QUANTIFYING SPATIAL UNCERTAINTY IN THE SPACE OF CURVES: STREAMLINE TRACTOGRAPHY

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Uncertainty in tractography. In medical imaging population studies, "data points" such as segmented organ surfaces or registration warps are themselves estimated from image data. These "points" are treated as fixed, but *are* in fact stochastic with their own uncertainty stemming e.g. from noise or lack of model fit in the estimation pipeline. Taking streamline tractography as a case, we estimate tract uncertainty with a tracking algorithm that returns a Gaussian Process over the resulting tract. This is philosophically different from probabilistic tractography, as our uncertainty lives in the space of curves, giving a spatial interpretation of the resulting uncertain curve. We discuss challenges with this current, simple model and outline future approaches to solving them.

In state-of-the-art tractography methods, structural brain connectivity to a point x_0 is estimated via a family of curves, or *streamlines*, that solve the initial value problem (IVP)

$$\frac{d}{dt}\gamma(t) = v(t), \quad \gamma(0) = x_0, \tag{1}$$

where $v(t)\colon \Omega \to \mathbb{R}^3$ is a vector field tangential to the underlying white-matter bundles, estimated from diffusion MRI. A deterministic or stochastic v leads to deterministic and probabilistic fiber tracking, respectively.

Seeding all streamlines in the same region, they are commonly summarized in a heatmap counting streamlines per voxel. The heatmap intensity is often interpreted as a measure of uncertainty, but while it varies spatially, the intensity does *not* capture the spatial variation of the underlying curves. This difference is crucial in applications where the *trajectory* of the white matter bundle is needed, e.g. surgical planning.

Tract uncertainty has [1, 2] been quantified over curves by estimating tracts as *Gaussian Processes* (GPs); intuitively "normal distributions over curves". These works employed *shortest-path tractography*, which struggled with numerical instability, and is less popular than streamline tracking.

We propose using a new probabilistic numerics IVP solver [3] to solve streamline tracking via the first order IVP (1). This solver uses a *Kalman filter* for the tracking, followed by a *smoother* which estimates the covariance of the final solution utilizing all visited locations. This algorithm is numerically advantageous, as the shortest path problem required a numerically less stable boundary value solver for a more complex, second order, differential equation. Moreover, uncertain streamline tracking is more likely to gather interest as this is currently by far the preferred type of algorithm.

Fig. 1 shows two single GP streamlines from the corticospinal tract, both visualized by drawing samples from the GP. The streamline tracking is performed on a single HCP subject based on DTI, giving a GP version of deterministic tracking.

Discussion and conclusion. Our current model is able to solve initial value problems quantifying numerical uncertainty in DTI. Incorporating data uncertainty as in [2] is our next research problem, but it is less clear how to correctly do so. On the other hand, extension to HARDI data should be straightforward. Future work moreover includes the incorporation of uncertainty estimates in population analysis [4].

1. REFERENCES

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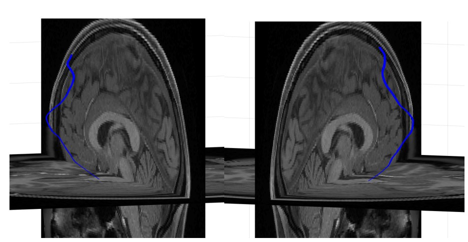

Fig. 1. Samples drawn from the GPs corresponding to two tracked, deterministic GP streamlines from the CST.

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