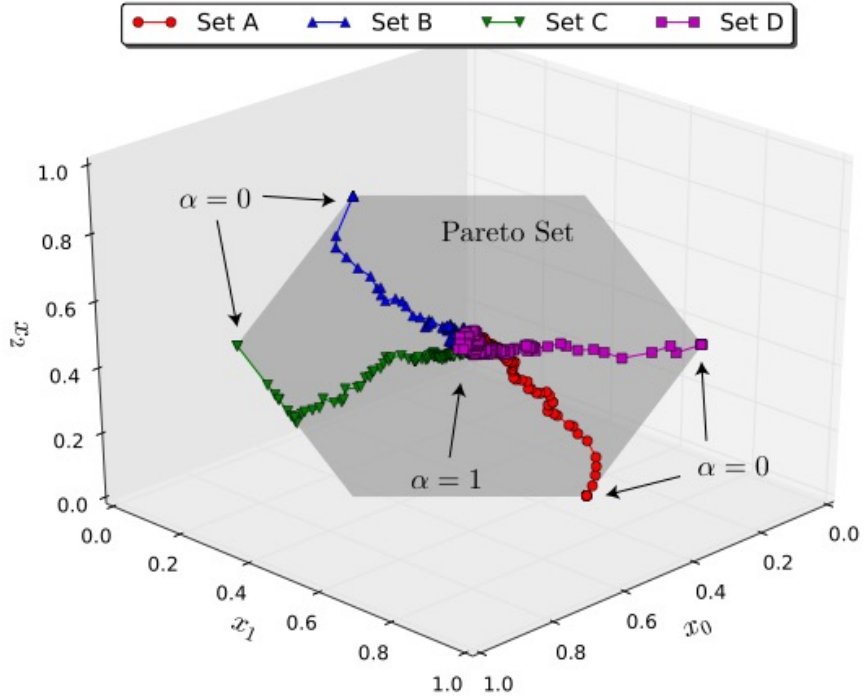
**Proof of Weak Pareto:**  $x^*$  is optimal for CVaR and thus  $\ll \mathbf{d}(x) \gg_{\alpha}^m \geq \ll \mathbf{d}(x^*) \gg_{\alpha}^m$  for any  $x \in \mathcal{X}$ . Assume  $x^*$  is *not* weakly Pareto optimal. This implies that there exists an alternative  $\bar{x} \in \mathcal{X}$  such that  $f_i(\bar{x}) < f_i(x^*)$  for all  $i \in \mathcal{O}$ . We thus have that  $\mathbf{w}_j^T \mathbf{f}(\bar{x}) < \mathbf{w}_j^T \mathbf{f}(x^*)$  for any  $\mathbf{w}_j$  with  $\mathbf{w}_j^{(i)} \geq 0$  and  $\sum_{i \in \mathcal{O}} \mathbf{w}_j^{(i)} = 1$ . Consequently,  $d_j(\bar{x}) < d_j(x^*)$  for all  $j \in \mathcal{S}$ . From previous Lemma we also have that  $\ll \mathbf{d}(\bar{x}) \gg_{\alpha}^m < \ll \mathbf{d}(x^*) \gg_{\alpha}^m$ . We thus have that the alternative  $\bar{x}$  cannot exist and we have a contradiction.

# Illustrative Example

$$\begin{aligned} \min_x \quad & (x_0, x_1, x_2) \\ \text{s.t.} \quad & x_0 + x_1 + x_2 \geq 1 \\ & 0 \leq x_0, x_1, x_2 \leq \frac{2}{3} \end{aligned}$$



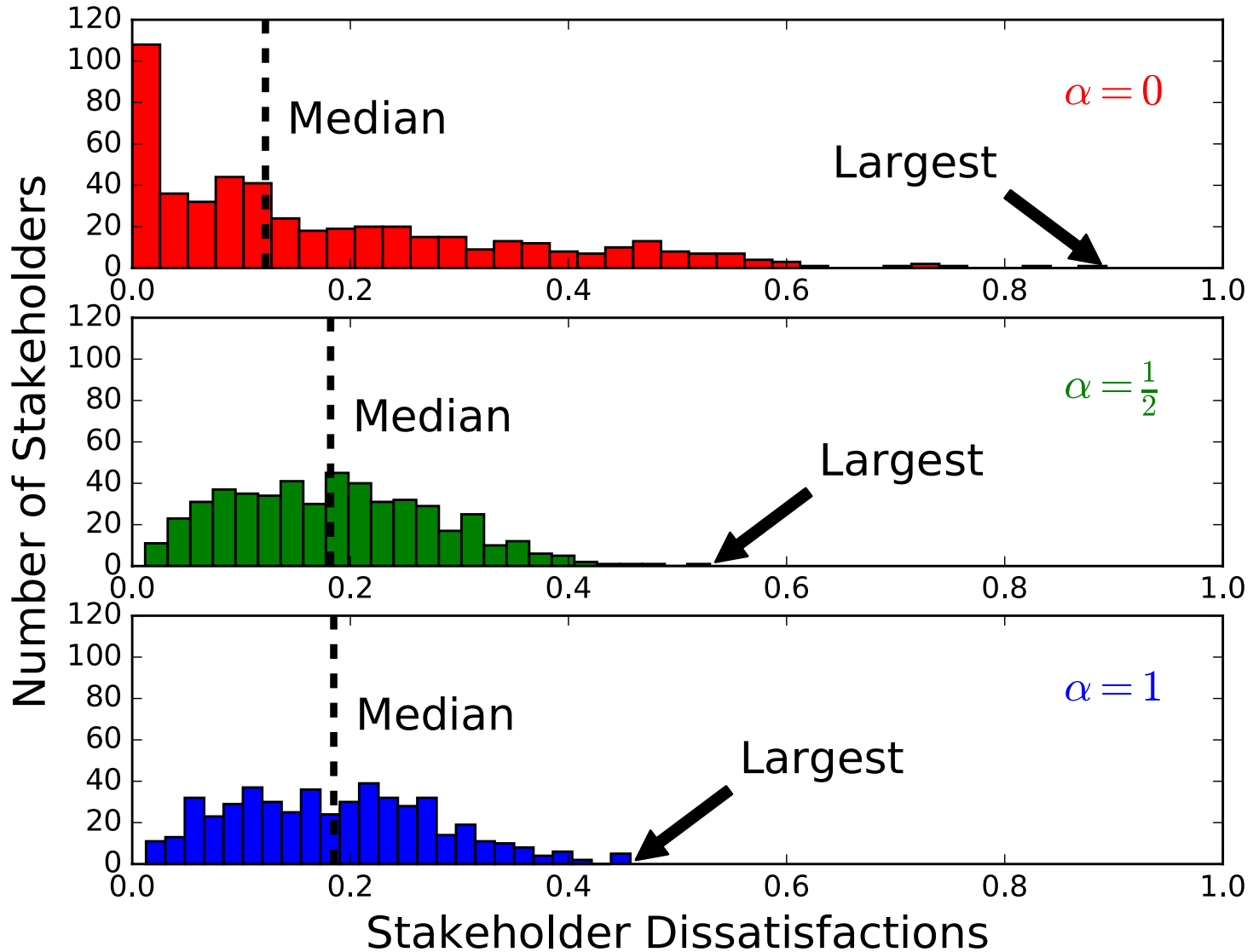
10 Stakeholders per Sample



500 Stakeholders per Sample



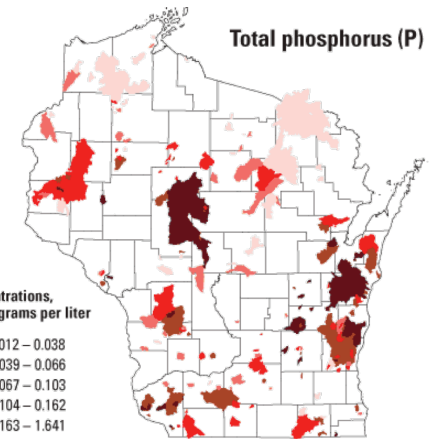
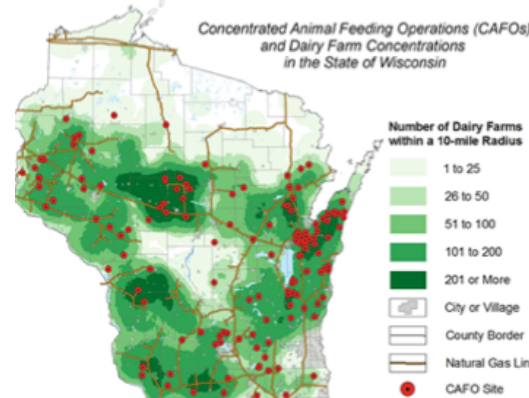
# Illustrative Example



# BioGas Facility Location



Source	Methane Potential (tonnes/yr)
Wastewater	2,339,339
Landfills*	2,454,974
Animal manure	1,905,253
IIC organic waste	1,157,883
<b>Total</b>	<b>7,857,449</b>



## Some Info:

U.S. Farm Animals Produce 2 Times the Amount of Waste of Entire Human Population

Single Dairy Cow Generates 20 tons of Waste/year

There are 9 Million Cows in the U.S. (1.2 Million in Wisconsin)

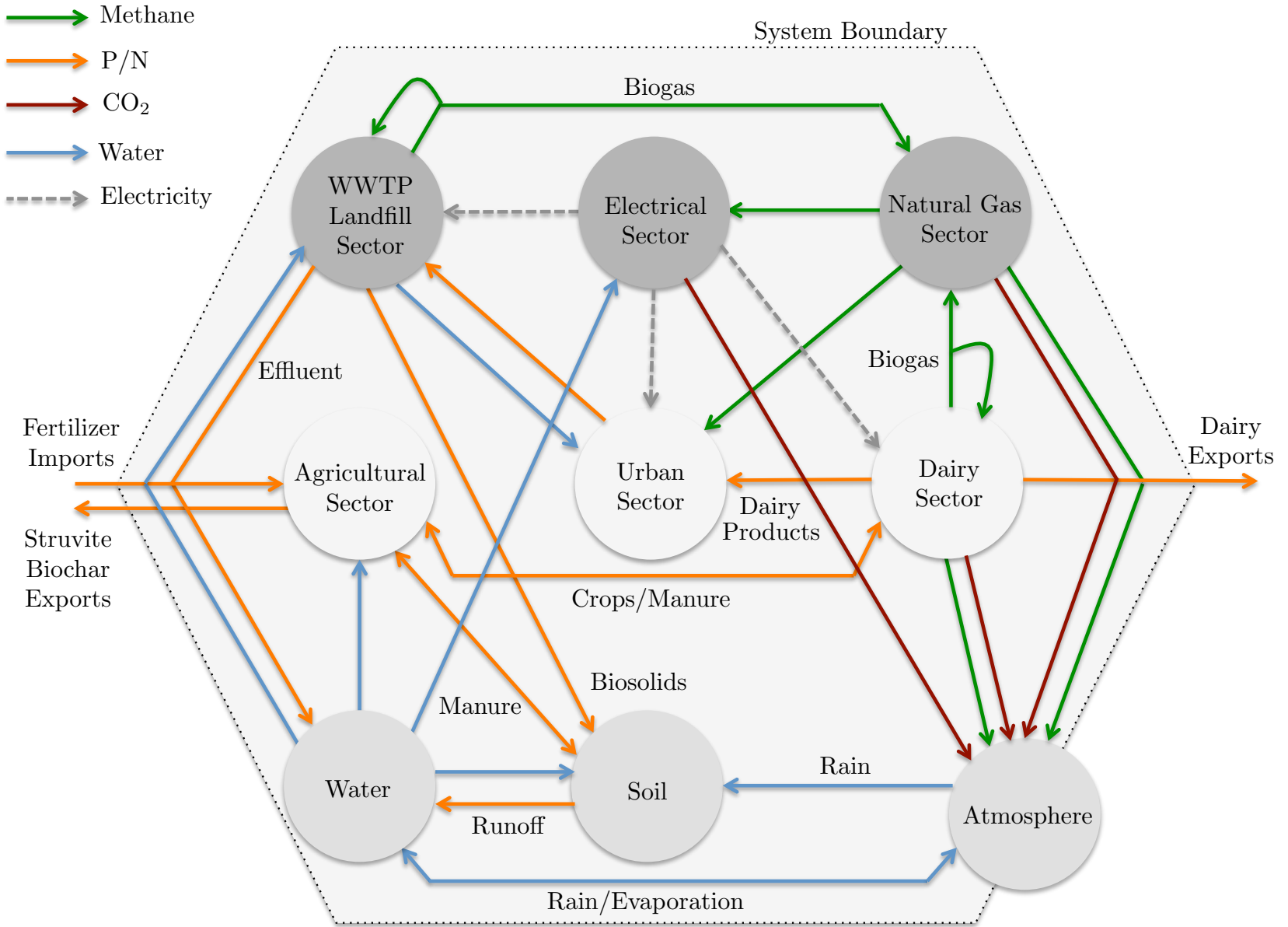
From EPA: 2,000 Farms Could Support Biogas from Waste (Less than 200 Installations)

## Challenges:

How to Reconcile Priorities (Emissions/Water/Health/Investment/Not-in-my-Backyard)?

How to Derive Fair Incentives/Regulations?

# BioGas Facility Location



# BioGas Facility Location (2 Objectives)

$$\max E = \sum_{j \in \mathcal{F}} E_j^P - \sum_{i \in \mathcal{F}, j \in \mathcal{B}} E_{i,j}^T - \sum_{j \in \mathcal{F}} E_j^U$$

**Emissions**

$$\max C = C^{e^-} - C^I - C^O - C^T$$

**Economics**

$$E^T = \sum_{i \in \mathcal{F}} \sum_{j \in \mathcal{B}} \alpha_{CO_2 Diesel} T_{i,j} d_{i,j}$$

**Transportation**

$$T_{i,j} = S_{i,j} / \bar{S}, \quad i \in \mathcal{F}, j \in \mathcal{B}$$

**Round Trips**

$$E^P = \sum_{j \in \mathcal{F}} \alpha_{CO_2 CH_4} \cdot \alpha_{CH_4 H} H_j^P$$

**Processed Waste**

$$E^U = \sum_{j \in \mathcal{F}} \alpha_{CO_2 CH_4} \cdot \alpha_{CH_4 H} H_j^U$$

**Unprocessed Waste**

$$C^I = \sum_{i \in \mathcal{T}} \sum_{j \in \mathcal{B}} c_i^I \cdot y_{i,j}$$

**Investment**

$$C^O = \sum_{i \in \mathcal{T}} \sum_{j \in \mathcal{B}} c_i^O \cdot W_{i,j}$$

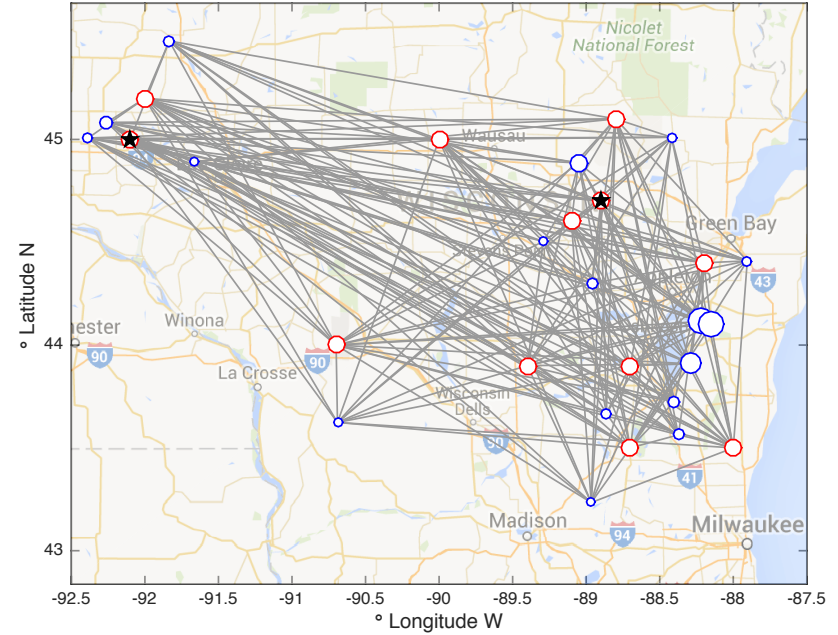
**Processing Cost**

$$C^T = \sum_{i \in \mathcal{F}} \sum_{j \in \mathcal{B}} c_{i,j}^T \cdot S_{i,j}$$

**Transportation Cost**

$$C^{e^-} = \sum_{i \in \mathcal{T}} \sum_{j \in \mathcal{B}} c_i^O \cdot c^{e^-} G_{i,j}$$

**Electricity Profit**



$$\sum_{j \in \mathcal{B}} S_{i,j} \leq \bar{F}_i, \quad i \in \mathcal{F}$$

**Balances**

$$\sum_{i \in \mathcal{F}} S_{i,j} = \sum_{i \in \mathcal{T}} W_{i,j}, \quad j \in \mathcal{B}$$

$$W_i^P = \sum_{j \in \mathcal{B}} S_{i,j}, \quad i \in \mathcal{F}$$

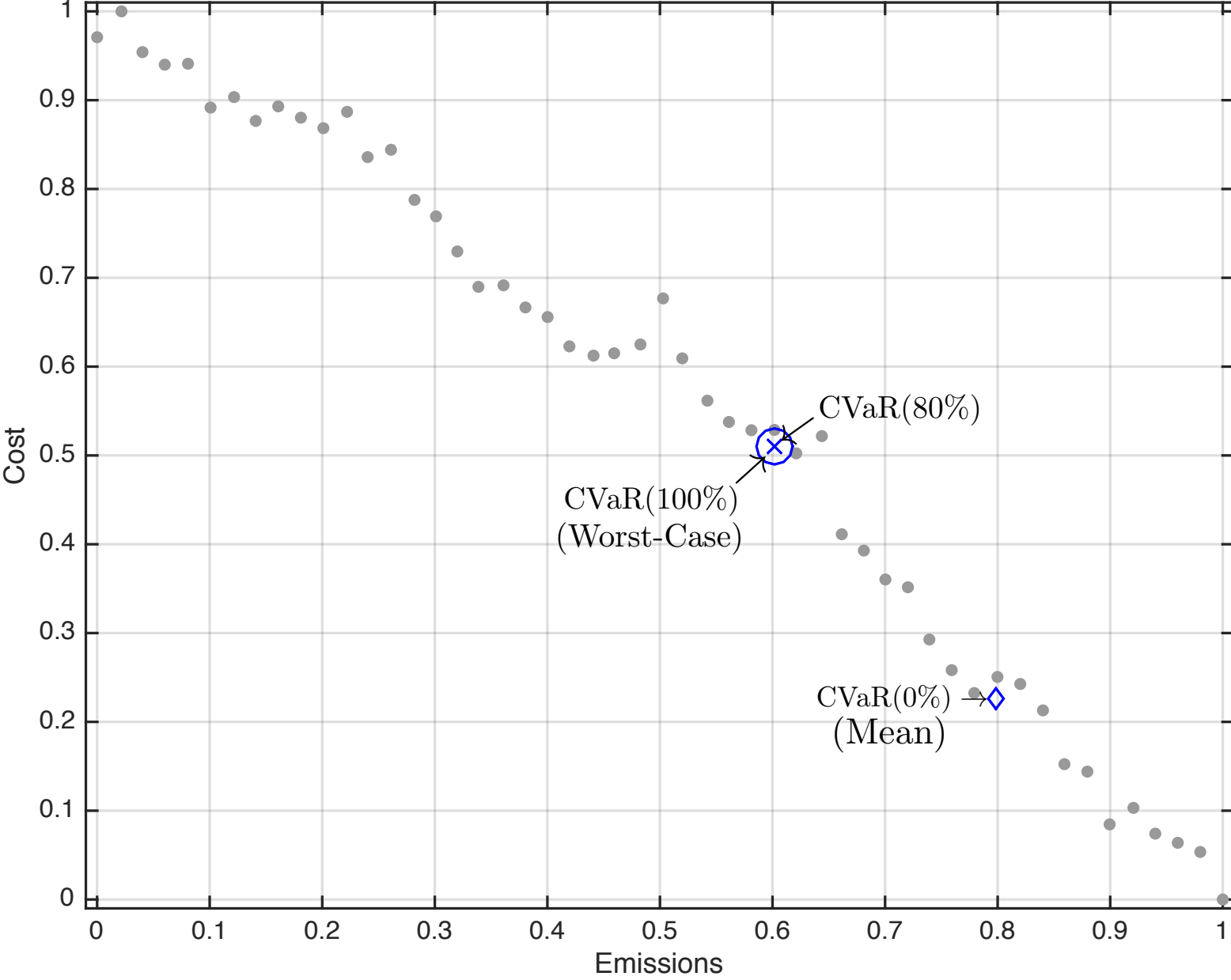
$$W_i^U = \bar{F}_i - \sum_{j \in \mathcal{B}} S_{i,j}, \quad i \in \mathcal{F}$$

$$G_{i,j} = \alpha_{GW} \cdot W_{i,j}, \quad i \in \mathcal{T}, j \in \mathcal{B}$$

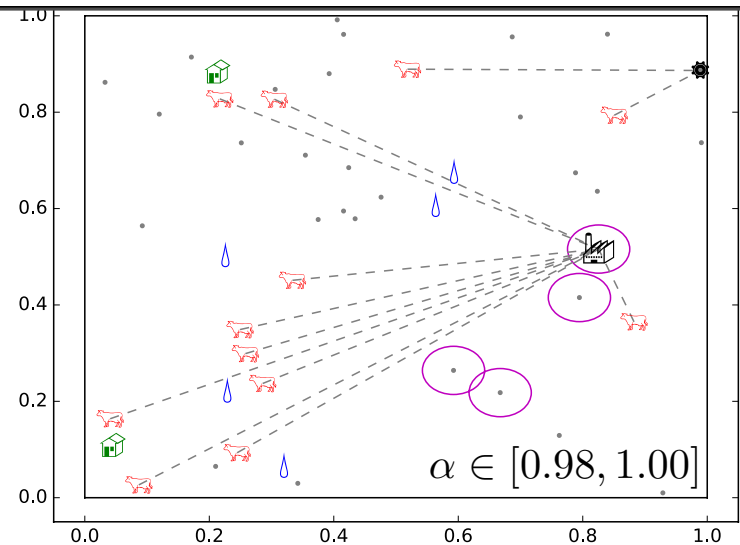
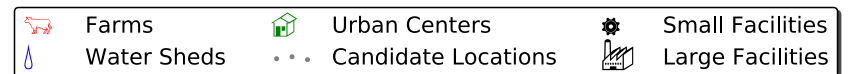
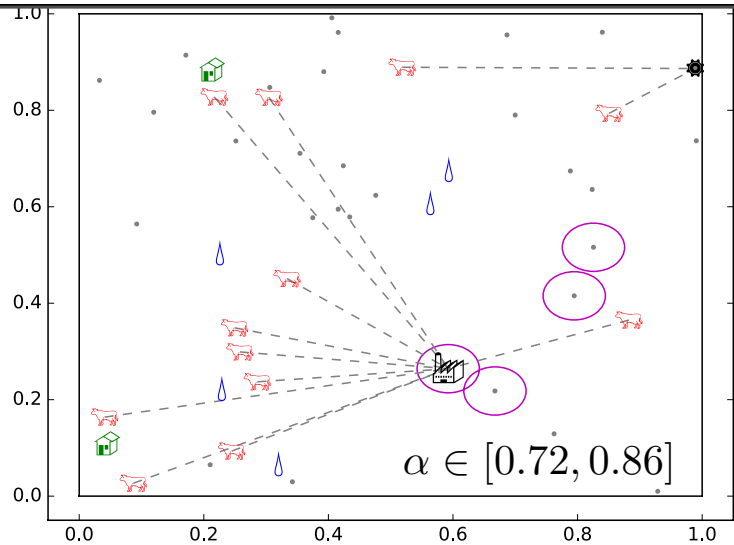
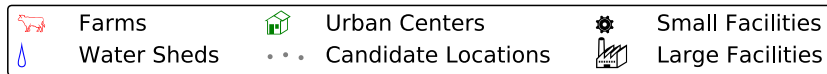
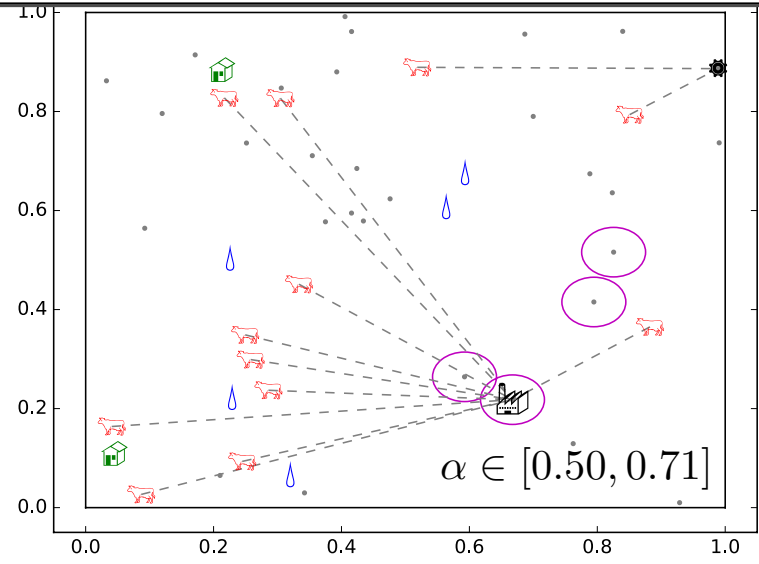
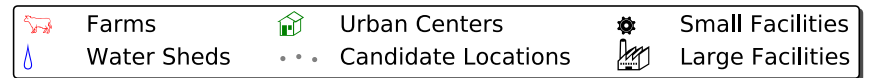
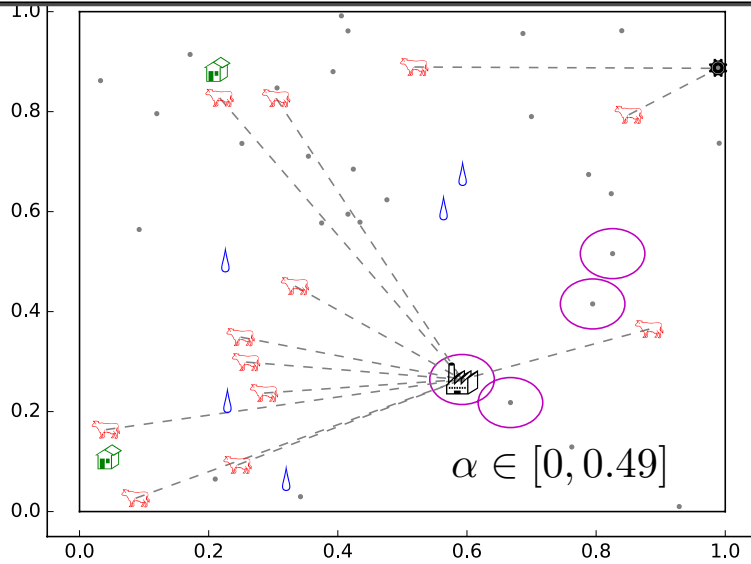
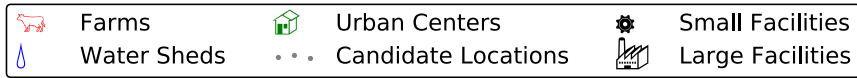
$$G_{i,j} \leq \bar{G}_i y_{i,j}, \quad i \in \mathcal{T}, j \in \mathcal{B}$$



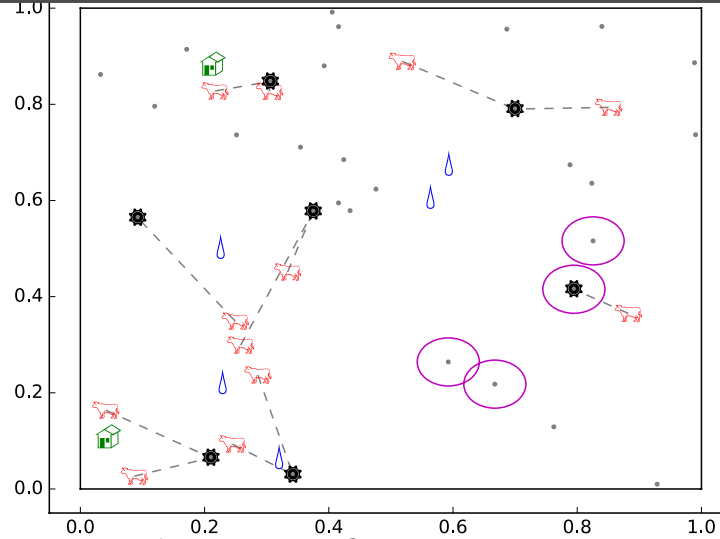
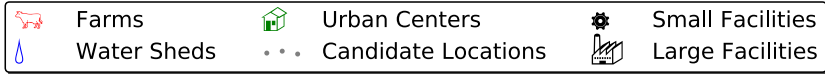
# BioGas Facility Location (2 Objectives)



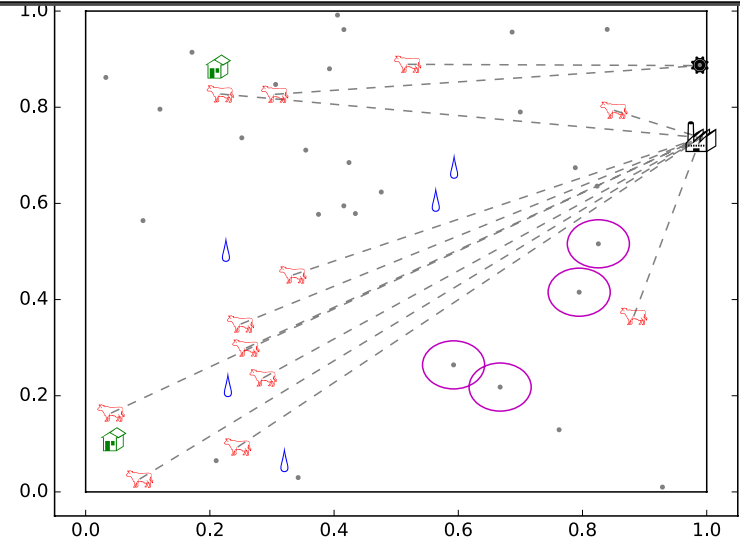
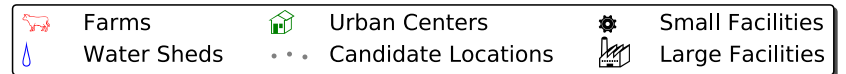
# Compromise Solutions (4 Objectives, 100 Stakeholders)



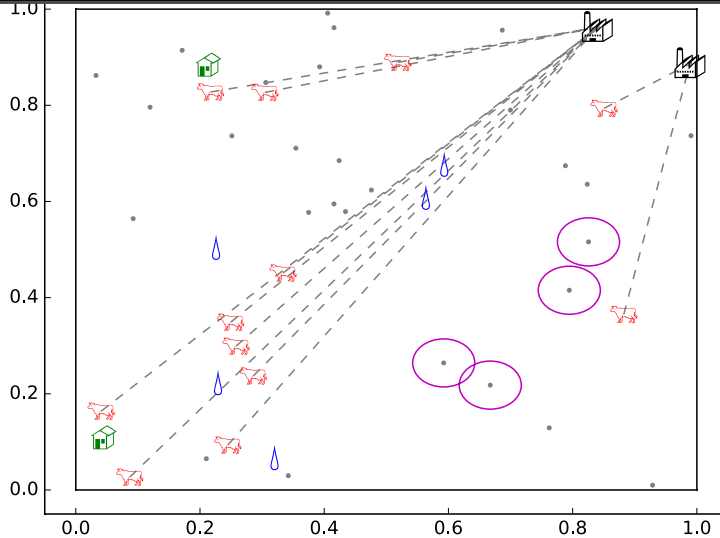
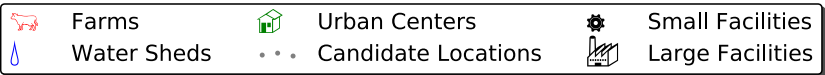
# Ideal Stakeholder Solutions



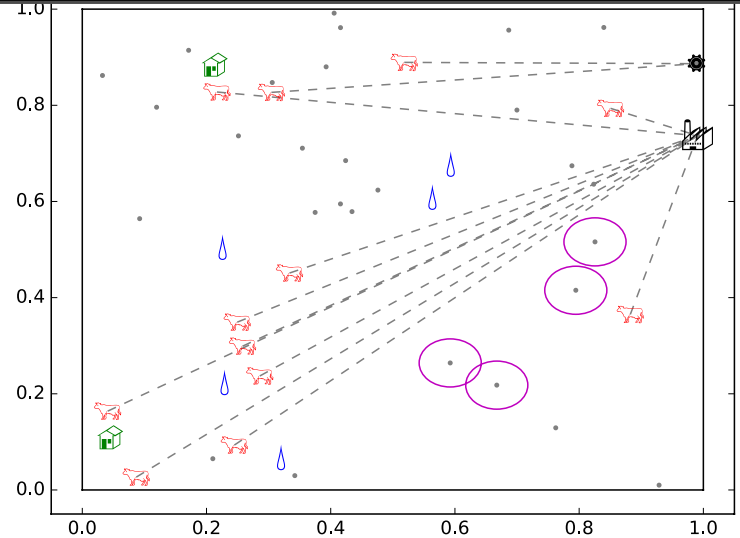
Distance from Farms



Distance from Watershed

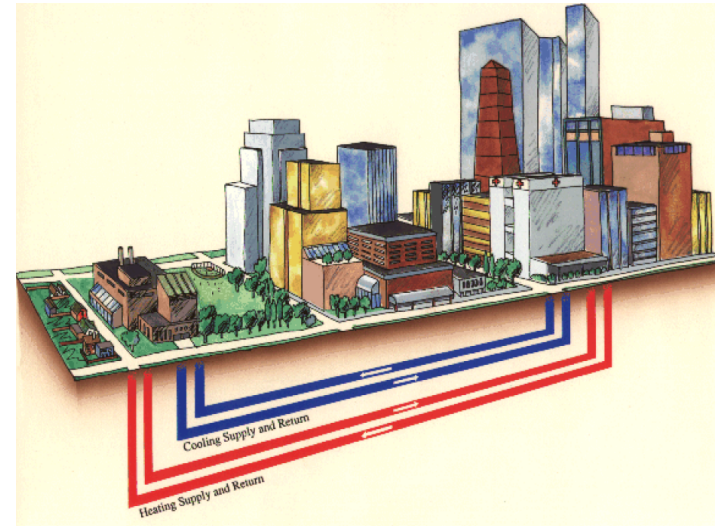
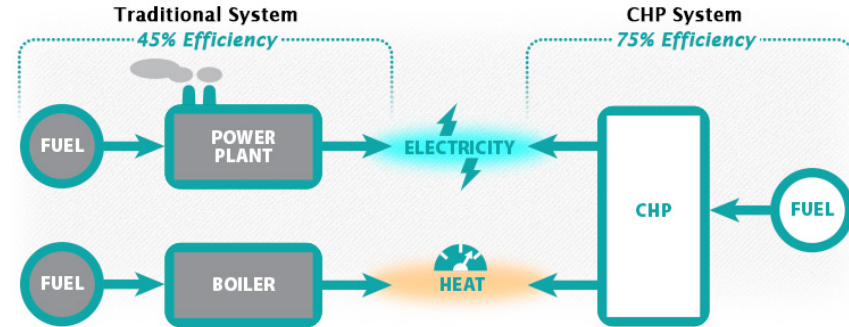
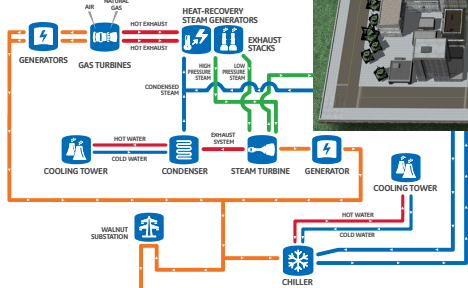
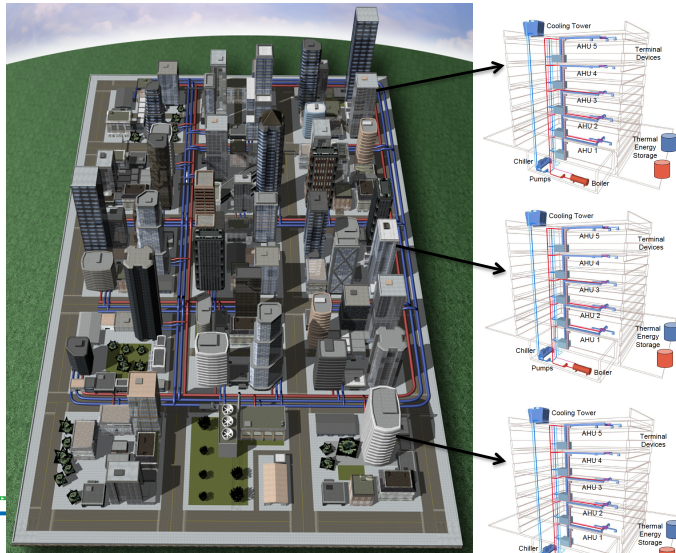


Distance from Urban



Capital Costs

# Combined Heat & Power (CHP) Units



## Some Info:

CHP Uses Heat Recovery to Simultaneously Provide Electricity, Heating, and Cooling

CHP Efficiency 70-80% vs. Traditional Power Plant Efficiency 40-50%

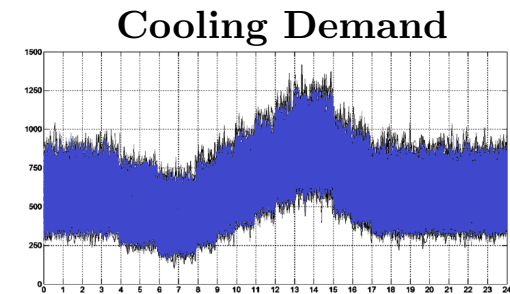
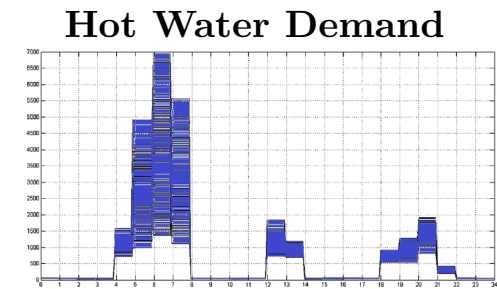
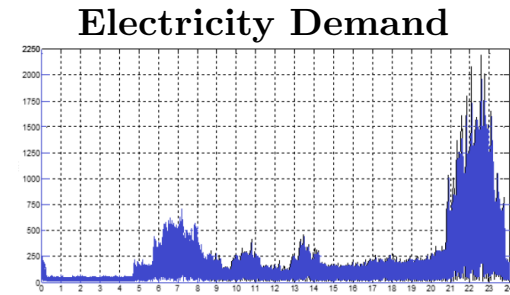
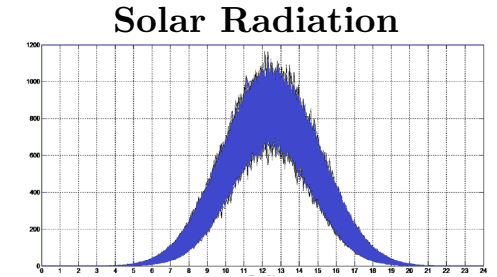
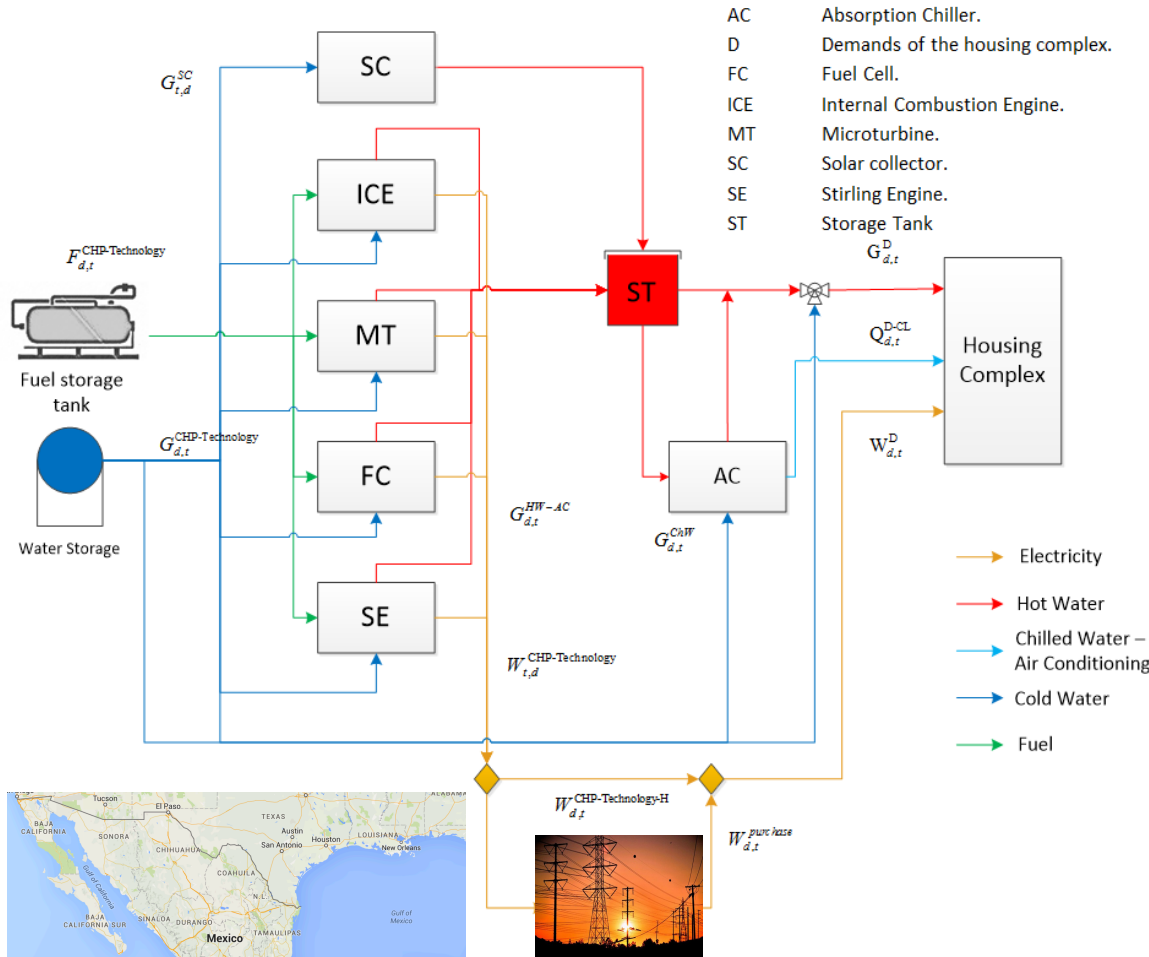
U.S. CHP Capacity To Increase from 80GW to 120 GW in 10 Years

## Design Challenges:

Capture Dynamic Patterns of Electricity/Cooling/Heating Demands

Many Emerging Technologies with Strong Trade-Offs (Investment, Emissions, Water)

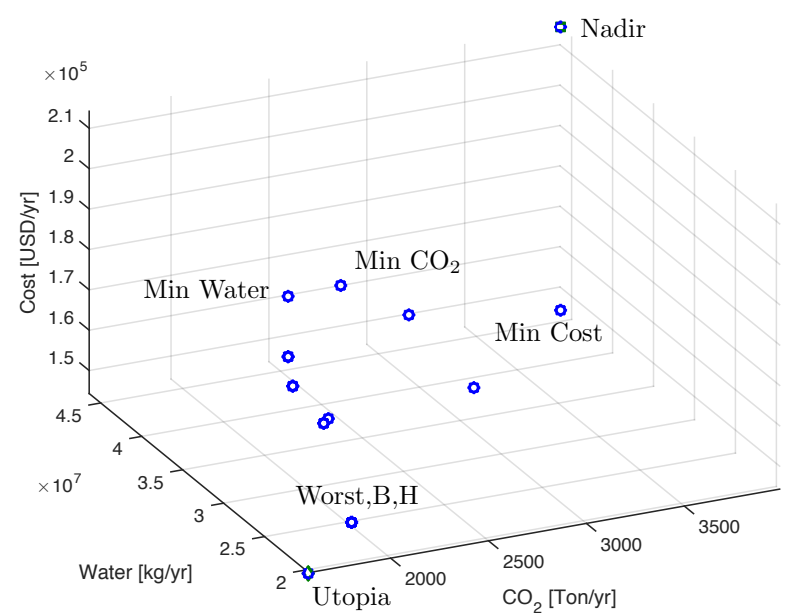
# Combined Heat & Power (CHP) Units



**Case Study in Pacific Coast of Mexico:**  
**Real Energy Demands & Weather Data for Housing Complex**  
**Housing Complex with 420 Units and 2,400 Inhabitants**

# CHP Units

Stakeholder	Cost	Emissions	Water
A	1/3	1/3	1/3
B	-	1/2	1/2
C	1/2	-	1/2
D	1/2	1/2	-
E	-	2/3	1/3
F	-	1/3	2/3
G	1/3	-	2/3
H	1/3	2/3	-
I	2/3	-	1/3
J	2/3	1/3	-



	Cost (USD/yr)	CO <sub>2</sub> (Ton/yr)	Water (Kg/yr)	CHP Tech	CHP Size (kWe)
Min Cost	144,307	3,987	46,411,000	ICE	335
Min Emissions	208,450	1,582	22,186,000	SE	182
Min Water	214,220	1,745	19,602,600	ICE	290
A	182,580	1,679	23,842,000	SE	180
B	144,310	2,016	24,771,000	ICE	285
C	147,120	3,168	37,260,000	ICE	280
D	180,630	1,655	19,603,000	ICE	287
E	193,340	1,582	22,186,000	SE	182
F	193,340	1,582	22,186,000	SE	182
G	184,910	2,482	28,952,000	MT	197
H	144,310	2,016	24,772,000	ICE	285
I	173,190	1,860	23,842,000	SE	181
J	180,630	1,655	19,603,000	ICE	287
Min Average	182,580	1,679	23,842,000	SE	180
Min Worst	144,310	2,016	24,771,000	ICE	285
Utopia	144,307	1,582	19,602,600		
Nadir	214,220	3,987	46,411,000		

# Coherent Risk Measures and Norms

## Coherency Conditions:

- **Homogeneity:**  $\rho(\lambda X) = \lambda\rho(X)$
- **Subadditivity:**  $\rho(X_1 + X_2) \leq \rho(X_1) + \rho(X_2)$
- **Normalized:**  $\rho(0) = 0$
- **Monotonicity:** If  $X_1 \leq X_2$  a.s. then  $\rho(X_1) \leq \rho(X_2)$

## Norm Conditions:

- **Homogeneity:**  $\rho(\lambda \mathbf{x}) = \lambda\rho(\mathbf{x})$
- **Subadditivity:**  $\rho(\mathbf{x}_1, \mathbf{x}_2) \leq \rho(\mathbf{x}_1) + \rho(\mathbf{x}_2)$
- **Normalized:**  $\rho(0) = 0$

$\iff$

## Incoherent Risk Measures:

- **Value at Risk:**  $\text{VaR}_\alpha(X) := \inf_{t \in \mathbb{R}} \{t : \Pr(X \leq t) \geq \alpha\}$
- **Mean-Standard-Deviation:**  $\text{M-SD}_\lambda = \mathbb{E}[X] + \lambda\sigma(X)^2$

(Violates Subadditivity)

(Violates Monotonicity)

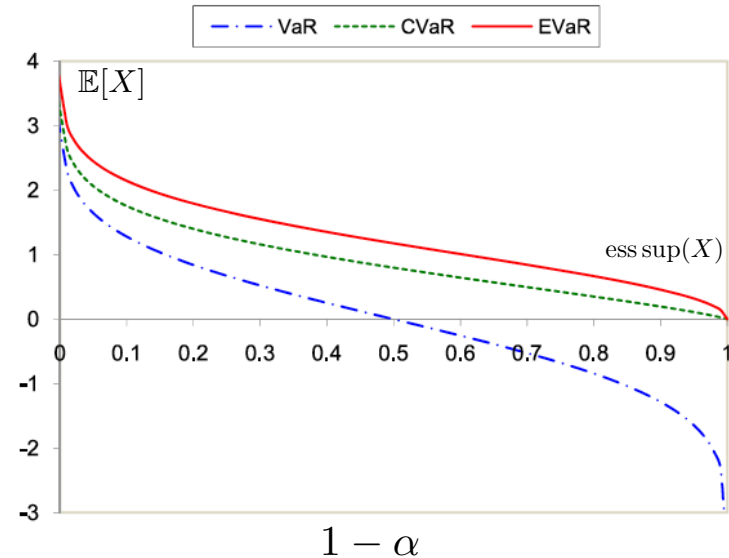
## Coherent Risk Measures:

- **Expected Value:**  $\mathbb{E}[X]$
- **Worst-Case Value:**  $\text{ess sup}(X)$
- **Conditional Value at Risk:**  $\inf_{t \in \mathbb{R}} \left\{ t + \frac{1}{1-\alpha} \mathbb{E}[(X - t)_+] \right\}$
- **Entropic Value at Risk:**  $\inf_{t > 0} \left\{ \frac{1}{t} \log \mathbb{E}[\exp(tX)] \right\}$

## Some Relationships:

$$\mathbb{E}[X] \leq \text{CVaR}_\alpha(X) \leq \text{EVaR}_\alpha(X) \leq \text{ess sup}(X)$$

$$\text{VaR}_\alpha(X) \leq \text{CVaR}_\alpha(X) \leq \text{EVaR}_\alpha(X)$$





# Generalized Entropy Index

## Generalized Entropy Index

$$GE_{\beta}(x) := \frac{1}{m\beta(\beta-1)} \sum_{i \in \mathcal{S}} \left( \left( \frac{s_i(x)}{\bar{s}(x)} \right)^{\beta} - 1 \right), \quad \beta \in [-1, 2]$$
$$= \frac{1}{m\beta(\beta-1)} \frac{1}{\bar{s}(x)^{\beta}} \sum_{i \in \mathcal{S}} (s_i(x)^{\beta} - \bar{s}(x)^{\beta})$$

## Mean Log Deviation $\beta = 0$

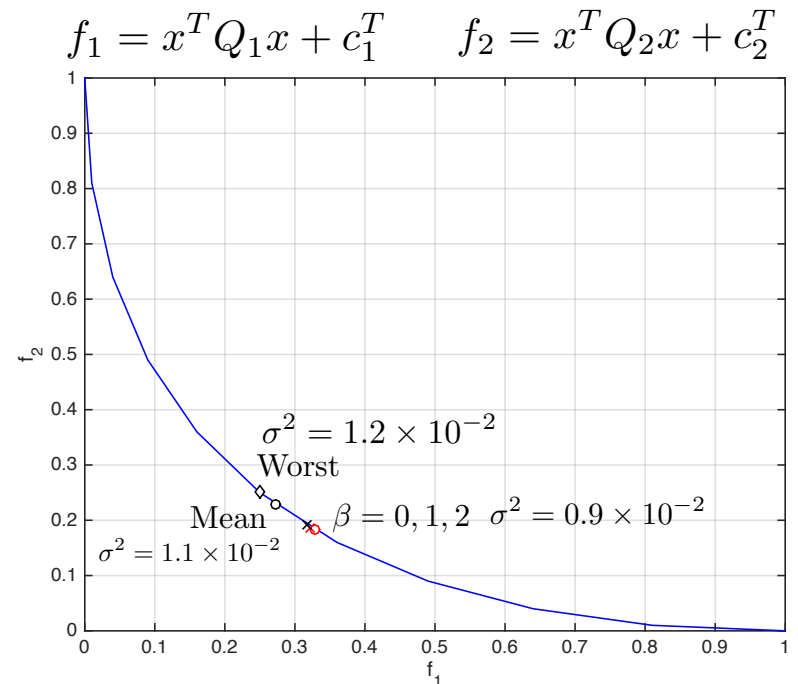
$$GE_0(x) = \log \bar{s}(x) - \frac{1}{m} \sum_{i \in \mathcal{S}} \log s_i(x)$$

## Theil Index $\beta = 1$

$$GE_1(x) = \frac{1}{m} \sum_{i \in \mathcal{S}} \frac{s_i(x)}{\bar{s}(x)} \log \frac{s_i(x)}{\bar{s}(x)}$$
$$= \frac{1}{\bar{s}(x)} \left( \frac{1}{m} \sum_{i \in \mathcal{S}} s_i(x) \log s_i(x) - \bar{s}(x) \log \bar{s}(x) \right)$$

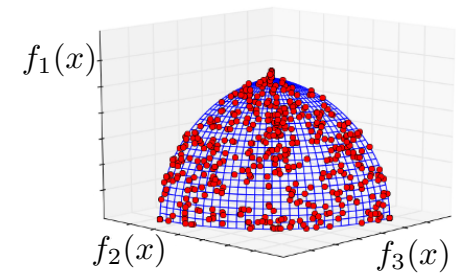
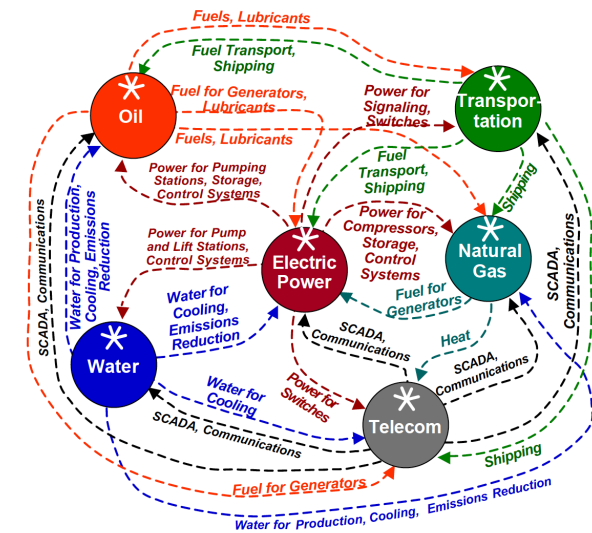
## Squared Coefficient of Variation $\beta = 2$

$$GE_2(x) = \frac{1}{2} \left( \frac{\sigma(x)}{\bar{s}(x)} \right)^2$$





# A Framework for Multi-Stakeholder Decision-Making and Conflict Resolution



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