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Reconstructing Objects with Sparse Boundaries:

Total Variation vs. Discrete Tomography

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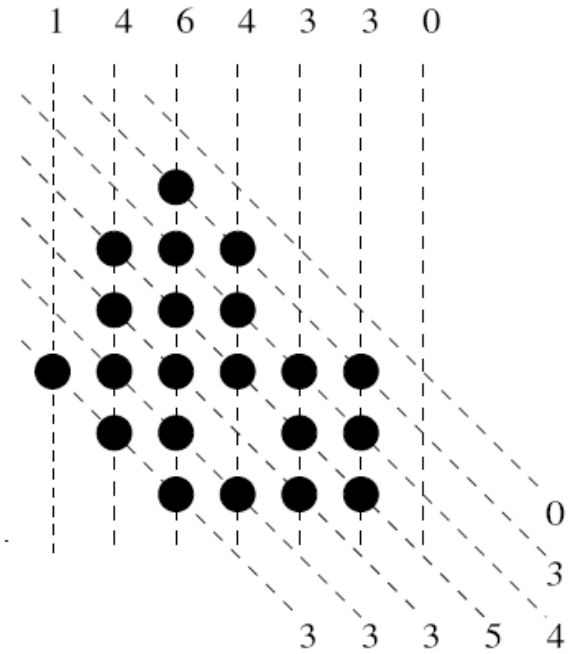
What is Discrete Tomography?

Classical definition: Reconstruction of lattice sets
(due to Larry Shepp)



Reconstructing lattice sets

Many theoretical results, since it is an elegant combinatorial setting

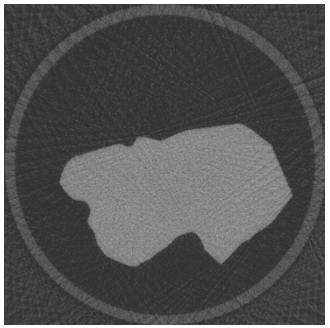
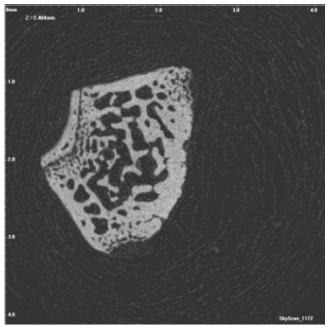
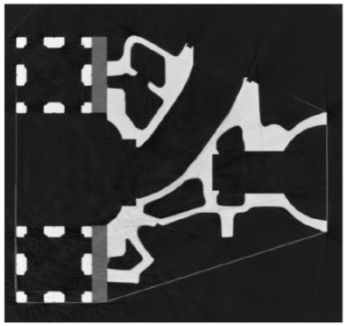
- Perfect with hv-convexity
- (In)stability
- NP-hardness

Very few applications.

What is Discrete Tomography?

Alternative definition: Reconstruction of images that have a small, discrete set of pixel values

(due to Herman & Kuba)

Potential advantages of DT

- Requires fewer projection images
  - Less radiation dose
  - Shorter scanning time
  - Can be the only solution if it is impossible to record many images
- Reconstruction is already segmented

Algorithms for DT

- Combinatorial algorithms
- Combinatorial optimization methods
- Stochastic algorithms
- Modified continuous reconstruction algorithms

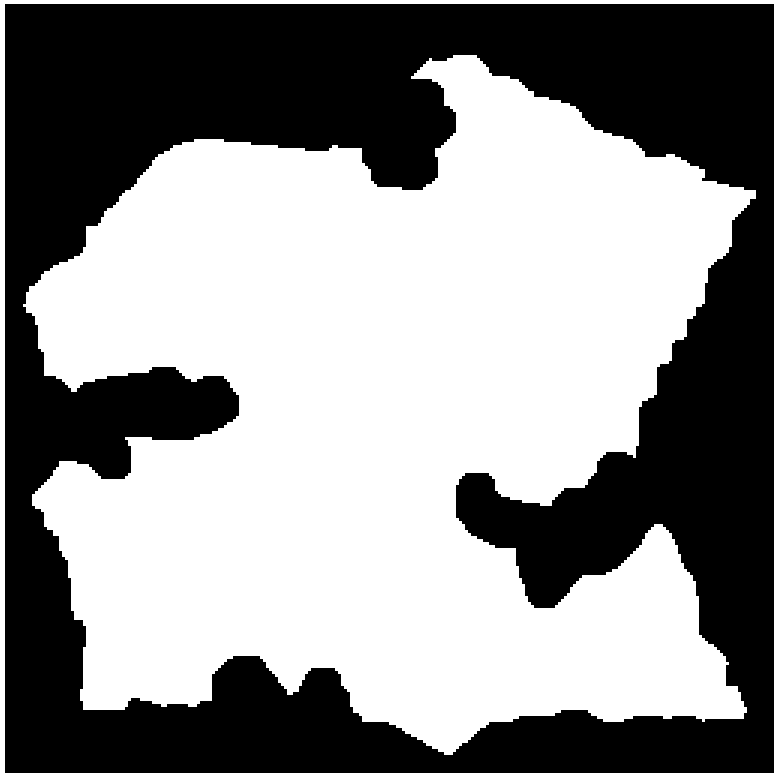
Algorithm: DART

DART: Discrete Algebraic Reconstruction Technique

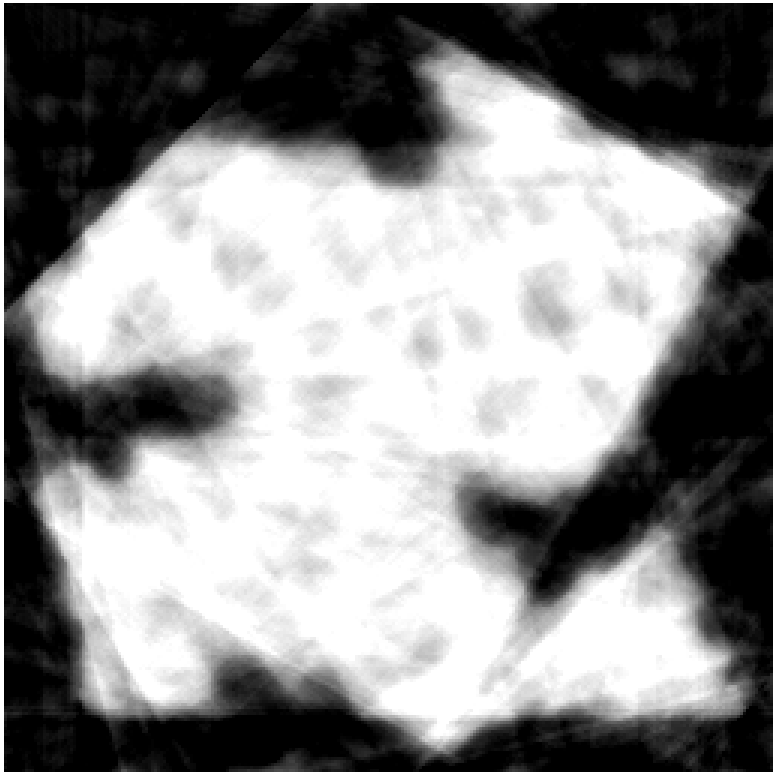
- Iterative method
- Input: projection images + set of intensities
- Output: segmented image

K.J. Batenburg, J. Sijbers, *DART: A Practical Reconstruction Algorithm for Discrete Tomography*, IEEE TIP, 2011

DART: Phantom



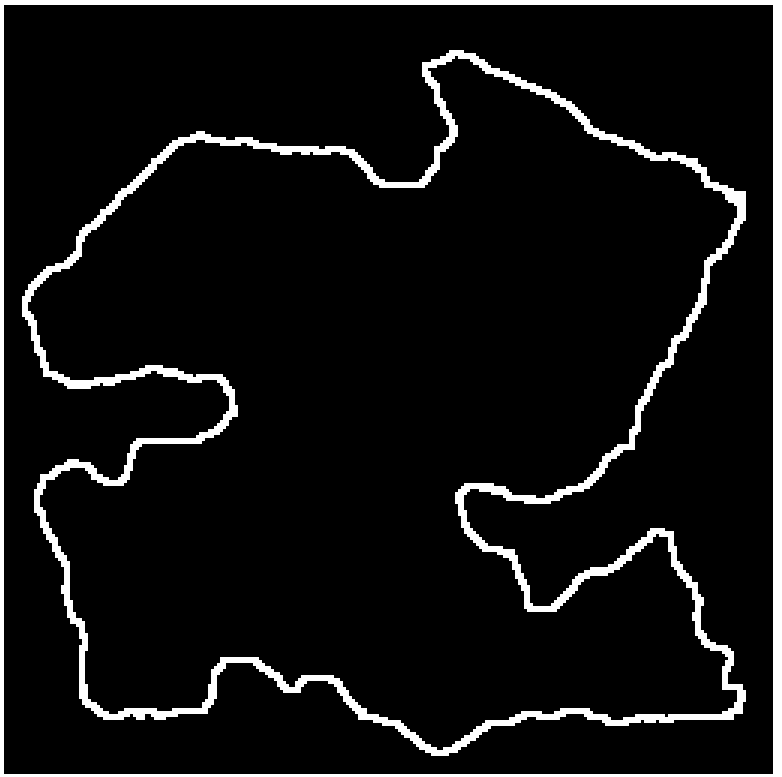
Phantom



SIRT reconstruction
from 12 projections

DART: Boundary

Thresholded SIRT reconstruction

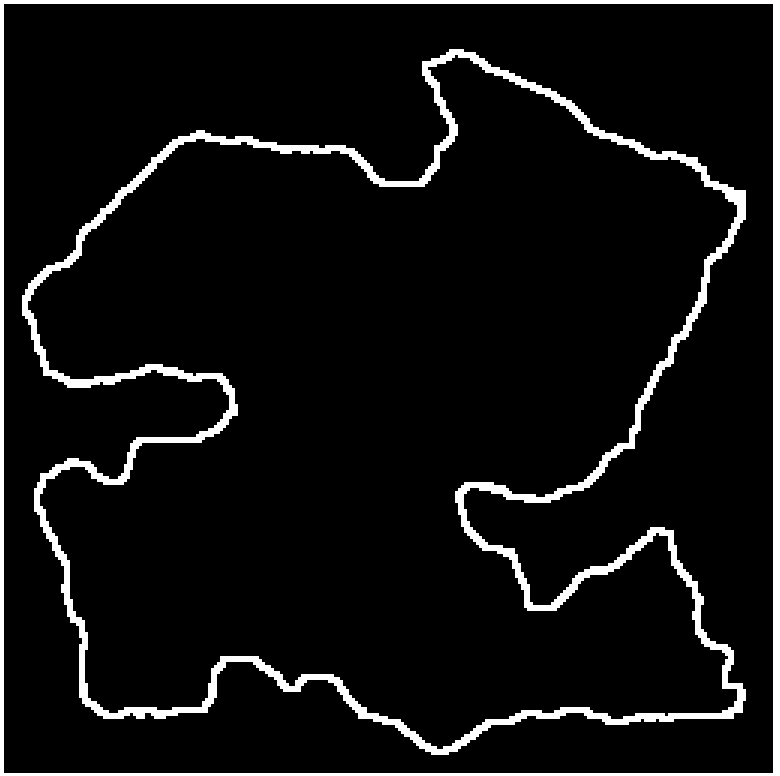


Boundary

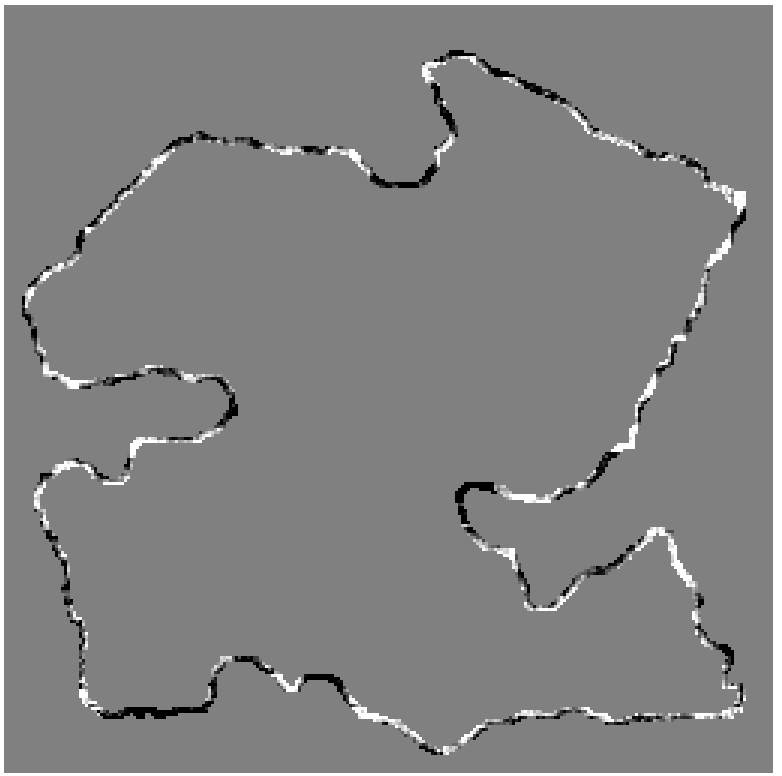
DART: Fixing pixels

- For the interior and exterior of the object, we can be quite confident about the grey level (either 0 or 1).
- Basic idea: fix the pixels in the interior and exterior at their known values (0 or 1).

DART: Continuous step

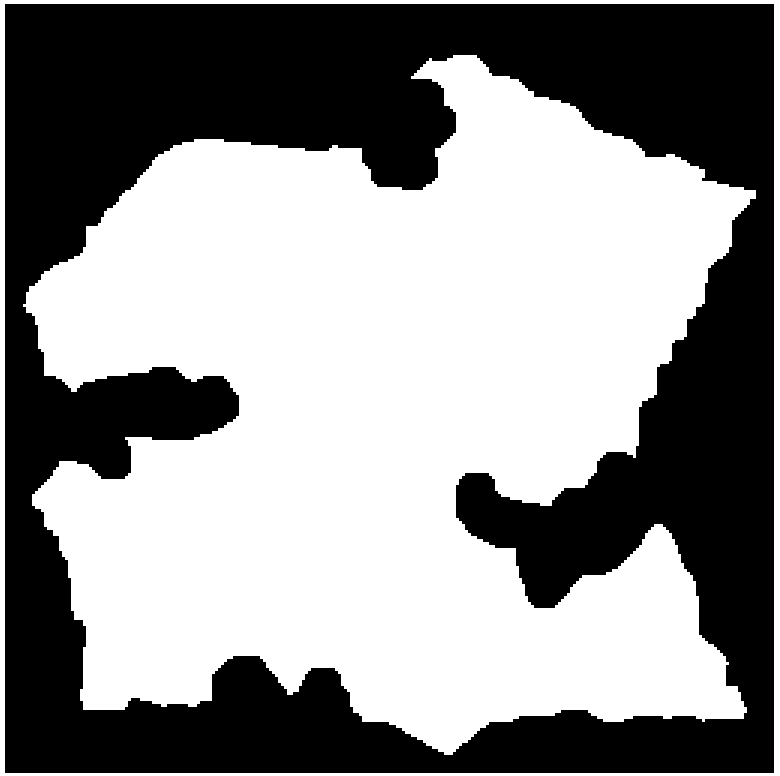


Boundary

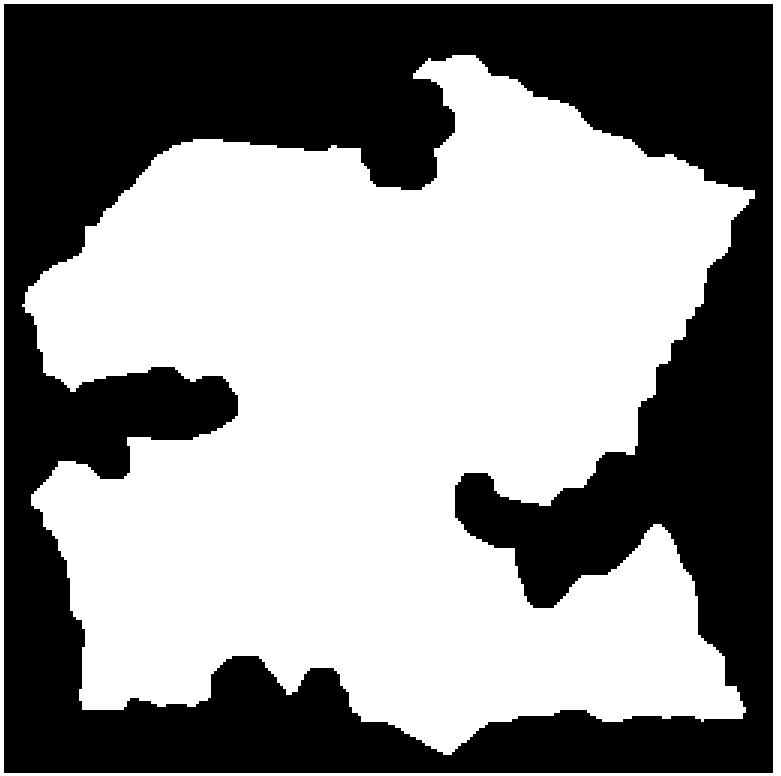


Boundary after SIRT iteration

DART: After three iterations

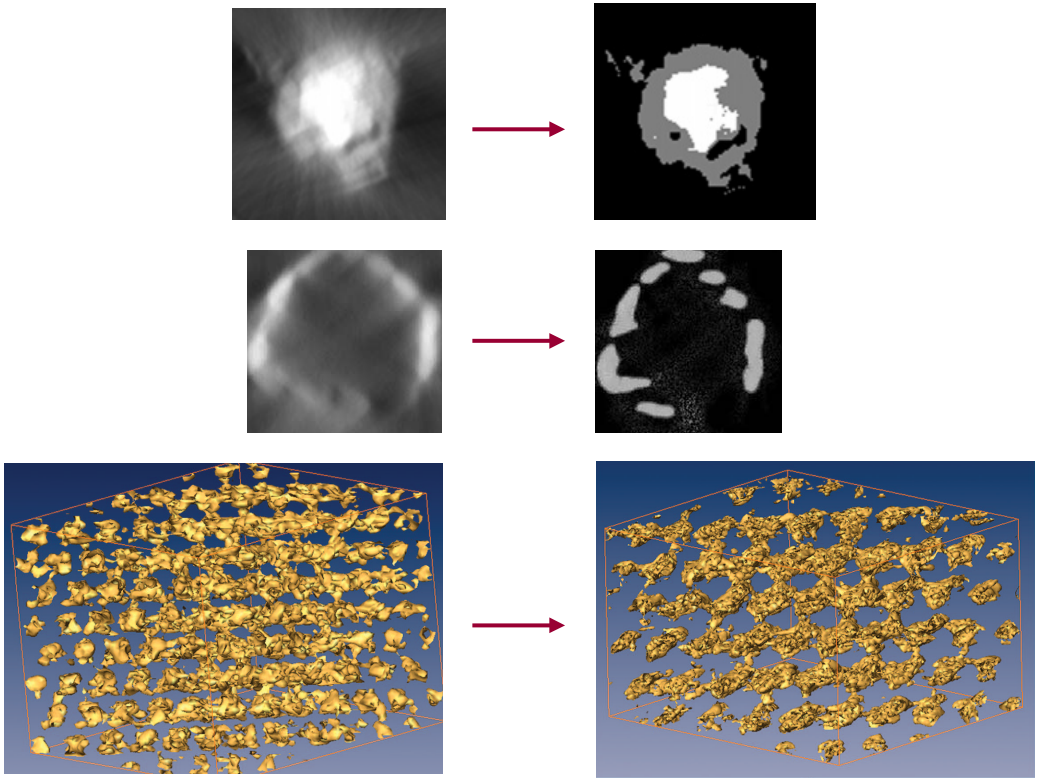


Phantom



DART, 3 iterations

DART: Applications in EM



Conventional Reconstruction

Discrete tomography

S. Bals, K.J. Batenburg et al.,
Nano Letters,
7(12), 3669-3674, 2007

S. Turner, S.M.F. Tavernier et al.,
J. Nanoparticle Research,
12(2), 615-622, 2009

S. Bals, K.J. Batenburg et al.,
J. Am. Chem. Soc.,
131(13), 4769-4773, 2009

Total Variation

- Many objects have sparse boundaries.
- Minimize the (absolute) gradient of the image.

TVmin vs DART: Similarities

- Large overlap in potential applications.
- Both methods focus on boundaries.

TVmin vs DART: Differences

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Total Variation minimization:

- Widely applicable

  (DART: Limited number of grey values is a big restriction)
- Only a few parameters

  (DART: The grey values and other minor parameters)
- Mathematical results

  (DART: Strictly heuristic)

TVmin vs DART: Differences

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DART:

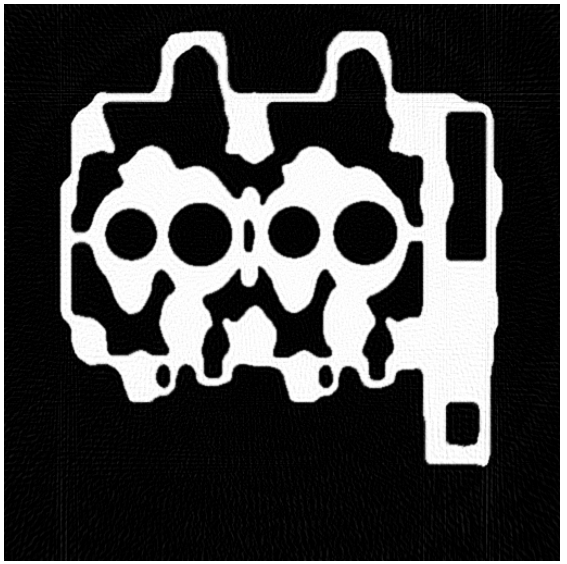
- Very strong prior
- Directly linked to physical property, and testable.

  (TVmin: hard to verify validity of prior)
- Output is a segmented image

  (TVmin: the boundary is less accurate if the interior is less accurate)

Reducing projection count

- 200 projections



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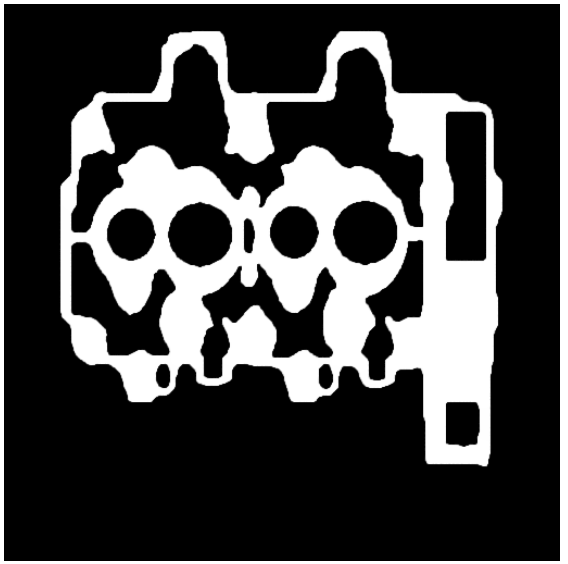
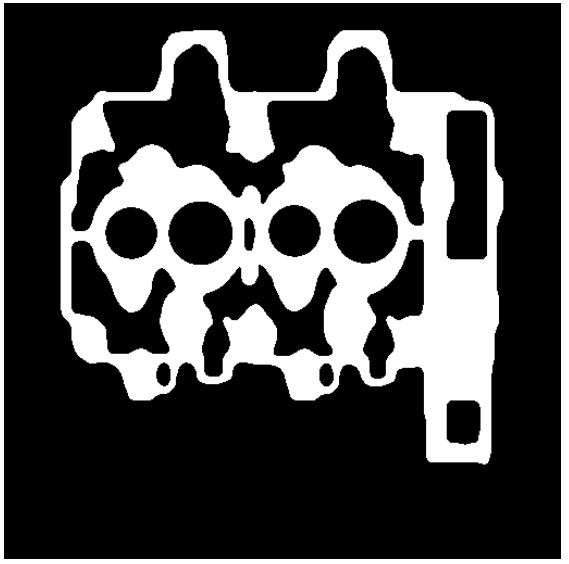


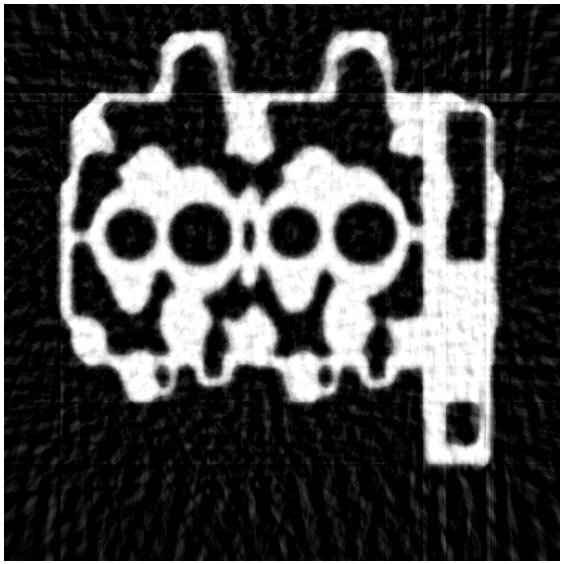
Tvmin (FISTA)



DART

Reducing projection count

- 50 projections



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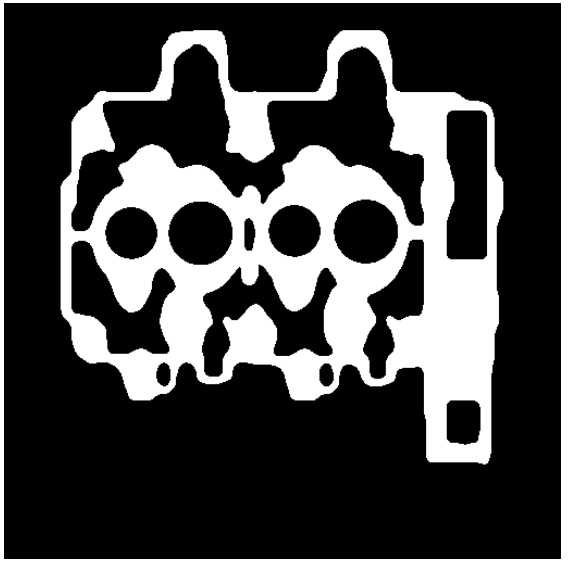
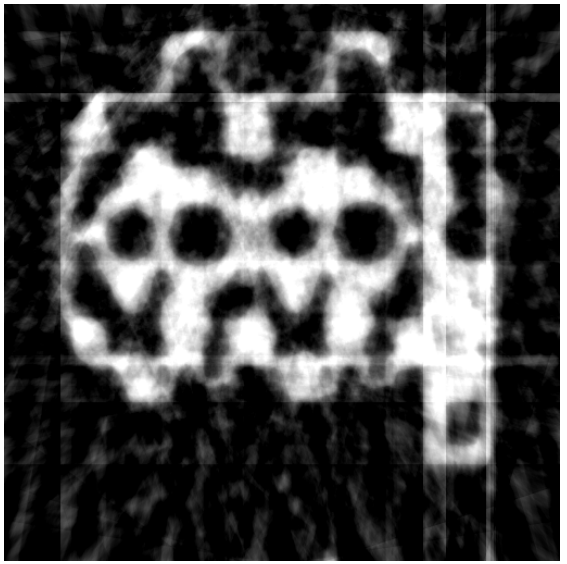
Tvmin (FISTA)



DART

Reducing projection count

- 20 projections



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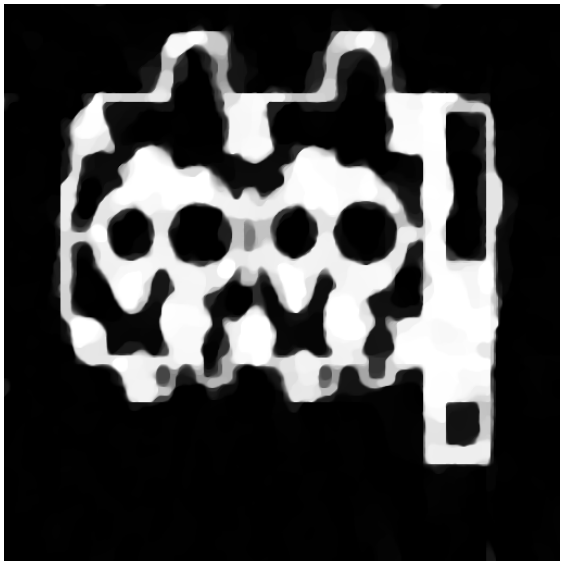
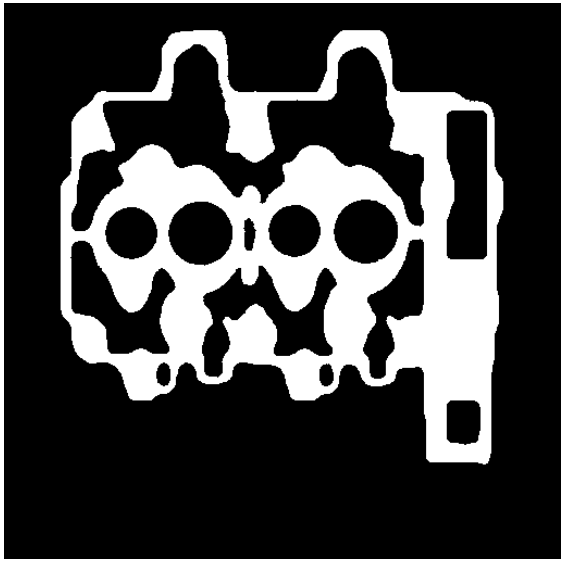


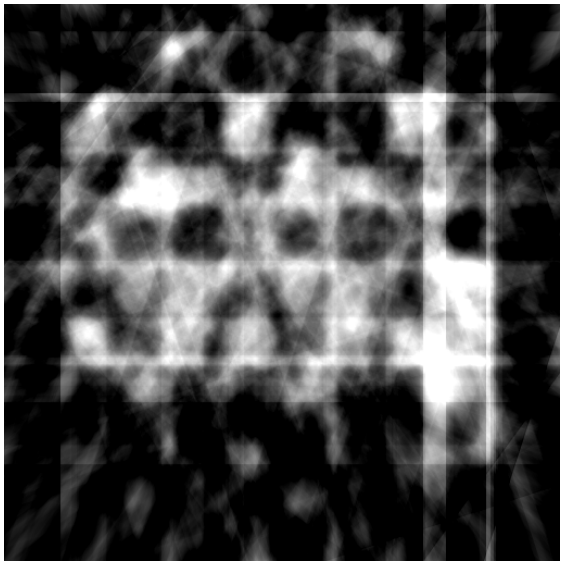
Tvmin (FISTA)



DART

Reducing projection count

- 10 projections



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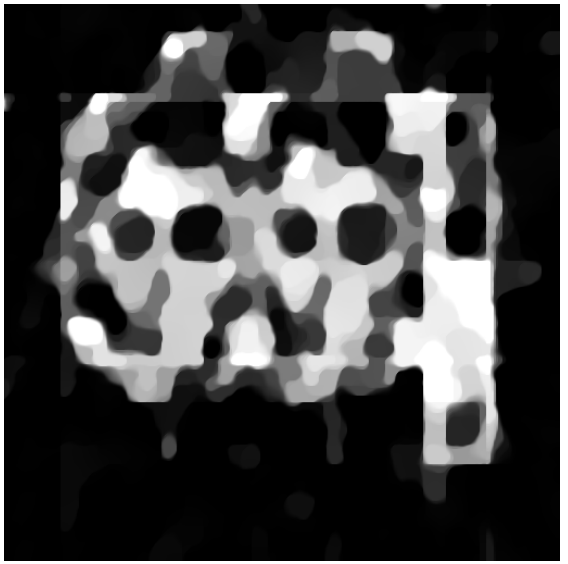
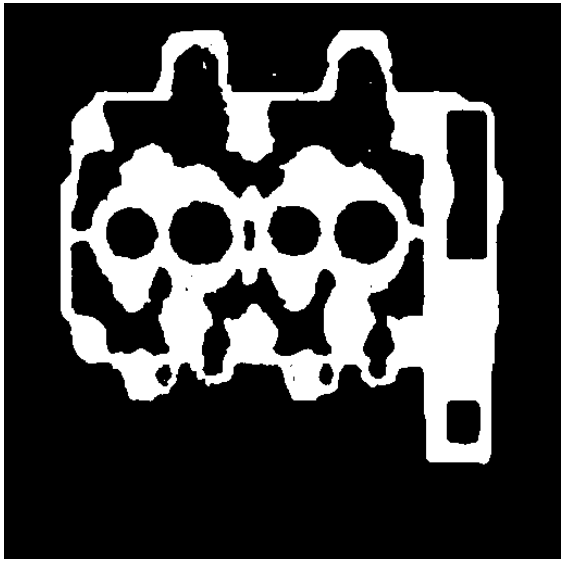


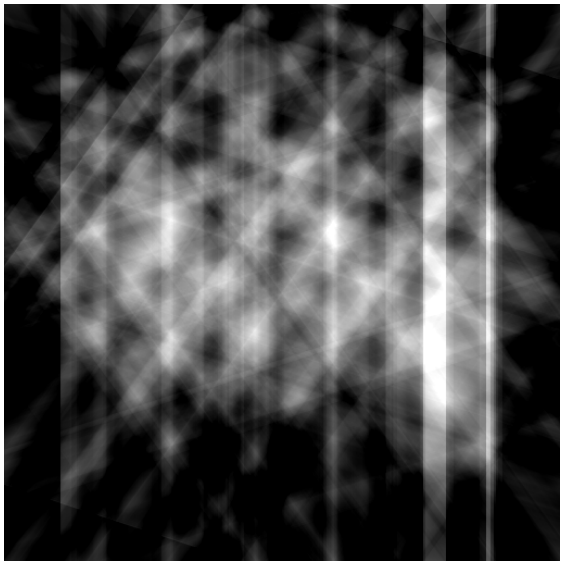
Tvmin (FISTA)



DART

Reducing projection count

- 5 projections



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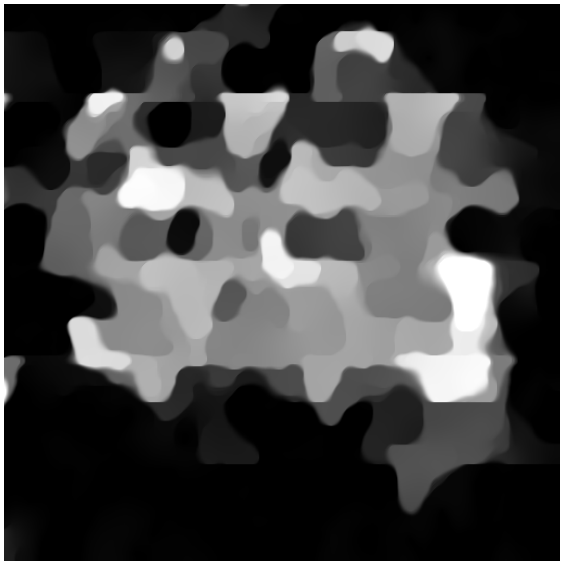
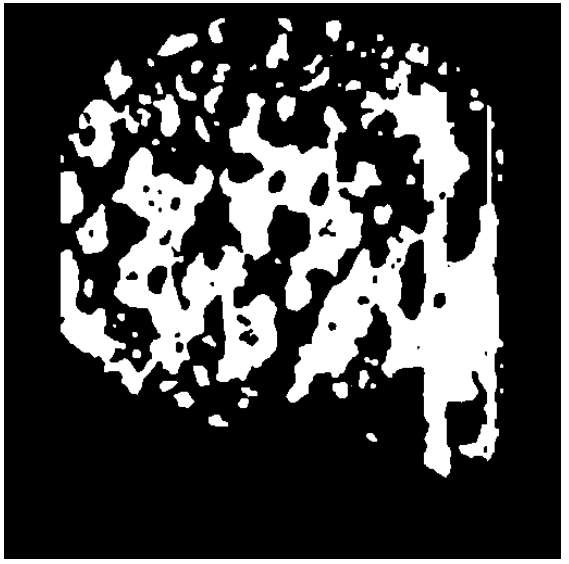
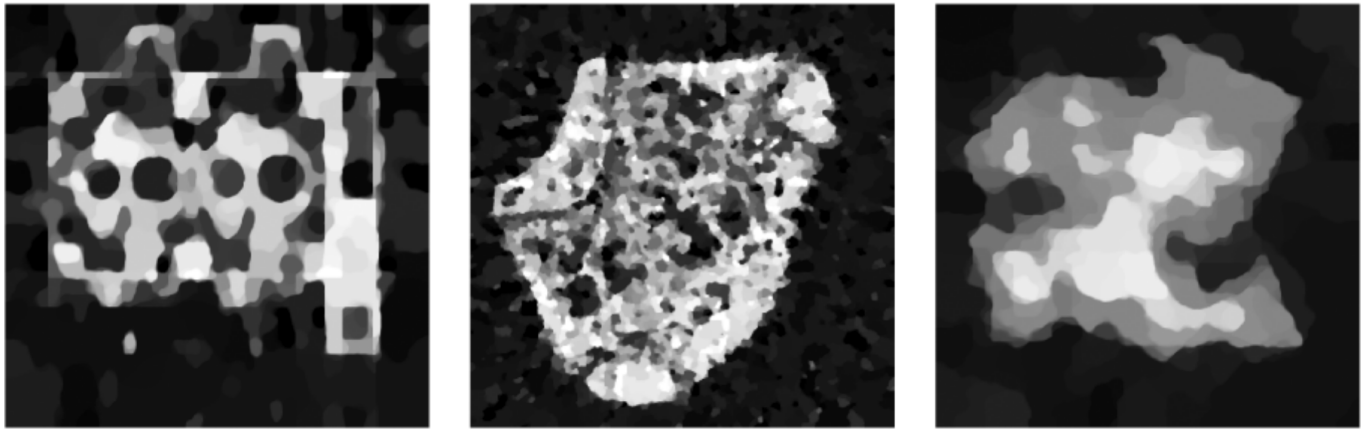


Tvmin (FISTA)



DART

TVmin vs. DART, graphically



(g) (h) (i)

(j) (k) (l)

TVmin + DART

- Also possibilities for combining the two:

  B. Goris et al., *Advanced reconstruction algorithms for electron tomography: From comparison to combination*, Ultramicroscopy, 2013

- Uses TVmin reconstruction as a method to determine grey values to be used with DART.

ASTRA Toolbox

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- Fast and flexible building blocks for 2D/3D tomography.

- Matlab toolbox for easy implementation of algorithms.
- Python wrapper also available.

- NVIDIA GPU support for high performance.

- Free and open source, for Windows and Linux.
- Developed by U. Antwerpen and CWI, Amsterdam.

DART with ASTRA

- The ASTRA Toolbox contains an implementation of DART.
- It includes sample matlab scripts for 2D and 3D.

Sparsity with ASTRA

- Combining the ASTRA, Spot and SPGL1 toolboxes for matlab for sparse wavelet reconstruction:

```
% Create a tomography Spot operator 'opTomo'
W = opTomo('cuda', proj_geom, vol_geom);

% can be used to create projection data as a vector
p = W*im(:);

% reconstruction using a Krylov subspace method
x = lsqr(W,p);
```

```
% Projection operator
W = opTomo('cuda', proj_geom, vol_geom);
B = opKron(opWavelet(n, 'Daubechies', [], levels), opWavelet(n, ...
    'Daubechies', [], levels));
y_spgl1 = spgl1(W*B', sinogram(:), [], 200,[] ,options);
```

Advertisements

- ASTRA:

  http://visielab.ua.ac.be/software

  astra@uantwerpen.be

- EXTREMA COST Action
  (European networking grant)

  http://extrema.ua.ac.be/