Technische Universität Braunschweig

Sparse and TV Kaczmarz solvers and the linearized Bregman method

Dirk Lorenz, Frank Schöpfer, Stephan Wenger, Marcus Magnor, March, 2014

Sparse Tomo Days, DTU

- **Motivation**
- **Split feasibility problems**
- **Sparse Kaczmarz and TV-Kaczmarz**
- **Application to radio interferometry**

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Underdetermined systems

- Seeking solutions of linear systems

$$Ax = b.$$

- Kaczmarz iteration:

$$x^{k+1} = x^k - \frac{a_{r(k)}^T x_k - b_{r(k)}}{\|a_{r(k)}\|_2^2} a_{r(k)}$$

  a_r^T: r-th row of A, $r(k)$: control sequence.

- Amounts to *iterative projection* onto hyperplane defined by $r(k)$-th equation. When initialized with 0: Converges to solution of $\min \|x\|_2^2$ such that $Ax = b$.

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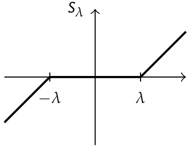

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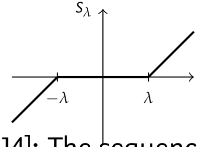

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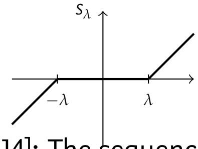

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  1. Very similar to Kaczmarz. Other “minimum-J-solutions” possible?
  2. Very similar to linearized Bregman iteration (replace first equation by $z^{k+1} = z^k - t_k A^T(Ax^k - b)$)

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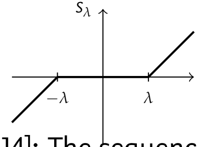

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- Approach: "Split feasibility problems" will answer the first and explain the second point.
- In a nutshell: Adapt the notion of "projection" to new objective.

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  Find x, such that

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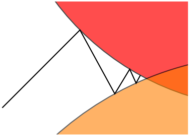

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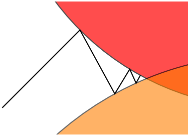

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- [1933 von Neumann (two subspaces), 1962 Halperin (several subspaces), Dijkstra, Censor, Bauschke, Borwein, Deutsch, Lewis, Luke...]

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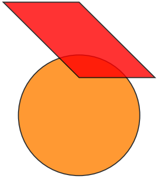

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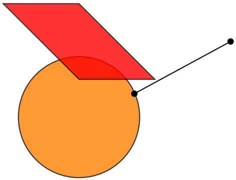

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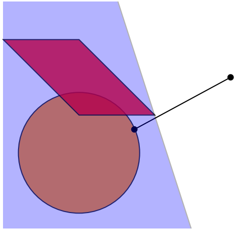

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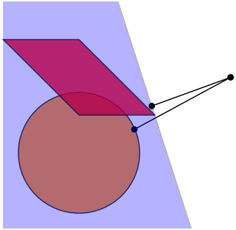

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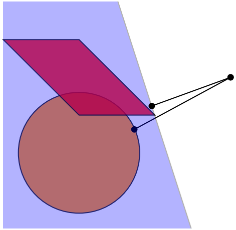

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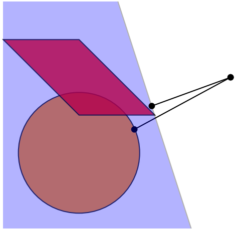

- Converges to feasible point.
- E.g.: $Q = \{b\}$: $x^{k+1} = x^k + t_k A^T(Ax^k - b)$
  $\rightsquigarrow$ minimum norm solution of $Ax = b$

Towards sparse solutions with generalized projections

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- Good news! Bregman projections onto hyperplanes $H = \{a^T x = \beta\}$ are simple:
  if $z \in \partial J(x)$

$$P_H(x) = \nabla J^*(z - \bar{t}a), \quad \bar{t} = \operatorname*{argmin}_t J^*(z - ta) + t\beta$$

  Moreover: $z - \bar{t}a \in \partial J(P_h(x))$ new subgradient in $P_H(x)$.

Convergence

- **Theorem:** [Schöpfer, L., Wenger 2013] Cyclic (or random) Bregman projections converge to a feasible point: $\bar{x} \in C_i$ and $A_i\bar{x} \in Q_i$.

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$$\min J(x) \text{ s.t. } Ax = b$$

Multiple possibilities, e.g.

1. only one "difficult constraints": $Ax \in Q = \{b\}$
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- In both cases: Convergence to minimum-J solution

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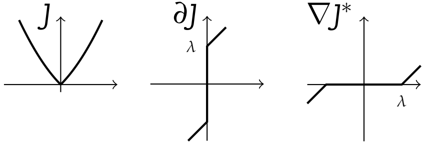

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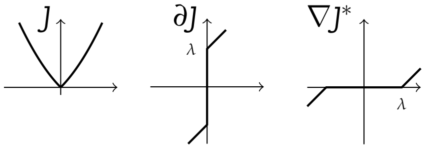

- $\nabla J^*(z) = S_\lambda(z)$

Basic algorithm and special cases:

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- $J(x) = \|x\|_2^2/2$, variant 1.: Landweber iteration
- $J(x) = \|x\|_2^2/2$, variant 2.: Kaczmarz method
- $J(x) = \lambda\|x\|_1 + \|x\|_2^2/2$, variant 1.: Linearized Bregman!
- $J(x) = \lambda\|x\|_1 + \|x\|_2^2/2$, variant 2.: Sparse Kaczmarz!

Inexact stepsizes are allowed

- Linearized Bregman:

$$t_k = \frac{\|Ax^k - b\|^2}{\|A^T(Ax^k - b)\|^2} = \frac{\|w^k\|^2}{\|A^T w^k\|^2}, \quad \text{or} \quad t_k \leq \frac{1}{\|A\|^2}$$

- However: To compute exact stepsize, solve one-dimensional piecewise quadratic optimization problem (can be done in $\mathcal{O}(n \log n)$, usually faster).

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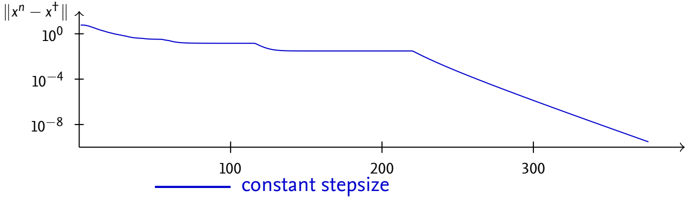
Stepsize comparison



$A \in \mathbf{R}^{1000\times 2000}$ Gauß, $x^\dagger$ 20 non-zeros (Gauß)

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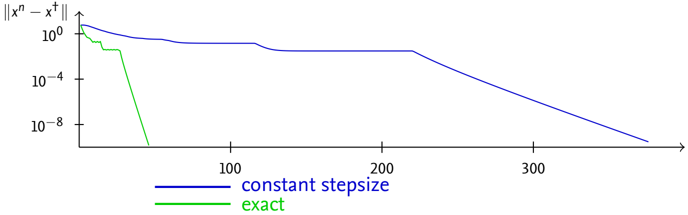

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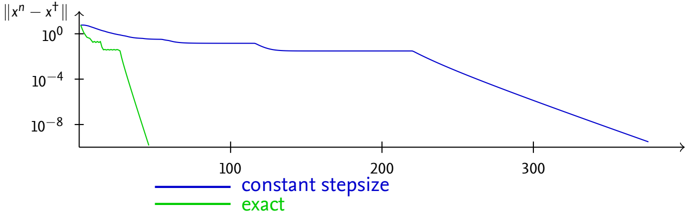

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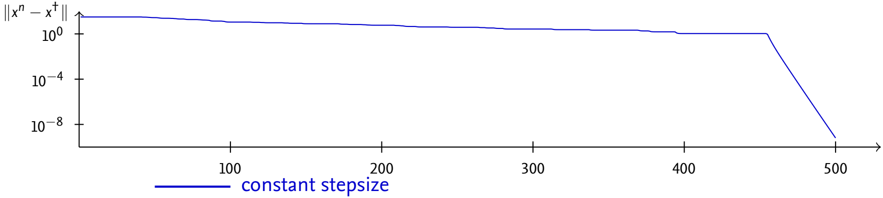

$A \in \mathbf{R}^{2000\times 6000}$ partial DCT, $x^\dagger$ 50 non-zeros (high dynamic range)

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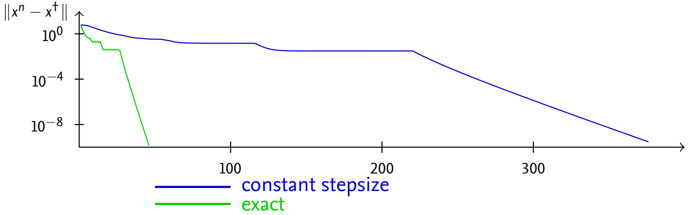

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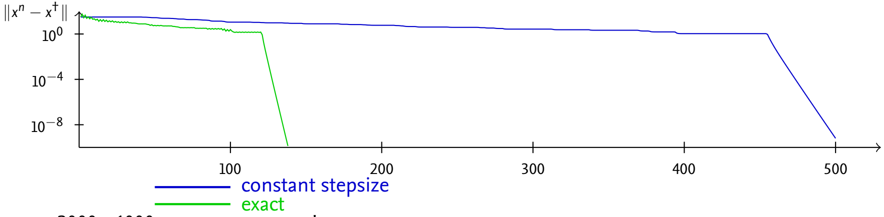

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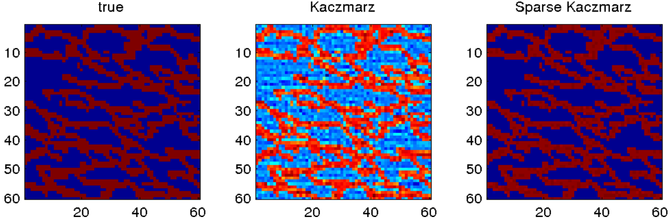
Really helps for sparse images

- `binarytomo.m` from AIRtools
- Standard Kaczmarz vs. Sparse Kaczmarz, 50 sweeps:



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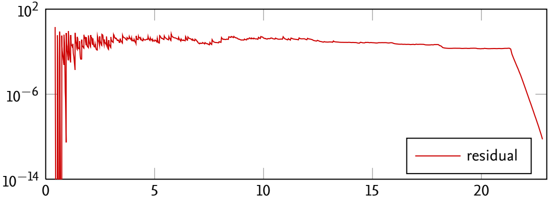

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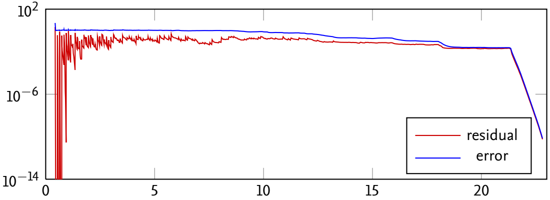

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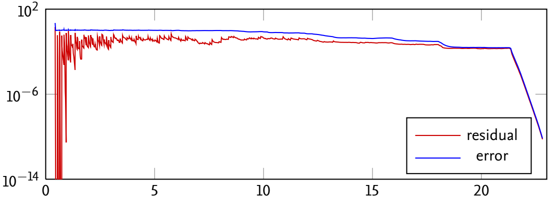


- Reconstruction error drops down precisely when residuum starts to stay small! Stop measuring when that happens

TV-Kaczmarcz

- How to treat

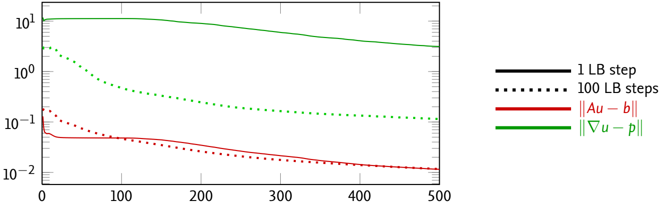
$$\min \||\nabla u|\|_1 \text{ subject to } Au = b?$$

- Introduce constraint $p = \nabla u$, add regularization:

$$\min_{u,p} \lambda \||p|\|_1 + \tfrac{1}{2}\left(\|u\|^2 + \|p\|^2\right) \quad \text{s.t} \quad Au = b,$$
$$\nabla u = p.$$

- Treat $Au = b$ by Kaczmarz ($u^{k+1} = u^k - \frac{a_{r(k)}^T u^k - b_{r(k)}}{\|a_{r(k)}\|^2} a_{r(k)}$)
- Treat $\nabla u - p = 0$ by linearized Bregman steps (with dynamic stepsize, uses two-dimensional shrinkage)

- Parallel beam geometry
- 16384 pixels, 3128 measurements
- 500 Kaczmarz sweeps



original

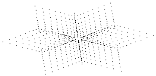
1 LB step per sweep

100 LB steps per sweep

- Motivation
- Split feasibility problems
- Sparse Kaczmarz and TV-Kaczmarz
- **Application to radio interferometry**

Radio interferometry

- Very Large Array telescope: a number of radio telescopes record radio emission from the sky. Each pair of telescopes gives one sample of the Fourier-transform of the image



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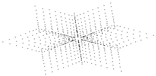
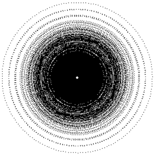


- After a small rotation of the earth, the sampling pattern also rotates. Half-day observation:



Compressed online radio interferometry

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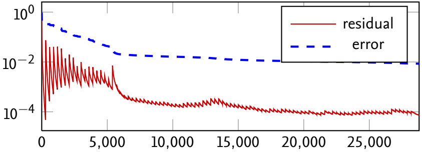
- Monitor the residual after new measurements have arrived



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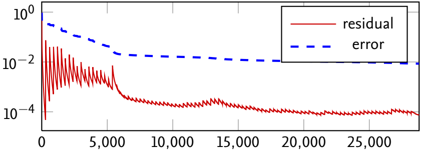
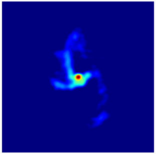
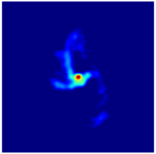


- Drop of the residual after 5,400 iterations (2.5 hours), no further increase of quality expected



Sagittarius A West



Reconstruction

Conclusion

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- New approach to sparse recovery via split feasibility problems
- Recover linearized Bregman with a different proof of convergence
- Exact stepsizes greatly improve convergence
- Obtained new sparse Kaczmarz solver
- Numerous generalizations possible, no new theory required

Technische Universität Braunschweig

Sparse and TV Kaczmarz solvers and the linearized Bregman method

Dirk Lorenz, Frank Schöpfer, Stephan Wenger, Marcus Magnor, March, 2014

Sparse Tomo Days, DTU

- **Motivation**

- **Split feasibility problems**

- **Sparse Kaczmarz and TV-Kaczmarz**

- **Application to radio interferometry**