# Tools supporting wind energy trade in deregulated markets

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# Abstract

A large share of the wind energy produced in Scandinavia is sold at deregulated electricity markets. The main market, Elspot, is a day-ahead market where energy is sold up to 36 hours before delivery. Failure in delivering exactly the quantity which was sold results in a fine, called regulation cost. As wind energy comes from an uncontrollable energy source - the wind - producers can not always fulfil their sales obligations and must, therefore, often pay high regulation costs. In this thesis it is examined how producers can increase their profit by bidding on the market in such a way that the regulation cost is minimised. The methods developed rely on new production forecasts which provide better probabilistic information about the prediction uncertainty than many forecasting systems currently in use.

The problem is formulated in two different ways. One, originally presented by John B. Bremnes, where only a part of the market is included, gives a simple method that can be applied using only statistical tools. The other method is more flexible at the cost of complexity. It uses both statistics and stochastic programming. This method can be changed and applied in other markets with a structure different from that of the Scandinavian market, NordPool.

Keywords: Electricity market, Wind energy, NordPool, Quantile Regression, Stochastic Programming.

# Preface

This thesis was written as a part of my studies for a Master degree at the Department of Informatics and Mathematical Modelling (IMM) of the Technical University of Denmark (DTU), under the supervision of professor Henrik Madsen and associate professor Henrik Aalborg Nielsen.

The project was carried out in the period from the 1st of February 2004 to the 1st of August 2005, evaluated as a 30 ECTS[1] point project.

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Úlfar Linnet

[1] European Credit Transfer System

# Acknowledgements

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# Abbreviations and notation

Upper case letters are used for random variables, sets and variables which have both subscript and super scripts. For instance:

$$E_{t-1,i}^{Q,B} = Elbas_{at\ time\ t-1, if\ production\ level\ i\ is\ observed}^{Quantity\ Bought} \tag{1}$$

Only variables related to electricity markets are written using this complicated notation.

The function names $f$ and $F$ are reserved for pdf and cdf respectively (see list of abbreviations).

The regulation cost function R{e} is denoted using curly brackets to avoid confusion with multiplication.

The date format used is ”yyyy-mm-dd hh”

Frequently used variables.

| | | |
|---|---|---|
| $\lfloor a \rfloor$ | : | Largest integer smaller than the real number $a$ |
| cdf | : | abbreviation for ”cumulative distribution function” |
| $\mathbb{E}[X]$ | : | The expected value of $X$ |
| $E_t^{ND}$ | : | Energy *Not Delivered* to the *E*lbas market |
| $E_{t-1}^{P,B}$ | : | The *P*rice of energy *B*ought at *E*lbas in hour $t-1$ |
| $E_{t-1}^{P,S}$ | : | The *P*rice of energy *S*old at *E*lbas in hour $t-1$ |
| $E_{t-1}^{Q,B}$ | : | The *Q*uantity *B*ought at *E*lbas in hour $t-1$ |
| $E_{t-1}^{Q,S}$ | : | The *Q*uantity *S*old at *E*lbas in hour $t-1$ |

| | | |
|---|---|---|
| $f_{X_t}^{t'}(x_t)$ | : | pdf estimated at time $t'$ for the production at time $t$ |
| $F_{X_t}^{t'}(x_t)$ | : | cdf estimated at time $t'$ for the production at time $t$ |
| $I_t$ | : | The income in hour $t$ |
| pdf | : | abbreviation for "probability density function" |
| $\mathbb{P}\{A\}$ | : | The probability of event $A$ |
| $R\{e\}$ | : | The cost of regulation when the need is $e$ (positive $e$: down regulation) |
| $R_t^D$ | : | Down regulation cost at time $t$ |
| $R_t^U$ | : | Up regulation cost at time $t$ |
| $RN_t$ | : | The total regulation need in hour $t$ |
| $OP_{p,t}$ | : | Original production plan of producer $p$ in hour $t$ |
| $CP_{p,t}$ | : | Changed production plan of producer $p$ in hour $t$ |
| $S_t^D$ | : | Energy *D*elivered to the *S*pot market |
| $S_t^P$ | : | The spot market price when energy is delivered |
| $S_{t'}^Q$ | : | The spot market bid quantity bidden at time $t'$ |
| $SSR$ | : | abbreviation for "sum of squared residuals" |
| $t$ | : | The time $t$, in that hour the energy is delivered. A period. |
| $t'$ | : | The time when first decision is taken (spot market) |
| $t-1$ | : | The hour before delivery |
| $X_t$ | : | A random variable describing the production at time $t$ |
| $x_t$ | : | Production at time $t$ |
| $x_{max}$ | : | Maximum production |
| $\alpha^S$ | : | The constant component of the linear Elbas price when selling energy |
| $\beta^S$ | : | The slope component of the linear Elbas price when selling energy |
| $\alpha^B$ | : | The constant component of the linear Elbas price when buying energy |
| $\beta^B$ | : | The slope component of the linear Elbas price when buying energy |

# Contents

Chapter 1

# Introduction

Production forecasts have been an inseparable part of wind power production for the last two decades. In the early days, production forecasts were only use to plan production but now, when many electricity markets have been deregulated, they are also used when the energy is being sold. This is because electrical energy is always sold in advance in order to avoid unstable prices. Although the forecasting systems are constantly being improved, will they never give forecasts with out errors. It is therefore important to know how precise the forecasts are. Currently methods to estimate the uncertainty of forecasts are being developed, i.e. methods which can provide accurate probabilistic information about future production.

The objective of this thesis is to investigate how the information about the uncertainty can be integrated into the sale process in order to increase producers profit. The block diagram in Figure 1.1 shows the main parts of the method that was developed. The market structure is combined with observations in order to create a mathematical model which describe the possible actions a producer can take when selling his energy (Box e, f and g). A forecast with out probabilistic information is combined with historical data and quantile regression applied to gain probabilistic information about the prediction uncertainty. The prediction is then discretised so that the problem can be solved using standard solvers (Box a, b, c and d). The mathematical model is combined with the discrete prediction and a price forecast and an optimal trade plan found by the use of a solver (Box

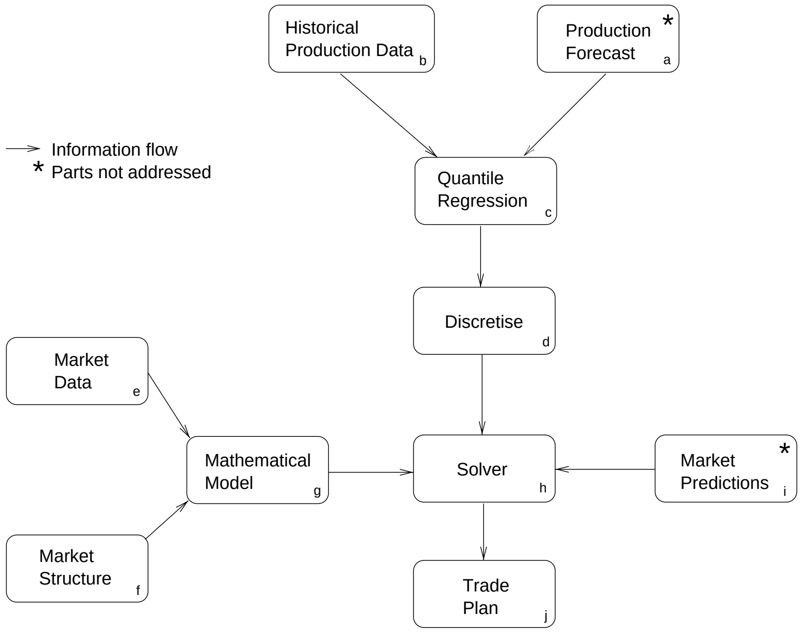


Figure 1.1: *Block diagram showing the main parts of the optimisation method developed.*

h, i and j). The results are a trade plan, stating how much energy should be sold or bought at different markets from the time when the first bid is placed to the time when the energy is actually delivered. Not all parts are addressed in this thesis, production and price forecasts are assumed to be provided.

## 1.1 Previous work

In [1] the consequences of the choice of criterion in short-term wind power prognosis is investigated. There the power curve of a wind farm is estimated using two different criteria: absolute error and minimum cost. It is observed that the criteria has effect on the estimate, resulting in different predictions. The authors conclude that the estimation should be a multi criteria problem.

In [2] Bremnes examines how bids should be placed at a market given probabilistic information. The main difference between his approach and the approach in [1] is that the model parameters are unchanged but model output statistics applied in order to find the optimal bid. He demonstrates how the method can increase the total income by approximately 7.6%.

In [3] Holttinen investigates as slightly different matter, that is how an optimal electricity market for wind power should be. Her analysis are mainly focusing on prediction errors given that all wind energy can be sold. The results are that shorter markets are better than long ones, because short predictions have lower error. However, the prices are not included so the results can not be applied directly to the situation at NordPool.

## 1.2 Thesis overview

The thesis is divided into parts intended to ease the selection of chapters. Readers who are new to the field of electricity markets should read part one and three and use part two as a mathematical reference. Readers who have a good understanding of the Scandinavian electricity markets should read part three and use part two as mathematical reference.

**Part I** A short introduction to production methods and the electricity market in Scandinavia.

**Chapter 2** provides general information about electricity as a commodity and the production methods used in Scandinavia[1].

**Chapter 3** lists the production methods applied in each of the Scandinavian countries. It contains also a description of the transmission capacity with in Scandinavia and to the rest of Europe[1].

**Chapter 4** covers the three electricity markets used in Scandinavia, Elspot, Elbas and the regulation market.

**Part II** Mathematical theory.

**Chapter 5** contains revision notes, listing the key mathematical theory used in part III [1].

**Part III** Bidding strategies developed.

**Chapter 6** The bidding strategy, originally suggested by John B. Bremnes described and tested. The chapter contains a detailed description of the data used in the tests and main test results.

**Chapter 7** contains a description of a new, robust, bidding strategy. The formulation is not as simple as in chapter 6, but it allows all the markets under NordPool to be included. The framework used is flexible and can be extended.

**Chapter 8** Conclusion.

Code appendix is omitted but all code is available up on request. The languages used were `R` and `S` for statistics and `GAMS` for optimisation. Please send emails to `ulfarlinnet@gmail.com`.

[1]The chapters marked are published under the GNU Free Documentation License. Part of the material they contain is taken from `www.wikipedia.org`

# Part I

# Background

Chapter 2

# Electricity production

## 2.1 Electricity as a commodity

The prices at electricity markets are expected to reflect production cost just as prices at other free commodity markets do. If not, new producers will enter the market or old ones fall out. However, they are three important things that make electricity different from other commodities [4].

- Electricity is by its nature difficult to store and has to be available on demand. Consequently, unlike for other products, it is not possible, under normal operating conditions, to keep it in stock or have customers queue for it[1]. Therefore, the generation of electric power must match the demand at all hours. If there is a large difference between supply and demand, the frequency of the network exceeds the allowed range and the stability is put at risk.

[1] A number of storage possibilities exist for electricity. In spite of that most of them are unusable in large power systems due to technical limitations or extremely high storage cost. The two most cost efficient methods that have been used with success on large scale are pumped hydro and compressed air energy storage. Both methods rely on special natural conditions and generation units. Therefore, they can not be easily applied and are not used in Scandinavia.

| Hydroelectric | Nuclear | Thermal | Wind |
|---|---|---|---|
| 46% | 28% | 24% | 2% |

Table 2.1: *The portion of total production capacity (363 TWh) at NordPool 2003 [6].*

- Transporting electrical power from generators to consumers requires a special infrastructure called a transmission system. This systems can not be used by any other commodity[2]. If there exists a transmission system, electricity can be transported a long distance in a split second without high losses[3]. There are, though, limitations to the amount of energy which can be transported simultaneously, and due to high building cost, transmission lines are often close to being fully utilised.

- The demand for electricity is inelastic. In other words, the consumers do not respond to changes in price. There can be many reasons for this, but in this context, only two possible causes are mentioned. One is that there is no other commodity that can easily replace electricity. The other is, that small consumers are normally not affected by the market price cause they have a price contract which is only revised once a year or so.

## 2.2 Methods for electric power production in Scandinavia

The intention here is to give a short description of the key production units in the Scandinavian power system, so the reader can better understand what controls the prices at NordPool. In table 2.1 the portion of available production capacity, grouped by type, is listed for the year 2003. The system is dominated be hydropower but the table does not tell the whole story as the situation in Norway is completely different from what it is in Denmark and transmission between the Scandinavian countries is limited.

[2] Some telecommunication companies offer data transfer through low voltage transmission lines but the technology is new and not widespread.

[3] Transmission and distribution losses in US were estimated to be around 7.2% in 1995 [5].

### 2.2.1 Thermal power

A thermal power plant converts energy stored in fossil fuels such as coal, oil, or natural gas successively into thermal energy, mechanical energy, and finally electric energy for continuous use and distribution. The size of the plants wary from $kW$ to $GW$ and they can either produced only electricity, or both electricity and hot water. Each plant is a highly complex, custom designed system. Starting a plant is normally quite expensive and plants can only be operated when the output is within in a limited range. The production is usually not cost efficient if it is close to zero. The price of both heat and electricity produced in a thermal plants is highly dependent on the fuel price. It can be expected that in the future price of emission quota will also influence the energy price but the use of quotas has just begun in a few countries, so as yet not much is known about its influence.

### 2.2.2 Nuclear power

Nuclear power involves converting the nuclear energy of fissable uranium into thermal energy by fission, from thermal to kinetic energy by means of a steam turbine, and finally to electric energy by a generator. Nuclear power provides steady energy at a consistent price. Production can only be changed slowly which is the reason why nuclear plants normally supply energy for the base load. Although nuclear generation of electricity does not produce carbon dioxide, sulphur dioxide or other pollutants associated with the combustion of fossil fuels, opponents of nuclear power argue against its use due to issues like the long term problems of storing radioactive waste and the potential for severe radioactive contamination by an accident. In Sweden, which has the highest nuclear power production capacity in Scandinavia, due to public protests, plans have been made to reduce its use, and instead focus on renewable energy. In the 1970s there was a strong debate in Denmark as to what extent nuclear power should be utilised, and consequently it was decided to stop all plans for nuclear power production. Currently the last experimental generator in Denmark is being shut down.

### 2.2.3 Hydroelectric power

Hydroelectric power from potential energy of the elevation of waters, now supplies about 19% of world electricity, and large dams are still being designed. Nevertheless, hydroelectric power produced in this way is probably not a major option for the future energy production in the developed world, because most of the major sites within the relevant countries with a potential for harnessing gravity in this way are either already being exploited or are unavailable for other reasons such as environmental considerations. This is, indeed, the case in Norway, Sweden and Finland, where the public opinion has turned against further use of hydropower. Hydroelectric power can be far less expensive than electricity generated from fossil fuel or nuclear energy. This applies especially in the spring when dams are overflowing. The price can get high in dry years, though, especially if it is uncertain whether the dams contain enough water for electricity production according to plans. Hydroelectric energy produces essentially no carbon dioxide, in contrast to the burning of fossil fuels or gas, hence it is classified as a renewable source of energy.

### 2.2.4 Wind turbines

A wind turbine converts the kinetic energy in wind into mechanical energy, which can then be transformed into electricity. Modern wind turbines can deliver about $3MW$ at maximum but this number is expected to increase. The total production over a whole year is on average 15% of installed capacity. A number of wind turbines is often collected into one unit, called a wind farm. Wind farms are both found on land and offshore. Wind turbines can not be controlled in a similar manner to many other production units, as electricity is only produced when the wind is blowing. Therefore, are wind forecasts or production forecasts normally used in order to plan the production in a system containing wind turbines. Denmark is a leading nation in design, production and use of wind turbines. Currently, wind power provides for approximately 15% of the total electrical energy used in the country per year with an installed capacity of 3 $GW$ [7].

#### 2.2.4.1 Wind power production forecasts

Wind power production forecasts are important both when planning system operation and when selling wind energy at a deregulated market. Prediction methods are therefore constantly being developed and improved. One of the latest improvement is better knowledge of the prediction uncertainty. Knowledge producers can use to manage their risk and exploit profit opportunities.

Many different prediction systems currently exists. They address a wide range of problems and have different prediction horizons. The forecasts used in this context are normally categorised in the literature as "short-term predictions".

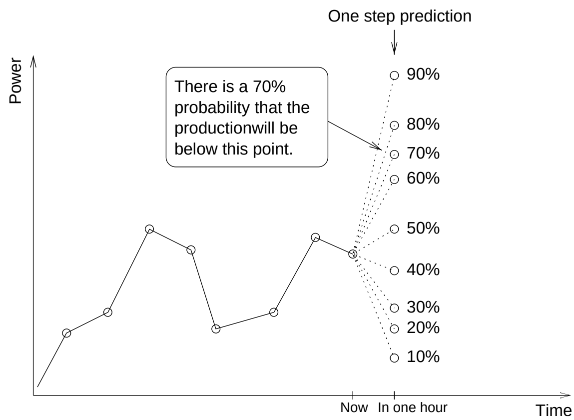


Figure 2.1: *An example of how probabilistic information can be included in a prediction. Not only one but a number of possible production levels is included in the prediction.*

Meaning that they usually have a prediction for the total production in each hour for the next 48 hours.

Such production forecast are based on a numerical weather predictions which cover a large area. Detailed, site specific, information is, therefore, not provided. Some forecasting systems solve this by including micro and meso-scale models that describe the surroundings of the wind farm. Others use mathematical, non physical, models to catch the site specific characteristics. Statistics are most often used to improve the forecasts.

The most common output are point predictions which state how much production is expected in each of the $n$ following hours. Some systems also provide information about the uncertainty, often done using confidence intervals or an estimate of the standard deviation. The latest addition is probabilistic information about the possible future production, see the example in Figure 2.1.

The best known simple forecasts are called persistence and mean. Persistence predicts future production to be equal to the current production. The mean forecast predicts that future prediction will be equal to the mean of historical observed production. Neither of these two predictions perform well but they are often used as benchmarks when testing other prediction methods. See [8] and [9] for further comments on production forecasting methods.

Chapter 3

# A brief description of production methods and transmission possibilities in Scandinavia

## 3.1 Production overview

The great differences in the landscape of Scandinavia are reflected in the power systems of the respective countries. The deep fjords in Norway create enormous possibilities for hydropower production, where as the flat landscape in Denmark has been the driving force behind a large wind power industry. Both Sweden and Finland rely on a mixture of hydroelectric, thermal and nuclear power production. Because of these marked differences on one hand and strong culture and economic ties on the other hand, an organisation called Nordel was founded in the 1960s making power trading between Norway, Sweden, Finland and Denmark possible. The interconnections, built on the initiative of Nordel, are now the basis for the modern deregulated electricity market in Scandinavia, called NordPool. The market was originally set up in Norway 1991 but in 2001 all the original members of Nordel had joined. The market has been quite successful and the underlying market ideas have been used as the basic concepts for the development of new markets around the world.[10]

### 3.1.1 Denmark

Almost all electrical energy in Denmark is produced in thermal plants although the system holds the highest share of wind power in the world. 15% of the energy is produced using renewable sources and the level of CHP[1] is now up to 30%. All major cities and large towns have a district heating system, supplying half of all hot water used in the country.

The Danish grid is split into two independent grids. The Western area (DK-1,DK-W) is comprised of Jutland and Funen but the Eastern area (DK-2,DK-E) comprises Zealand and the islands north of the Great Belt. [11], [6], [10]

### 3.1.2 Finland

The Finnish power production mixture contains hydroelectric, thermal and nuclear power. The largest share, 60%, is produced by using thermal and CHP plants. 25% is produced by using nuclear power and 15% by the use of hydropower. District heating has developed rapidly since the 1950s and covers now more than 40% of the heating demand. [11], [6]

### 3.1.3 Norway

Hydroelectricity is absolutely dominating in the Norwegian power pool and hardly any electricity is produced in thermal plants. The cheep hydroelectricity has given electricity-intensive industry a possibility to flourish and made the used of electric heating widespread. District heating systems are not common.[11], [6]

### 3.1.4 Sweden

Like in Finland the main power production is by the use of hydroelectric, thermal and nuclear sources. In 2003 half of the energy came from nuclear plants, 40%

[1] Combined heat and power (or CHP) is the use of a power station to simultaneously generate both heat and electricity

from hydro-dams and 10% from thermal plants. There is a long tradition for district heating systems in Sweden but due to low electricity prices many of the heating systems are driven by electric heat pumps. In the beginning of the 1980s loup protests against nuclear power began to gain ground and a long term operation with the goal to limit the number of nuclear plants in Sweden to 12, was started. Smaller nuclear plants have therefore slowly been shut down and the focus has been moved to renewable energy sources such as wind. [11],[6]

## 3.2 Transmission between the Scandinavian countries

In Figure 3.1 the transmission capacity between the Scandinavian countries and to other parts of Europe is shown [12]. The dotted line brakes the area into a hydro and a thermal part. The interconnections allow hydropower to be transported from North to South in periods of sufficient reservoir leves and the other way when reservoir levels are low. Energy is therefore often transported trough Denmark because of its geographical localisation between the hydro area and the rest of Europe. Cheap nuclear power is imported from Russia to Finnland.

The transmission system is not a static system. Maintenance, breakdowns and limitations often change the situation so that no or little energy can be transported trough individual connections. For instance was it impossible to transmit electricity form Sweden to East Denmark during the coldest part of March 2005 due to internal transmission limitations in Sweden [13]. Such limitations can have a high effect on the price, depending on the season and the hour of the day. Another example of limited transmission is the reconstruction of the Kontek connection between East Denmark and Germany. It was so extensive that the connection was on and off during a six month period from May to October 2004 [14].

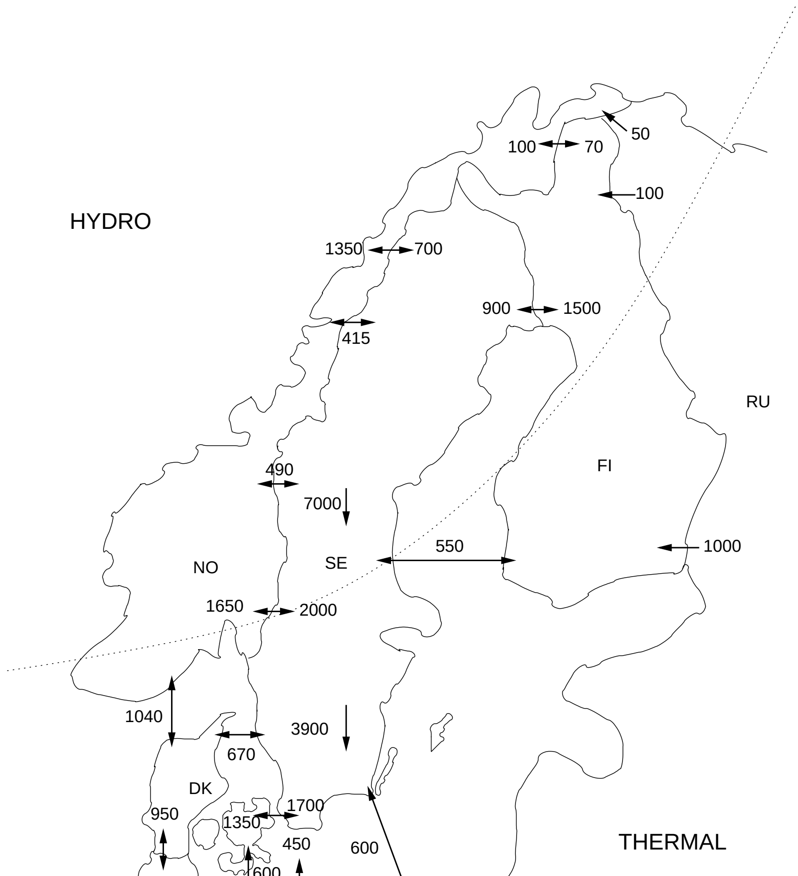


Figure 3.1: *Transmission capacity within and out of Scandinavia [12]. It is also shown how the area is split into a hydroelectric and a thermal part.*

Chapter 4

# The Scandinavian electricity market

## 4.1 The historical co-operation

### 4.1.1 Nordel

Before the deregulation of the electricity markets state owned enterprises dominated the power sector in Norway, Sweden and Finland. Although the situation was not identical in all the countries, they all shared the same characteristics. One large enterprise dominated the whole process from generation to retail. In the 1960s these nations formed Nordel in order to make trading of electricity between the borders possible. The main idea behind Nordel was that each country had enough generation capacity to be self-sufficient but trading would be a tool for operating the whole Scandinavian system in an optimal way. Investments in interconnection between countries were generally reasoned by expected savings. The countries traded by informing each other with marginal production costs and in the case of possible savings the price was set as the average of the two costs involved.

This structure lead to over investment and a poor return to investors. But the competition had a positive effect on the utilities, where no significant efficiency problems were observed.

Nordel still exists, now with a different purpose and new countries have been included. [15],[10]

### 4.1.2 Steps towards a deregulated market

Competition in electricity production and distribution was stared in 1990 when the electricity system in England and Wales was deregulated under Margaret Thatcher's government. Since then, many other countries have followed suite and the attention has been focused on these matters in Europe. Norway was the first Scandinavian country to deregulate its electricity market when the new electricity act came into force in 1991. The idea was to reduce the differences in power cost between regions and to increase operational efficiency in generation and distribution. Norway's market, NordPool, opened in 1993 and has since been the foundation, along with Nordel's transmission lines, for a common Scandinavian electricity market. The Scandinavian method is similar to what has been done in other countries, for instance Germany. The electricity system is split into four main parts: Generation, transmission, distribution and retail[1]. Competition is allowed both in generation and retail but transmission and distribution is considered to be a natural monopoly. A non-profit state enterprise takes therefore the responsibility for transmission, distribution and system stability. That enterprise is called the transmission system operator or TSO in short.

The initial step in all the Scandinavian countries was to separate the existing state enterprise into a generation unit and transmission unit. Then the transmission and distribution network was opened to other producers and a new fee structure, minimising discrimination, implemented. This was an extensive change, specially for small producers who had been in the shadow of the large enterprises.

[1] The following definitions are used: Generation is the actual generation of electricity. The transmission grid allows large generation facilities to produce large quantities of energy which is then deliver it to distribution networks. Delivery is the part between transmission and user purchase from an electricity retailer. Electricity retailing is the final process in the delivery of electric power from generation to the consumer, it includes metering and billing.

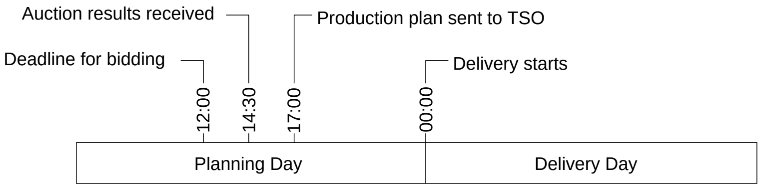


Figure 4.1: *A timeline showing the process of biding, accepting and planning at NordPool's spot market.*

## 4.2 NordPool

NordPool is the common Scandinavian electricity market. It was originally founded in Norway and operated for the first time in 1993. All the large nations, Sweden, Finland and Denmark, had joined by the first of October 2000. Today NordPool comprises more than electricity markets, emission allowances and financial products are also traded. The focus here will though only be on the three electricity markets; Elspot, Elbas and the regulation market. All these markets are currently growing, although the growth is not as it was the first years. From 1993 to 2004 the total energy turned over in the spot market grew from $10TWh$ to $167TWh$. [16],[6]

### 4.2.1 Elspot

Elspot is a physical power market, organised by NordPool. Energy is both sold and bought in the market and participation is free (producers are not forced to sell in the market[2]). Anyone who has signed the necessary agreements with Elspot and fulfils the set requirements can act on the market. Elspot is a day-ahead electricity market which means that all purchasing and selling is carried out the day before delivery. Each day is divided into 24, one hour long contracts and no special contracts exist for base or peak load. Bids for each of the 24 contract periods must be submitted to NordPool before noon, see timeline in figure 4.1. Three different bid types can be submitted

[2] In some electricity markets, producers are forced to bid energy in order to secure that enough energy is available at all times.

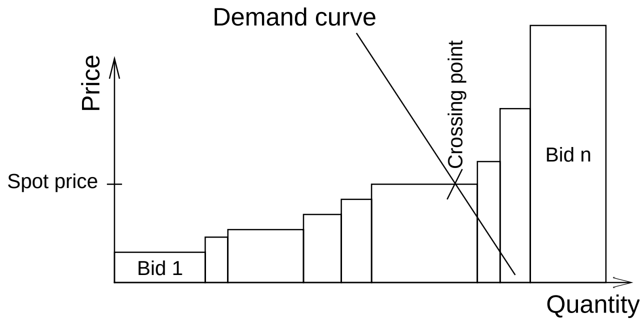


Figure 4.2: *Supply and demand curves are used to settle the price at NordPool. The demand curve is actually made of bids, just as the supply curve but it is not elastic and thus drawn as a line.*

| | |
|---|---|
| Basic | A basic bid is either a sell or a buy bid, valid for one specified hour in one area. Both energy price and quantity are listed. |
| Block | A block bid is a series of $n$ basic bids valid in $n$ adjacent hours. The series can not be split so either all the bids in the block are accepted or rejected. Average energy price and quantity for the hours is listed, not a specific price for each hour. |
| Flexible | A flexible bid is a basic bid without any specified hour. Instead the hour is set as the hour with the highest price where the bid is accepted. If no hour has a price higher than the price of the bid, the bid is rejected. |

When all bids have been collected, demand and supply curves are created by ordering all buy bids in an decreasing price order and all sell bids in an increasing price order. The point where the curves cross defines the system price, see figure 4.2. Now only one price exists for the whole NordPool area but it is possible that transmission constraints are violated. In order to solve this problem the NordPool area is divided into predefined sub areas[3] until no transmission constraints are violated. The results could be different price in all the areas. One example of this is shown in table 4.1. There the system price in Denmark-East and Sweden is listed for two different cases. First when transmission capacity between the countries was zero and then when it was at its maximum. The numbers show how different the prices can be in neighbouring areas if the interconnection is not available . At this stage, when a solution that does not violate any transmission constraints has been found, participants are informed which of

[3] The NordPool area is divided into sub areas, so that in each area no transmission constraints are observed in normal operation. These areas are; Norway, Sweden, Finland, Denmark West (West of the Great Belt) and Denmark East (East of the Great Belt).

| Day | Transmission capacity | DK-E price | SE price |
|---|---|---|---|
| 2005-03-09 8:00 | 0 | 708.21 | 226.03 |
| 2004-10-31 2:00 | Maximum | 138.01 | 138.01 |

Table 4.1: *The prices (DKK) show how the area price is connected as long as there is enough transmission capacity between two areas. Without a interconnection the price difference can get extremely high.*

these bids were accepted and which were not. Producers must then plan their production and inform the TSO how it will be carried out. This plan will from now be called "the original plan". The plan is final from Elspot's point of view cause all accepted bis are binding, some changes can though be made, but those changes are in connection to the regulation market not the spot market, see section 4.2.3. [17]

### 4.2.2 Elbas

The time between bidding and delivering at Elspot can be up to 36 hours. During this period the realised consumption and production can deviate from what was expected at the auction time. Consequently producers and consumers may find a need for trading during these 36 hours. This is where the Elbas market steps in. It is open at all hours, giving the participants an opportunity to trade energy down to one hour in advance. The market consists of one hour contracts only and the price mechanism is different from Elspot's. Bids are order by price and if two bids have the same price, the time when the bid was received breaks the tie, see figure 4.3. If a participant accepts a bid on the market, an agreement is signed between him and the bidding participant, and the bid is removed from the line. Elbas is not active in all areas, currently Finland, Sweden and Denmark-East participate.[6]

### 4.2.3 The Regulation Market

The regulation market is used to balance generation and load in real-time. The market is not harmonised in the NordPool area although some effort has been made in order to form one common regulation market for all the countries [18]. The physical part, though, is identical in all the areas, the main difference lies in price settlements and participation fees. No matter what price structure is examined, the aim is always that:

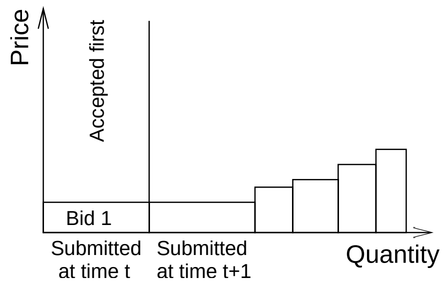


Figure 4.3: *Participants at Elbas can only accept the bid with the highest priority. The bid with the lowest price has the highest priority, and if two or more bids have the lowest price, the oldest one is put first. When that bid has been accepted it is removed from the sequence and the next bid can be accepted.*

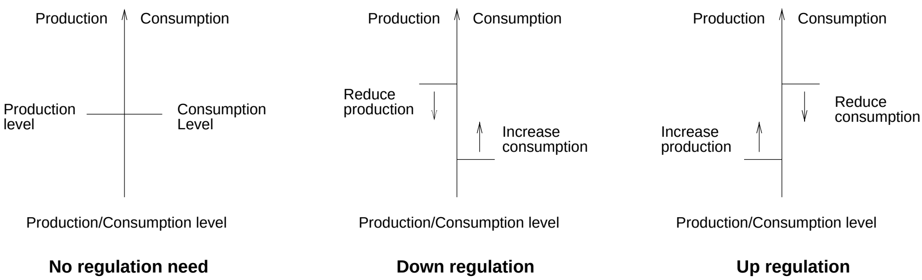


Figure 4.4: *Possible regulation scenarios.*

- Prices should reflect production costs.
- Prices should discourage producers to plan imbalances.

Up regulation is performed by increasing generation and down regulation by decreasing generation, see figure 4.4. Regulation can also be performed, technically, by increasing or decreasing consumption but that would require special contracts between retailers and consumers allowing the retailer to put the consumer off-line. Currently, such contracts are not available to the public and therefore, no regulation performed on the consumption side.

Up regulation bids are made of a quantity which the producer can deliver with a few minutes notice and the minimum price he must receive for it. Down regulation bids are made of the quantity which the producer is willing to stop producing and how high payment he requests for stopping. The TSO accepts

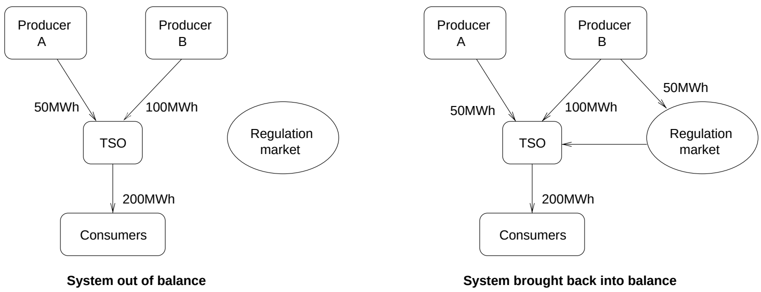


Figure 4.5: *The figure demonstrates how the regulation market is used to bring the system back into balance. When producer A fails to produce the 100 $MWh$ he was supposed to, producer B is selected to produce the missing energy. Producer B is selected because he had the best offer in the regulation market.*

bids, either up or down, in order to keep the system in balance. He buys the contract with the lowest price when regulating up but sells the contract with the highest price when regulating down. In other words, if extra production is needed the most cost efficient production is started and if less production is needed the least cost efficient production is stopped.

The way the price is determined and how imbalance is calculated depends on the market area. Here the imbalance between the original plan of the producer and actual production is considered as the imbalance. Imbalances on the consumption side are not described here as they lie out of the scope of this project. The two possible price settlements are examined in section 4.2.3.1 and 4.2.3.2.

The regulation need depends on the original production plan and a corrected version of it. Producers are allowed to correct the original plan because many things can happen between the time of bidding and delivering, so forcing the producers to keep their plans 100% could jeopardise the grid stability. Thus, producers can request a permission to produce less or more than originally planned. This is probably best explained by an example, see figure 4.5. If producer A has to change his production for some reason, the whole system is brought out of balance and the stability is at risk. Some other producer must therefore increase or decrease his production in order to bring the system back into balance. As it is the TSO's job to keep the system in balance, the TSO accepts bids from the regulation market in order to select a producer to adjust his production so that producer A can be allowed to deviate from the original plan. After the change, producer A has a changed plan but the other producer, who responded, is following his original plan and selling regulation power. Producer A pays for the additional cost involved.

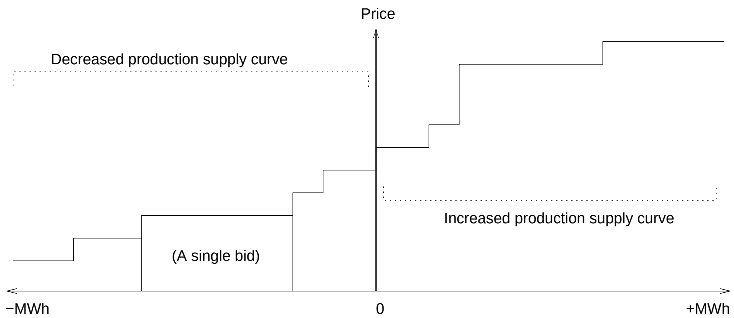


Figure 4.6: *The bids in the regulation market are ordered so that they form two different supply curves. One for decreased production (on the left hand side of the 0) and one for increased production supply (on the right hand side of the 0).*

The total regulation need $RN_t$ at time $t$ is defined as the total difference between the original plans and changed plans of all producers.

$$RN_t = \sum_{p \in \langle P \rangle} OP_{p,t} - CP_{p,t} \tag{4.1}$$

$P$ is the set of all producers, $OP_{t,p}$ is the quantity producer $p$ originally planned to produce at time $t$ and $CP$ is the quantity he can actually produce. $RN_t$ can be, and actually sometimes is zero, although all producer have changed their plans.

Bids for increased and decreased production are accepted with positive and negative signs respectively. Two different supply curves are formed. One for decreased production and another for increased consumption. The construction of the curves is demonstrated in figure 4.6. When regulation power is needed, bids are accepted going from 0 to the amount of power needed (x-axis). The price seen by the producer offering regulation power and the producer responsible for the regulation need is calculated in two ways. Norway, Sweden, Finland and East Denmark have agreed on a marginal pricing but in West Denmark the price is determined as the weighted average price of all offers accepted in that hour. [19],[16],[20],[21]

The function of the regulation market was extremely clear on the 8th of January 2005, when a large front passed over Jutland, the wind speed went over 25 $m/s$ and wind turbines had to be stopped in order to protect them from mechanical breakdown. As a result of this wind power generation fell rapidly from covering all the consumption to covering less than 5%. The response of the regulation market can be seen in figure 4.7. This is an extreme case, not seen frequently,

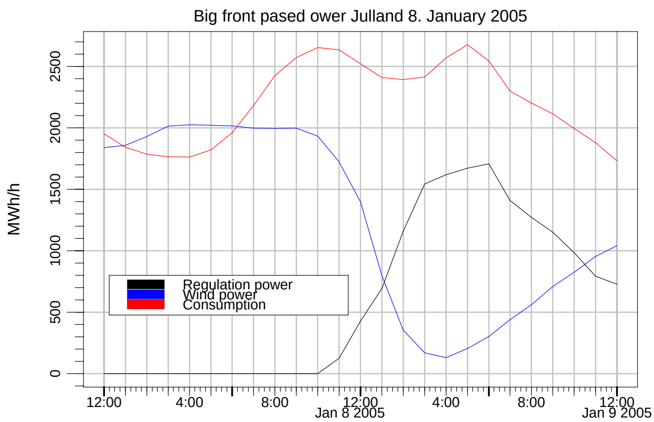


Figure 4.7: *A big front passed over Jutland the 8th of January 2005. Around 12:00 AM, the wind speed went over* $25m/s$*, in that situation normal wind turbines must be shut down in order to protect machinery. As a result of this wind power generation fell rapidly from covering all the consumption to covering less than 5%. The regulation market responded quickly.*

but it demonstrates well the function of the regulation market. [22]

System balance determines whether a producer is charged for his imbalance or not. If the whole system needs energy and a producer is producing too much. Regulation is not charged because the producer is bringing the system back into balance. Put differently, a producer is only charged for regulation if his imbalance has the same sign as the balance of the whole system-price.

#### 4.2.3.1 Marginal regulation prices

Regulation prices in some NordPool areas are set as the marginal regulation price as long as it is in correct relation to the spot price, other wise it is set equal to the spot price. The correct relation is defined as: down regulation bids must have a price lower than the spot price and up regulation bids must have a higher price than the spot price. This ensures that producers responsible for a regulation need never gain from their imbalances.

A free regulation price $P_{rf}$, disregarding the correct relationship, is defined as:

$$P_{rf}(RN_t) = \begin{cases} p^-(RN_t) & \text{if } RN_t < 0 \\ p^+(RN_t) & \text{if } RN_t > 0 \end{cases} \tag{4.2}$$

Where $p^{\cdot}$ is the supply curve, $p^-$ for down regulation and $p^+$ for up regulation. $P_{spot}$ is the spot price. The actual regulation price $P_r$, holding the correct relation, is defined as:

$$P_r(RN_t) = \begin{cases} \min(P_{rf}(RN_t), P_{spot}) & \text{if } RN_t < 0 \\ \max(P_{rf}(RN_t), P_{spot}) & \text{if } RN_t > 0 \end{cases} \tag{4.3}$$

This price settlement is shown for down regulation in figure 4.8. Using this system producers offering regulation power receive at minimum what they offered but the producers responsible for the need must pay for the most expensive regulation offer accepted.

#### 4.2.3.2 Weighted average regulation prices

Regulation prices in some NordPool areas are set as the weighted average price of all offers accepted as long as it is in correct relation[4] to the spot price, otherwise it is set equal to the spot price. This ensures that producers responsible for regulation need never gain from their imbalances.

The free regulation price $P_{rf}$, disregarding the correct relationship, is now defined as:

$$P_{rf}(RN_t) = \begin{cases} \frac{-\int_0^{-RN_t} p^-(v)dv}{-RN_t} & \text{if } RN_t < 0 \\ \frac{\int_0^{RN_t} p^+(v)dv}{RN_t} & \text{if } RN_t > 0 \end{cases} \tag{4.4}$$

As before is $p^-$ the down regulation supply curve, $p^+$ the up regulation supply curve and $P_{spot}$ the spot price. The actual regulation price $P_r$ is then defined as:

$$P_r(RN_t) = \begin{cases} \min(P_{rf}(RN_t), P_{spot}) & \text{if } RN_t < 0 \\ \max(P_{rf}(RN_t), P_{spot}) & \text{if } RN_t > 0 \end{cases} \tag{4.5}$$

Using this system the producers offering regulation power receive exactly what they offered but the producers responsible for the need pay the average regulation cost in that hour. Figure 4.8 and 4.9 show two possible price settlement scenarios, both for down regulation.[23],[24]

[4] See definition in section 4.2.3.1

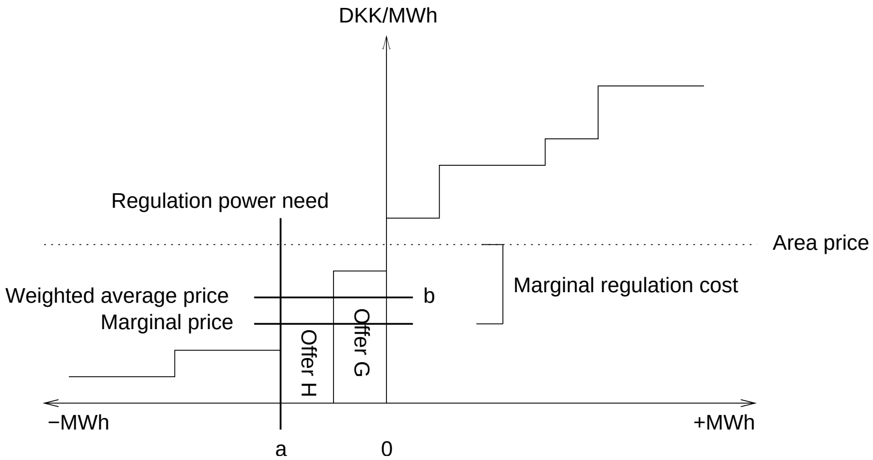


Figure 4.8: *The regulation price in each hour can either be set as the marginal price or the weighted average of the regulation offers that are accepted. In this example two offers are accepted (G and H) to satisfy the regulation need a. The price depends on the system.*

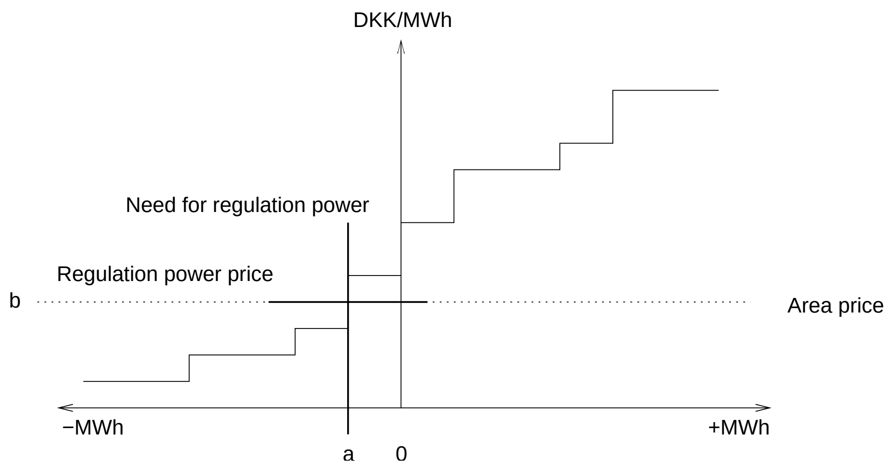


Figure 4.9: *When the market regulation price is on the wrong side of the spot price (above for down regulation and below for up regulation) it is set equal to the spot price. This makes it impossible for the buyer to gain by planning imbalances.*

#### 4.2.3.3 Regulation cost and price

Regulation cost is defined as the penalty a participant responsible for regulation need must pay for every $MWh$ which he adds to the system imbalance. The regulation cost is not equal to the regulation price. For a regulation price $P_r$ and a spot price $P_s$ the regulation cost, $C_r$, is defined as the difference between the two:

$$C_r = |P_s - P_r| \tag{4.6}$$

### 4.2.4 The status of wind power at NordPool

Energy produced by wind turbines in Denmark has been priced and handled in many different ways since the first machines were installed. In the early days all wind energy could be sold at a high price as a prioritised dispatch[5]. This was done in order to strengthen the industry. Now the aim is to make the wind energy competitive and the pricing has therefore been moved closer to what applies for energy generated in traditional ways. The main tools used to control wind energy prices and transmission are:

| | |
|---|---|
| Priority | Is the energy prioritised or not. |
| Price | Is the price predefined as a constant or does it follow the spot price. |
| Tariff | Is there a feed in tariff, for instance could the price be the spot price plus a feed in tariff. |
| Transmission | Are the transmission fees subsidised. |
| Regulation | Has the producer got to pay for regulation. |

Wind turbines are currently divided into different classes and the energy coming from each class is handled in a different way. For instance, the energy coming from new on-shore wind turbines is not priced the same way as the energy coming from new off-shore wind turbines. The price of the offshore energy is kept higher in order to encourage investors to participate in offshore projects.

Here it is assumed that wind energy must be sold on the spot market, but that the producer is not responsible for his balance. This means that all additional energy can be sold in the case when less energy is bidden than produced and vice versa. However, the wind power producer must pay for the regulation.

[5] Consumers must buy all prioritised energy although it might be more expensive than non-prioritised energy.

# Part II

# Theory

CHAPTER 5

# Key theory

## About this chapter

In this chapter the main mathematical tools used in the following part will be presented. The discussion is, though, only intended to cover the most basic elements. References should be looked up for a more complete description.

## 5.1 Random variables

The power output from wind turbines is ever-changing, and it is therefore well described using random variables. Consequently do random variables play an important role when analysing how wind energy should be sold. Here the main tools used to describe and evaluate the properties of random variables are listed. A complete description of discrete and continuous random variables can be found in [25].

### Probability

**A probability density function** is any function $f(x)$ that describes the probability density in terms of the input variable $x$. For a valid probability density function equations (5.1) and (5.2) hold.

$$\int_{-\infty}^{\infty} f(x)\,dx = 1 \tag{5.1}$$

$$f(x) \geq 0 \tag{5.2}$$

**Probability mass functions** give the probability that a discrete random variable is exactly equal to some value. For a valid probability mass function that describes the random variable $X$, which belongs to the set $H_1 = \{x_1, \ldots, x_N\}$ the following must hold.

$$\sum_{i=1}^{N} f(x_i) = 1 \tag{5.3}$$

$$f(x_i) > 0 \qquad \text{for all } x_i \text{ in } H_1 \tag{5.4}$$

**A cumulative distribution function** completely describes the probability distribution of a real-valued random variable, X. For a real number x, the cumulative distribution function is defined as:

$$F(x) = \mathbb{P}\{X \leq x\} \tag{5.5}$$

The right-hand side represents the probability that the variable X takes on a value less than or equal to $x$. The probability that $X$ lies in the interval $(a, b)$ is therefore $F(b) - F(a)$ if $a < b$. It is conventional to use a capital $F$ for a cumulative distribution function, in contrast to the lower-case $f$ used for probability density functions and probability mass functions.

If the cumulative distribution function $F$ of $X$ is continuous, then $X$ is a continuous random variable; if furthermore $F$ is absolutely continuous, then there exists a probability density function $f(x)$ such that

$$F(b) - F(a) = \mathbb{P}\{a \leq X \leq b\} = \int_a^b f(x)\mathrm{d}x \tag{5.6}$$

Furthermore, we have

$$\lim_{x \to -\infty} F(x) = 0 \tag{5.7}$$

$$\lim_{x \to +\infty} F(x) = 1 \tag{5.8}$$

If $X$ is a random variable describing the power output from a wind farm[1] which can at maximum provide $x_{max}$ units then the following holds:

$$\lim_{x \to 0^-} F(x) = 0 \tag{5.9}$$

$$\lim_{x \to x_{max}^+} F(x) = 1 \tag{5.10}$$

Where $0^-$ means that zero is approached from below and $x_{max}^+$ than $x_{max}$ is approached from above.

### Expectation

If the probability distribution of $X$ admits a probability density function $f(x)$, then the expected value of $X$ can be computed as:

$$\mathbb{E}[X] = \int_{-\infty}^{\infty} x f(x) dx \tag{5.11}$$

The expected value of a function $g$ of the random variable $X$ given that $X$ admits a probability density function $f(x)$ can be calculated as

$$\mathbb{E}[g(X)] = \int_{-\infty}^{\infty} g(x) f(x) dx \tag{5.12}$$

If $g$ is a function of two dependent random variables $X_1$ and $X_2$ which admit a joint probability density function $f(x_1, x_2)$, then the expected value can be computed as

$$\mathbb{E}[g(X_1, X_2)] = \int_{-\infty}^{\infty} g(x_1, x_2) f(x_1, x_2) dx_1 dx_2 \tag{5.13}$$

If $X$ is a discrete random variable taking $N$ values from the set $H = \{x_1, \ldots, x_N\}$ and corresponding probabilities $x_p, \ldots, p_N$ which add up to 1, then the expected value of $X$ can be computed as:

$$\mathbb{E}[X] = \sum_{i}^{N} p_i x_i \tag{5.14}$$

[1] Wind farm is a group of wind turbines labeled as one.

## 5.2 Quantile regression

In [26] a method to add probabilistic information to an existing wind prediction system is presented. It relies on quantile regression [27] which is also known as percentile regression.

The $\tau$ quantile $Q(\tau)$ for the random variable $Y$ is a function which admits the following relation:

$$\mathbb{P}\{Y < Q(\tau)\} = \tau \tag{5.15}$$

Or, put differently:

$$F(Q(\tau)) = \tau \tag{5.16}$$

where $F$ is the cumulative distribution function describing $Y$.

In quantile regression the quantile $Q(\tau)$ is modelled as a linear function of $p$ known regressors $x$ and unknown coefficients $\beta$

$$Q(\tau) = \beta_0(\tau) + \beta_1(\tau)x_1 + \cdots + \beta_p(\tau)x_p \tag{5.17}$$

Given $N$ observations on the form $(y_i, x_{i,1}, \ldots, x_{i,p})$ the unknown coefficients $\beta(\tau)$ can be evaluated for a given value of $\tau$. A check function is defined as:

$$p_\tau(e) \begin{cases} \tau e & \text{if } e \geq 0 \\ (\tau - 1)e & \text{if } e < 0 \end{cases} \tag{5.18}$$

The value of $\beta.$ is estimated by minimising the sum of the check function value for the given observations:

$$\min_{\beta.} \sum_i^N p_\tau \left(y_i - (\beta_0 + \beta_1 x_{i,1} + \cdots + \beta_p x_{i,1pi,1})\right) \tag{5.19}$$

The minimisation problem can be solved using linear programming when the function $p$ has been transformed using the transformation presented in Section 5.3.4.

Note that $Q(0.5)$ is better known as the median. Replacing $p_\tau(e)$ by $p(e) = e^2$ gives the least squares error estimate.

## 5.3 Linear, quadratic and stochastic programming

### 5.3.1 Linear programs

Linear programming problems are optimisation problems in which the objective function and the constraints are all linear. An example of such a problem is:

$$\begin{aligned} z &= \max_x cx \\ Ax &\leq b \\ x &\geq 0 \end{aligned}$$

Where $c$ is a $n$ dimensional row vector, $A$ is a $m$ by $n$ matrix, $b$ is a $m$ dimensional column vector and $x$ is and $n$ dimensional column vector of variables with unknown value.

The function that is maximised or minimised is called the objective function. A linear program is said to be unbounded if the vector $x$ which maximised $z$ contains one or more infinite values. If a linear program is not unbounded, the optimal solution can be found with certainty.

Constrains can also be formulated as:

$$\begin{aligned} a_i x &\geq b_i \\ a_i x &= b_i \end{aligned}$$

where $a_i$ is the $i$'th row in $A$ and $b_i$ is the $i$'th value in $b$.

### 5.3.2 Quadratic programs

A quadratic program is like a linear program except that quadratic terms are allowed in the objective function. The quadratic formulation is often necessary if the price of some commodity that is begin purchased depends linearly on the amount bought.

An example of a quadratic program is:

$$\begin{aligned} z &= \max_x x^T Q x + xcx \\ Ax &\leq b \\ x &\geq 0 \end{aligned}$$

Where $x^T$ is the vector $x$ transposed and Q is a $n$ by $n$ dimensional matrix.

### 5.3.3 Stochastic programs

Stochastic programming is a framework for modelling optimisation problems that involve uncertainty.

An optimisation problem has a objective function defined as:

$$g(x, A) \tag{5.20}$$

where x is the decision vector and A is a set of future events. If there are $N$ possible future events $A_i$, the optimisation problem can be transformed into a stochastic program by redefining the objective function as:

$$\sum_{i}^{N} p_i g(x, A_i) \tag{5.21}$$

where $p_i$ is the weight of the event $A_i$. A common configuration of the weights is $p_i = \mathbb{P}\{A_i\}$. Using that configuration the stochastic formulation in Eq. (5.21) is in fact maximisation or minimisation of the expected value of $g$.

An extensive description of Stochastic Programming can be found in [28].

### 5.3.4 Formulation of V-shaped functions

A V-shaped piecewise linear function is defined as:

$$R\{e\} = \begin{cases} c_1 e & \text{if } e \leq 0 \\ c_2 e & \text{if } e > 0 \end{cases} \tag{5.22}$$

This function can not be inserted directly into a linear program but a simple transformation exists so that is can be used. The optimisation problem:

$$\max_{x} x - R\{x - a\} \tag{5.23}$$

can be solved using linear programming by transforming it into

$$\max_{x,d,u} x - c_1 d - c_2 u \tag{5.24}$$

Subject to:

$$-d \leq x - a \tag{5.25}$$

$$-d \leq 0 \tag{5.26}$$

$$x - a \leq u \tag{5.27}$$

$$0 \leq u \tag{5.28}$$

$$d, u, x \geq 0 \tag{5.29}$$

In the case when $x - a < 0$ constraint (5.25) and (5.28) become active. Decreasing $u$ increases the income so in an optimal solution $u$ must be equal to zero. Decreasing $d$ also increases the income so in an optimal solution $d$ must be as small as possible, that is $d = x - a$. In the case when $x - a > 0$ constraint (5.26) and (5.27) become active. Decreasing $u$ increases the income so in an optimal solution $u$ must be set to $u = x - a$. Decreasing $d$ also increases the income so in an optimal solution $d$ must be as small as possible, that is $d = 0$.

# Part III

# Study

Chapter 6

# Optimal bidding using quantile regression

## 6.1 Theoretical solution

### 6.1.1 Overview

In [2], John Bremnes suggests a simple bidding strategy for wind power producers who sell their energy at a day-ahead energy market. In this section the method will be described, extended and a case study performed, using resent predictions and prices in East-Denmark.

### 6.1.2 Problem definition

A wind power producer sells his energy at a spot market. Bids for tomorrows production must be delivered before noon today, so there are 12 to 36 hours between bidding and delivering. If the generated electricity does not match the bid exactly, regulation cost must be paid. Up regulation if the production is below the bid, down regulation if the production is above the bid. The producer receives the spot price for every energy unit delivered to the network.

A probability density function describing the possible production levels observed $n$ hours ahead is available at noon for the following day. Now the producer wants to know how much he should bid in order to minimise the regulation cost.

### 6.1.3 Finding the optimal bid

It is assumed that the producer has a forecasting system that can provide a probability density function $f_{X_t}^{t'}$ calculated at time $t'$ describing the production, $\{X_t\}$, at time $t$:

$$f_{X_t}^{t'}(x_t) \tag{6.1}$$

It is known that the following holds:

$$f_{X_t}^{t'}(x_t) = 0 \qquad \text{for } x_t < 0 \tag{6.2}$$

$$f_{X_t}^{t'}(x_t) = 0 \qquad \text{for } x_t > x_{max} \tag{6.3}$$

where $x_{max}$ is the installed generation capacity of the wind turbines included in the prediction.

A general formulation of the income at time $t$ is:

$$I_t(\mathbf{u}_t, X_t) \tag{6.4}$$

where $\mathbf{u}_t$ is a vector of decision variables. The expected income $z$ is then (see Section 5.1) calculated as:

$$z = \mathbb{E}[I_t] = \int_0^{x_{max}} I_t(\mathbf{u}_t, x_t) f_{X_t}^{t'}(x_t) dx_t \tag{6.5}$$

The maximum total income over a period of time, labeled as t, can be gained by adjusting the decision parameter $\mathbf{u}_t$ at each time $t$ so that the expected income $z$ is maximised:

$$z^* = \max_{\mathbf{u}_t} \int_0^{x_{max}} I_t(\mathbf{u}_t, x_t) f_{X_t}^{t'}(x_t) dx_t \tag{6.6}$$

#### 6.1.3.1 A simple model including a spot and regulation market

Lets now assume that a producer will place a bid on the spot market between zero and the installed capacity of his wind farm. The bid is always accepted no matter how high it is, and it does not influence the system price[1].

In that case the income at time $t$, can be defined as:

$$I_t = S_t^P x_t - R\{x_t - S_{t'}^Q\} \tag{6.7}$$

Where the variables are described as:

$$\begin{aligned}
S_t^P &: \text{The } \mathit{S}\text{pot market } \mathit{P}\text{rice when energy is delivered} && (6.8)\\
x_t &: \text{The produced electricity at time } t && (6.9)\\
S_{t'}^Q &: \text{The } \mathit{Q}\text{uantity bid in the } \mathit{S}\text{pot market at time } t' && (6.10)\\
R\{e\} &: \text{The regulation cost} && (6.11)\\
R_t^D &: \text{Down regulation cost at time } t && (6.12)\\
R_t^U &: \text{Up regulation cost at time } t && (6.13)
\end{aligned}$$

And the regulation cost function is defined as:

$$R\{e\} = \begin{cases} R_t^D e & e \geq 0 \quad \text{(Down regulation)} \\ -R_t^U e & e < 0 \quad \text{(Up regulation)} \end{cases} \tag{6.14}$$

The optimal bid, maximising his expected income at time $t$ is then:

$$\begin{aligned}
z_t^* &= \max_{S_{t'}^Q} \left[ \int_0^{x_{max}} (S_t^P x_t - R\{x_t - S_{t'}^Q\}) f_{X_t}^{t'}(x_t) dx_t \right] \\
&= \max_{S_{t'}^Q} \left[ \int_0^{S_{t'}^Q} R_t^U (x_t - S_{t'}^Q) f_{X_t}^{t'}(x_t) dx_t - \int_{S_{t'}^Q}^{x_{max}} R_t^D (x_t - S_{t'}^Q) f_{X_t}^{t'}(x_t) dx_t \right] \\
&\quad + S^P \int_0^{x_{max}} x_t f_{X_t}^{t'}(x_t) dx_t
\end{aligned} \tag{6.15}$$

The spot price does not influence the decision because it is assumed that all the energy is sold at the spot market no matter what the regulation cost is[2]. The decision, how much is bid, is only influenced by regulation cost. The calculations can therefore be simplified by removing the income from the equation and

[1]System price is defined in Section 4.2.1. The assumption is reasonable because the marginal production cost of electrical energy using wind power is lower than by using most other energy sources.

[2] It is assumed that the down regulation cost is never higher than the spot price. This is a natural assumption because a down regulation cost higher than the spot price implies a production cost lower than zero.

minimise the expected regulation cost $\mathbb{E}[R\{S^Q_{t'}\}]$ instead.

$$\begin{aligned} z^r_t &= \min_{S^Q_{t'}} \mathbb{E}[R\{S^Q_{t'}\}] \\ &= \min_{S^Q_{t'}} \left[ \int_0^{S^Q_{t'}} R^U_t (S^Q_{t'} - x_t) f_{X_t}(x_t) dx_t \right. \\ & \quad \left. + \int_{S^Q_{t'}}^{x_{max}} R^D_t (x_t - S^Q_{t'}) f_{X_t}(x_t) dx_t \right] \end{aligned} \tag{6.16}$$

One way to find a solution to the minimisation problem defined in Eq. (6.16), is to calculate the derivative of the expected regulation cost with respect to the bid, find the stationary points of the derivative and compare the cost in each of them. The derivative can be found by the use of Leibniz integral rule[3]:

$$\frac{\partial}{\partial z} \int_{a(z)}^{b(z)} f(x,z) dx = \int_{a(z)}^{b(z)} \frac{\partial f}{\partial z} dx + f(b(z), z) \frac{\partial b}{\partial z} - f(a(z), z) \frac{\partial a}{\partial z} \tag{6.17}$$

The derivative of the expected regulation cost is:

$$\frac{\partial \mathbb{E}[R\{S^Q_{t'}\}]}{\partial S^Q_{t'}} = R^D_t - F(S^Q_{t'}) \left[ R^D_t + R^U_t \right] \tag{6.18}$$

Only one stationary point exists[4]:

$$F(S^Q_{t'}) = \frac{R^D_t}{R^D_t + R^U_t} \tag{6.19}$$

and the optimal bid is found by inserting into the inverse of $F$:

$$S^Q_{t'} = F^{-1} \left( \frac{R^D_t}{R^D_t + R^U_t} \right) \tag{6.20}$$

However, the producer can not know the regulation price at time $t$ when he is placing the bid at time $t'$ because the regulation prices are based on bids which are received after $t'$. He must therefore predict the price in order to find the optimal bid, and the optimal bid is therefore only optimal if the prediction is correct.

[3] Also known as differentiation under the integral sign

[4] $F$ is monotonically increasing function.

One way to predict the regulation cost is by describing it using stochastic processes. For instance, $\{R_d\}$ for down regulation cost and $\{R_u\}$ for up regulation cost. Lets assume that $\{R_d\}$ and $\{R_u\}$ admit the following probability density functions:

$$f_{R_D}(r_d) \quad : \quad \text{Up regulation} \tag{6.21}$$

$$f_{R_U}(r_u) \quad : \quad \text{Down regulation} \tag{6.22}$$

Then the expected income can be maximised by solving the following equation:

$$\begin{aligned} z_t^* \quad = \quad & S_t^P \mathbb{E}[X_t] - \min_{S_{t'}^Q} \left[ \int_0^{S_{t'}^Q} \int_0^{\infty} r_u (S_{t'}^Q - x_t) f_{R_U}(r_u) f_{X_t}^{t'}(x_t) \mathrm{d}r_u \mathrm{d}x_t \right. \\ & \left. + \int_{S_{t'}^Q}^{x_{max}} \int_0^{S_t^P} r_d (x_t - S_{t'}^Q) f_{R_D}(r_d) f_{X_t}^{t'}(x_t) \mathrm{d}r_d \mathrm{d}x_t \right] \end{aligned} \tag{6.23}$$

Or, given that the price does not depend on the wind power production level, simply by evaluating the expected cost of up and down regulation and inserting

$$R_t^D = \mathbb{E}[R_d] \tag{6.24}$$

$$R_t^U = \mathbb{E}[R_u] \tag{6.25}$$

into Eq. (6.16), se following proof.

**Proof:** Shown only for the up regulation cost.

$$\int_0^{S_{t'}^Q} \int_0^{\infty} r_u (x_t - S_{t'}^Q) f_{R_U}(r_u) f_{X_t}^{t'}(x_t) \mathrm{d}r_u \mathrm{d}x_t \tag{6.26}$$

$$= \int_0^{S_{t'}^Q} \int_0^{\infty} r_u f_{R_U}(r_u) \mathrm{d}r_u (x - S_{t'}^Q) f_{X_t}^{t'}(x_t) \mathrm{d}r_u \mathrm{d}x_t \tag{6.27}$$

$$= \int_0^{S_{t'}^Q} \mathbb{E}[R_t^U](x - S_{t'}^Q) f_{X_t}^{t'}(x_t) \mathrm{d}x_t \tag{6.28}$$

$$\tag{6.29}$$

Note that $R_t^U$ is a point prediction for the up regulation cost and should not be confused with the random variable $\{R_u\}$.

#### 6.1.3.2 A model taking system balance into account

In Denmark, only those producers who bring the system out of balance are charged for regulation. This means that if the system needs energy and a wind producer produces to much, he does not have to pay regulation cost for that extra production but receives the spot price unchanged instead.

Lets first assume that the system balance is unrelated to producers balance. Two new binary random variables $T_U$ and $T_D$ are introduced.

$$T_U = \begin{cases} 0 & \text{If up regulation is not charged} \\ 1 & \text{If up regulation is charged} \end{cases} \tag{6.30}$$

$$T_D = \begin{cases} 0 & \text{If down regulation is not charged} \\ 1 & \text{If down regulation is charged} \end{cases} \tag{6.31}$$

with the associated probabilities

$$\mathbb{P}\{T_U = i\} = \pi_i^U \qquad i \in \{0, 1\} \tag{6.32}$$

$$\mathbb{P}\{T_D = i\} = \pi_i^D \qquad i \in \{0, 1\} \tag{6.33}$$

$$\tag{6.34}$$

The optimisation problem is now formulated as:

$$\begin{aligned} z_t^r &= \min_{S_{t'}^Q} \mathbb{E}[R\{S_{t'}^Q\}] \\ &= \min_{S_{t'}^Q} \Bigg[ \sum_{t_u=0}^{1} t_u \pi_{t_u}^U \int_0^{S_{t'}^Q} R_t^U (S_{t'}^Q - x_t) f_{X_t}^{t'}(x_t) \mathrm{d}x_t \\ &+ \sum_{t_d=0}^{1} t_d \pi_{t_d}^D \int_{S_{t'}^Q}^{x_{max}} R_t^D (x_t - S_{t'}^Q) f_{X_t}^{t'}(x_t) \mathrm{d}x_t \Bigg] \end{aligned} \tag{6.35}$$

$$\begin{aligned} &= \min_{S_{t'}^Q} \Bigg[ \pi_1^U \int_0^{S_{t'}^Q} R_t^U (S_{t'}^Q - x_t) f_{X_t}^{t'}(x_t) \mathrm{d}x_t \\ &+ \pi_1^D \int_{S_{t'}^Q}^{x_{max}} R_t^D (x_t - S_{t'}^Q) f_{X_t}^{t'}(x_t) \mathrm{d}x_t \Bigg] \end{aligned} \tag{6.36}$$

And the optimal solution is found to be

$$F(S_{t'}^Q) = \frac{\pi_1^D R_t^D}{\pi_1^D R_t^D + \pi_1^U R_t^U} \tag{6.37}$$

In a system where the portion of wind power is high, it is unlikely that the system balance is unrelated to the wind power producers imbalance . In other words, if all wind power producers use the same forecasting system and bid the forecasted quantity at the spot market. The probability of having an imbalance in the same direction as the whole system, increases as the forecasting error increases. One way to model this behaviour is by looking at the regulation need, not the forecasting error. If we have a function $\pi_1^U(e)$ defining the probability that up regulation cost must be paid given an regulation need $e$, the behaviour might be formulated to some extent as:

$$R\{e\} = \begin{cases} -\pi_1^D(e)R_t^D e & e > 0 \\ 0 & e = 0 \\ \pi_1^U(e)R_t^U e & e > 0 \end{cases} \tag{6.38}$$

Using this formulation the "active" regulation cost is a function of the need. In the following section, 6.1.3.3, a method to find the optimal bid, given such a regulation cost function is derived. Note that this formulation can only handle the case, when other producers have a bidding strategy that is related to the bidding strategy which the producer applying the method uses. A more general solution will not be presented here.

#### 6.1.3.3 Regulation cost formulated as a piecewise linear function

If the regulation cost, depends on the amount of needed regulation, a continuous piecewise linear cost function can be used to approximate the real cost function. That way, a simple analytical solution can be derived for the derivative of the expected income and like before, stationary points examined to find the optimal bid.

Lets start by defining a continuous cost function $C(r)$ of the regulation need $r$.

$$C(r) = \begin{cases} c_0 r + a_0 & \text{if} \quad r \leq d_1 \\ c_1 r + a_1 & \text{if} \quad d_1 < r \leq d_2 \\ \quad\vdots & \quad\vdots \\ c_n r + a_n & \text{if} \quad d_n < r \end{cases} \tag{6.39}$$

Where $\lim_{r \to d_i} C(r) = C(d_i)$ for all $d_i$.

Using this regulation cost function the expected regulation cost is calculated as:

$$\begin{aligned}\mathbb{E}[R\{S_{t'}^Q\}] &= \int_0^{S_{t'}^Q+d_1} \Big[(x_t - S_{t'}^Q)c_0 + a_0\Big] f_{X_t}^{t'}(x_t)\mathrm{d}x_t \\ &+ \sum_{i=1}^{n-1} \int_{e_b+d_i}^{e_b+d_{i+1}} \Big[(x_t - S_{t'}^Q)c_i + a_i\Big] f_{X_t}^{t'}(x_t)\mathrm{d}x_t \\ &+ \int_{S_{t'}^Q+d_n}^{m_p} \Big[(x_t - S_{t'}^Q)c_n + a_n\Big] f_{X_t}^{t'}(x_t)\mathrm{d}x_t \end{aligned} \tag{6.40}$$

Using Leibnitz rule (6.17), the derivative is found to be:

$$\begin{aligned}\frac{\partial\mathbb{E}[R\{S_{t'}^Q\}]}{\partial S_{t'}^Q} &= \Big[((S_{t'}^Q + d_1) - S_{t'}^Q)c_0 + a_0\Big] f_{X_t}^{t'}(S_{t'}^Q + d_1) \\ &+ \int_0^{S_{t'}^Q+d_1} c_0 f_{X_t}^{t'}(x_t)\mathrm{d}x_t \\ &+ \sum_{i=1}^{n-1} \Big[((S_{t'}^Q + d_{i+1}) - S_{t'}^Q)c_i + a_i\Big] f_{X_t}^{t'}(S_{t'}^Q + d_{i+1}) \\ &- \sum_{i=1}^{n-1} \Big[((S_{t'}^Q + d_i) - S_{t'}^Q)c_i + a_i\Big] f_{X_t}^{t'}(S_{t'}^Q + d_i) \\ &+ \sum_{i=1}^{n-1} \int_{S_{t'}^Q+d_i}^{S_{t'}^Q+d_{i+1}} c_i f_{X_t}^{t'}(x_t)\mathrm{d}x_t \\ &- \Big[((S_{t'}^Q + d_n) - S_{t'}^Q)c_n + a_n\Big] f_{X_t}^{t'}(S_{t'}^Q + d_n) \\ &+ \int_{S_{t'}^Q+d_n}^{m_p} c_n f_{X_t}^{t'}(x_t)\mathrm{d}x_t \end{aligned} \tag{6.41}$$

or simplified:

$$\begin{aligned}\frac{\partial \mathbb{E}[R\{S_{t'}^Q\}]}{\partial S_{t'}^Q} &= \sum_{i=1}^{n} \left[d_i(-c_i + c_{i-1}) + a_{i-1} - a_i\right] f_{X_t}^{t'}(S_{t'}^Q + d_i) \\ &+ \sum_{i=1}^{n} (c_{i-1} - c_i) f_{X_t}^{t'}(S_{t'}^Q + d_i) \\ &+ c_n \end{aligned} \tag{6.42}$$

Note that if the forecast is given by a probability density function, the cumulative distribution function can be calculated and vice versa. A numerical search algorithm can be used to find the stationary points.

#### 6.1.3.4 More general use of the method

Although the formulation here has been focusing on selling wind energy with out balance responsibility, the method can be used in some other situations. The up regulation cost is in fact the loss of not being able to produce enough energy, just as the down regulation cost is the loss of not placing a bid high enough at the spot market. If the wind power producer is balance responsible and can therefore not sell extra production, the down regulation cost can be set equal to the spot price. Or in other words, all extra production can be formulated as worthless. If the producer, on the other hand has some other way to sell his energy, for instance, produce hydrogen which might become a valuable fuel in the future. The down regulation cost could be replaced by the difference between the spot market price and the hydrogen price, as long as the hydrogen price is below the spot price. Other actions than selling all the energy at the spot market can therefore be included in the formulation, as long as the spot market has the highest price and there is only one or no other alternative. A more flexible formulation is described in next chapter.

### 6.1.4 The role of Quantile Regression

The method described can be applied in practice although many modern forecasting systems are not able to provide the probability density function $f_{X_t}^{t'}$. In [26] a method to add probabilistic information to point prediction systems is described. The only requirement is that historical production forecasts, observed production levels and weather related observations are available.

The statistical tool used is named Quantile Regression[5], described shortly in Section 5.2 and completely in [27]. It can be used to find a non-parametric estimate of the cumulative distribution function $F_{X_t}$.

[5]Also known as precentile regression.

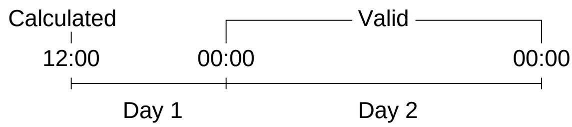


Figure 6.1: *A timeline showing the relation between calculation time, and the time when the predictions supplied were valid.*

## 6.2 Implementing and testing the optimal quantile bidding strategy

In this section the method just described will be implemented and tested. Firs the data used in the tests will be described and analysed. Then quantile regression will be applied in order to find the optimal bid. And finally the performance of the method will be tested both using artificial and historical prices.

### 6.2.1 Data

The data used comes from two sources.

- Group of windmills in Denmark[6]
- NordPool, trough Elkraft's home page.

Predictions and observed power production by a group of wind turbines located in East Denmark from April '04 to April '05. The predictions were the newest available predictions at noon for every hour of the following day, see Figure 6.1 and Table 6.1.

The idea behind deregulated markets such as NordPool is that information about the markets should be made official to the public. That is why results from bidding rounds, such as prices and total quantities, are available though NordPool's FTP server. The data is, however, not on a standard format and much work must be done in order to transfer it into modern databases or statistical programs. This is why market data was not taken directly from the original source, but taken from Elkraft-system's home page instead. There the

[6] The ownership is confidential. They are spread ower Zealand with a maximum power output around 300MW

| Variable | Unit/Format | Description |
|---|---|---|
| t | yyyy-mm-dd hh | Date containing time with the resolution of one hour. |
| ProductionForecast | $MWh$ | Production Forecast calculated at last noon ($t'$), stating how much electricity production is forecasted at time $t$. |
| ObservedProduction | $MWh$ | Observed wind power production at time $t$. |
| Spot price | $DKK/MWh$ | The NordPool spot price in East Denmark at time $t$. |
| Down regulation cost | $DKK/MWh$ | The cost of down regulation in East Denmark at time $t$. |
| Up regulation cost | $DKK/MWh$ | The cost of up regulation in East Denmark at time $t$. |
| System balance | $MWh$ | The balance in East Denmark at time $t$. |
| First observation | 2004-04-01 01 | |
| Last observation | 2005-04-30 23 | |

Table 6.1: *The variables which the data set contained.*

data from NordPool has been collected into Elkraft-System's data system and is published on a more convenient format. The variables used are listed in Table 6.1.

### 6.2.2 Spot market prices

Describing the behaviour of the spot market price at NordPool precisely is extremely complicated due to the number of elements which affect it. The approach here will therefore mainly be through examples. Observed spot prices in the period are shown in Figure 6.2 and summary statistics in table 6.2.

In the period, stable prices, prices showing changes between night and day and price spikes were observed. Comparing the actual prices (Fig. 6.2.A) to daily averages (Fig. 6.2.B) shows that in some periods there is a high difference between nigh and day but in other cases prices have little daily variation. The weekly mean and median express clearly how extremely high price spikes can be observed at the spot market. In March 2005, the median does no indicate any abnormality although the mean price is close to being doubled. This behaviour is observed at electricity markets through out the world.

| | $DKK/MWh$ |
|---|---|
| Min | 0 |
| Median | 213.6 |
| Mean | 214.2 |
| Max | 1000 |

Table 6.2: *Summary for the spot price in East Denmark, April 2004-2005. When all thermal plants that can not be shut down are running at minimum, the price can go down to zero. This is an example of how strange the behaviour of electricity prices can be.*

In Figure 6.3 prices for selected subintervals of the whole period are shown. The analysis is based on Elkarft-System's market reports which are published every month.

**January to March 2004** In January, 2004 prices were stable, though, with exceptions. In one instance the price jumped up to 750 $DKK/MWh$ when the transmission to Germany failed. There was a net export, during the period, from East Denmark to Sweden and to Germany. This is often the case during the last moths of winter[7].

The situation in February was similar to the situation in the previous months. German prices tended to be high during the day and low during the night. Electricity was therefore imported from Germany in night-time and exported to Germany during the daytime.

The transmission to Sweden was reduced during 18% of the time in March. The effect was not drastic except in one case when the system price in Denmark, fell down to $0DKK/MWh$. That hour high wind production was combined with reduced export possibilities. [29],[30],[31]

**Spring 2004** The situation in May was completely different from what it was in March. In spring time, when the snow is smelting, rivers get filled with so much water that the dams can not store it all. Hydroelectricity is therefore cheep during this time but the price rises again when the flow falls down to normal. This slow change from low to normal price can be seen in Figure 6.3.B.

[7] There is little flow into dams during the winter, the water level is therefore often low the first months of the year. This can cause high prices in systems dominated by hydroelectric power.

There is , though, one exception, the connection to Sweden was down the second week in June, causing the Danish price to follow the German price closely. This explains a price range of 8 to $700 DKK/MWh$ and high variation within each day. In other periods where there was no connection to the German system and prices became stable again. [14],[32]

**Winter 2004** The prices were stable during the winter months, October to December. The Danish price, followed the low German price during the night and the Swedish price during the day, as long as the transmission capacity was not reduced. See Figure 6.3.C for reference. [33],[34],[35]

**March 2005** Dramatic prices were observed in March 2005. Cold weather in Europe caused increased electricity consumption. This combined with a decreased transmission capacity from Sweden to Denmark resulted in extremely high prices and price differences. In Germany the maximum price went up to 2500 $DKK/MWh$ whereas the Danish price peaked at 1000 $DKK/MWh$. At the same time was the price in Sweden 226 $DKK/MWh$, giving a price difference of 774 $DKK$. The reason for decreased transmission capacity from Sweden to Denmark was increased consumption in Sweden and local transmission limitations. The price series is shown in Figure 6.3.D. [13]

These four cases show that there are many elements which affect the system price. The season, the temperature, the interconnections and the time of the day all have great influence. Price spikes normally are observed when something unexpected happens or when things that tend to lift the price are combined. It is also clear that the system is extremely complicated and expertise knowledge is needed to give a good description of the price behaviour.

### 6.2.3 Regulation prices

In this section the term price is for the regulation price as it is published by NordPool whereas the term regulation cost is used for the difference between the spot price and the regulation price. One some graphs the down regulation cost has a negative sign, that is only to ease the comparison of the prices, is does **not** mean that the down regulation cost increases the profit.

The regulation price fluctuates even more than the spot price. Summary statistics for the regulation cost show that it was 2384 $DKK/MWh$ when it was at its maximum and that the median is almost zero although the mean is around 15 $DKK/MWh$, see Table 6.3.

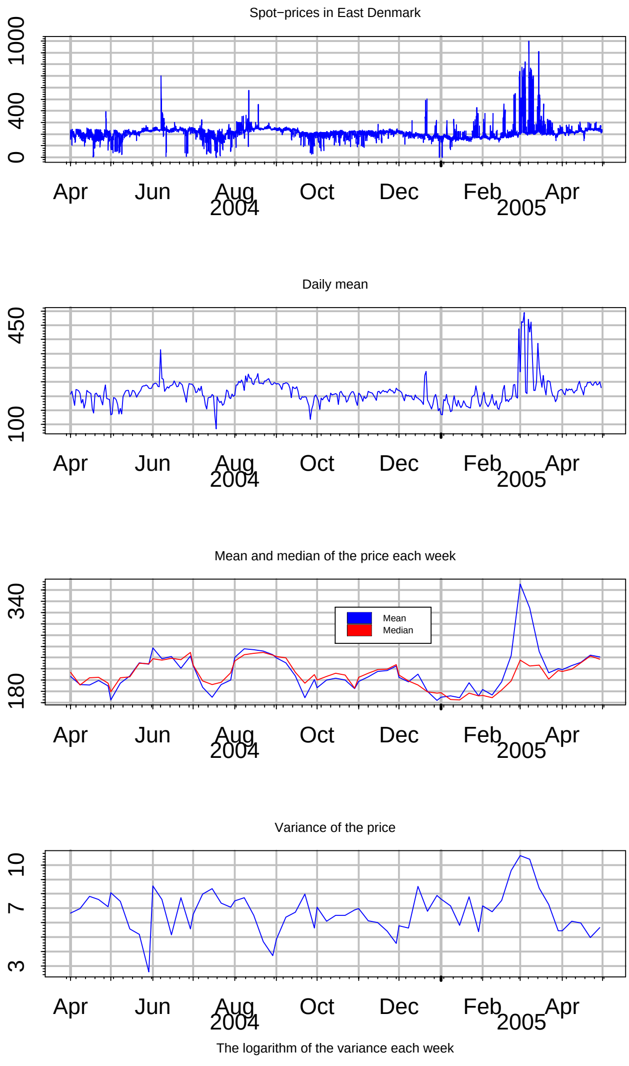


Figure 6.2: *From top: A,B,C,D. The spot prices in east Denmark, for the whole data set.*

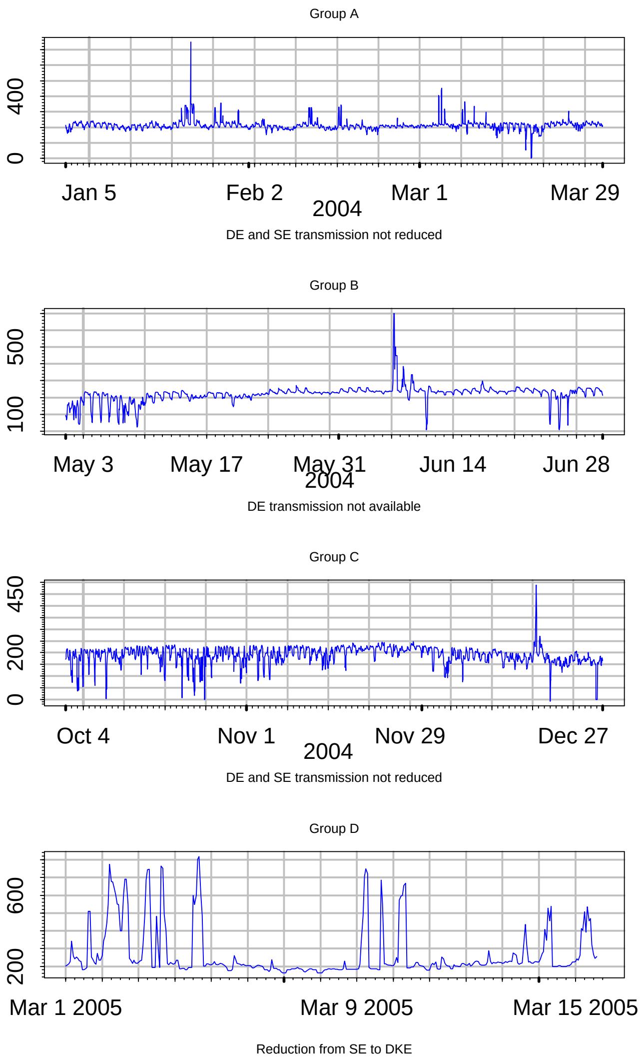


Figure 6.3: *From top: A,B,C,D. The spot price in four different situations.*

| | Up cost $[DKK/MWh]$ | Down cost $[DKK/MWh]$ |
|---|---|---|
| Min | 0.0 | 0.0 |
| Median | 0.2 | 0.0 |
| Mean | 14.7 | 18.5 |
| Max | 2384.5 | 813 |

Table 6.3: *Summary for the prices in East Denmark, April 2004-2005.*

In Figure 6.4 both up and down regulation prices are showed for the whole period. The median indicated unstable prices, because it is separated from the mean at all times. It can also be read from the figures, that there is no well defined relation ship between the up and the down regulation price as Morthorst [36] reported in Vest Denmark, 2002. There he observed that the down regulation price was always above the up regulation price and assumed this was because down regulation producers are loosing revenues. However in East Denmark, the mean up regulation prices goes from being 30 $DKK$ higher than the down regulation price in January 2005 to begin 30 $DKK$ lower in March 2005. This is a contradiction to Morthorst observations. It should , though, be noted that energy markets are still developing and the situation can change from year to year. Regulation is also priced differently in East Denmark where the marginal cost is charged unlike the weighted average which is charged in West Denmark (see Section 4.2.3).

Examining the connections between regulation prices and other variables, Figure 6.5 shows that there is no clear link between the price and the regulation volume needed. In fact are there no extreme prices observed when the need for regulation is high. The data should , though, not be used to pose new hypotheses on this matter cause the number of observations in these extreme cases is low. Plotting the relationship between the spot price and the regulation price shows a two fold relationship. Either the regulation price is hardly different from the spot price, or it follows roughly a linear relationship. Where the density of observations is high, as it is for "normal" spot prices, extremely high regulation costs are detected and the pattern between the spot price and the down regulation price becomes unclear.

### 6.2.4 Price forecasts

The model presented assumes that the up and down regulation prices are known. It is therefore essential in order to used the model to forecast the prices. However, the previous examples showed that the main elements affecting the price are quite complicated and time varying. Some methods were tried but they all

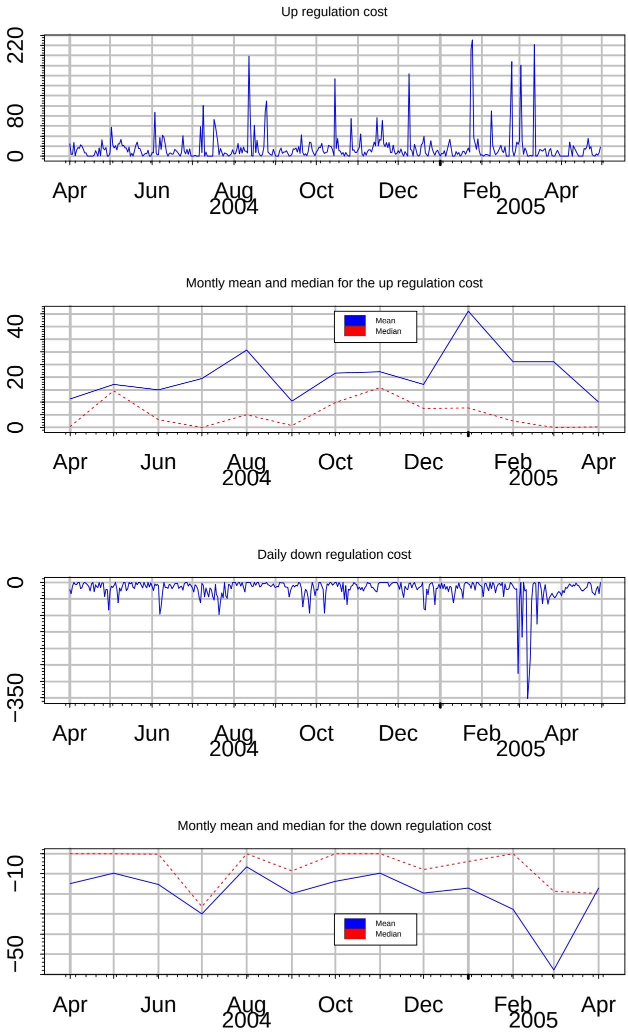


Figure 6.4: *From top: A,B,C,D. Daily and monthly averages of down and up regulation prices.*

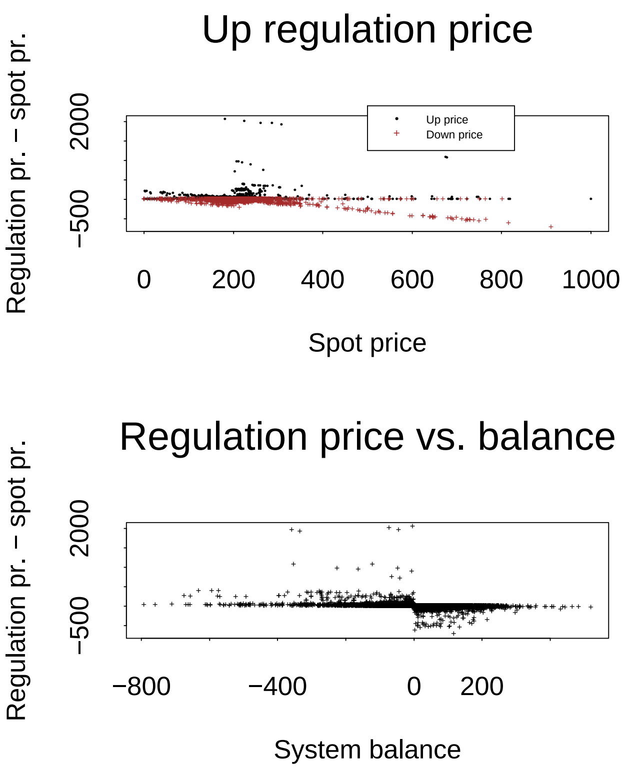


Figure 6.5: *From top: A,B. The relation between regulation cost and other variables. Note that the regulation cost is the absolute value of the difference between the spot price and the regulation price.*

failed, included were persistence, mean and median of $n$ last variables, linear regression[37] and prototype matching [38]. The fact that they were all linear[8] and the lack of deep system understanding are probably the two main reasons for their failure.

### 6.2.5 Applying quantile regression

In order to be able to use the method presented the forecast system must be able to provide the optimal bid $S^Q_{t'}$ so that

$$F_{X_t}(S^Q_{t'}) = \tau \tag{6.43}$$

Output from forecasting systems is not standardised and the information they provide can be different from one system to another. However, most forecasting systems, for instance WPPT[9] which the owner of the wind turbines in question used in the period, provide point forecasts. That is, one number stating what the production after $n$ hours will be. For that reason, the focus here will be one quantile regression, which can be used to extend a point forecast with probabilistic information, so that the optimal bid can be found for any quantile $\tau$ [26].

What data is needed to extend the system is site specific and no attempt will be made here to address the problem of selecting all the relevant variables. On the other hand, it is a known fact that the relationship between wind speed and power output from a wind turbine is not linear. An artificial power curve is shown in Figure 6.6. For that power curve a wind prediction in the interval `A` can be expected to give a more accurate power forecast than a wind prediction in interval `B`, because the curves do not have the same slope in both intervals[10]. Based on this property, the uncertainty of a forecast can be estimated as long as enough historical data, containing predictions and observed production, exists.

#### 6.2.5.1 The regression performed

The purpose of the regression is to create a probabilistic connection between the predicted value to the observed production. But before that can be done a linear regression model combining these two variables must be found.

[8] A linear distance measure was used for prototype matching.

[9] Wind Power Prediction Tool, developed and distributed by the Technical University of Denmark.

[10] A small change in wind, in interval `B` causes a greater change in power production than a change of same size in interval `A`.

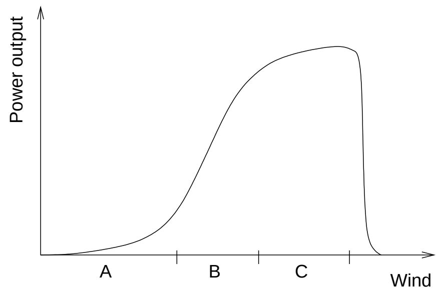


Figure 6.6: *The power output from a wind turbine highly depends on the wind speed. In extremely high winds, turbines must be shut down in order to protect the machinery.*

To do that a natural spline base was created, containing as many spline functions $B_k(x)$ as degrees of freedom $df$. The relation ship between the observed production $y$ and the predicted production $x$ was formulated as:

$$y_i = \beta_0 + \sum_{k=1}^{df} \beta_k B_k(x_i) + \epsilon_i \tag{6.44}$$

The horizon was not included in the model for two reasons; a sufficient number of observations is needed for the tests in the following section and in [26] the horizon is not found to be an important variable, at least not when the horizon is long.

In order to find out how large the spline base (describing the power curve) had to be, the model defined in Eq. (6.44) was fitted for 1 to 30 degrees of freedom. The value of the parameters $\beta_k$ was estimated, minimising the sum of squared error ($SSR$):

$$SSR = \sum_{i}^{n} \epsilon_i^2 \tag{6.45}$$

Figure 6.7 shows the $SSR$ value for all the fits. Based on the results 10 degrees of freedom are assumed to be a sufficiently high number. Examination of the residuals, $\epsilon$, shows that they are correlated, see Figure 6.8. This effect can, however, not be removed as all the predictions for a given day are based on the same weather forecast. The errors in one weather prediction are correlated but there is hardly any correlation between the errors of two different weather forecasts. It is therefore not possible for a model to catch the error with in a day. The figure also shows that if residuals are examined for a given prediction horizon, that is when only predictions valid for instance at noon are examined,

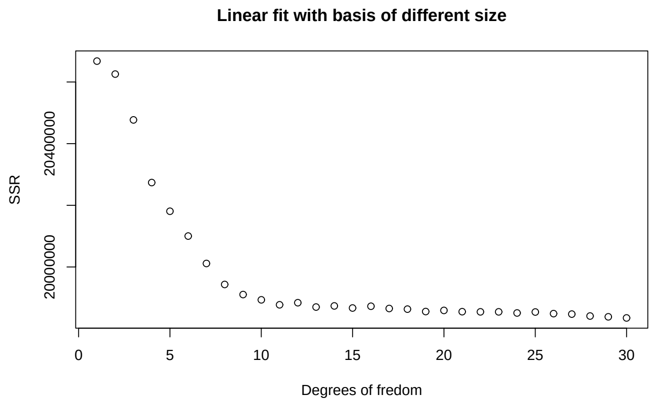


Figure 6.7: *The performance, in terms of sum of squared residuals, of a linear model, fitting a curve connecting forecasted production and observed production.*

the residuals appear to be white noise. Based on this the model was assumed to be applicable.

When the model has been determined, quantile regression can be used in order to estimate different values for $\beta$. so that the relationship in Eq. (6.46) holds for a $i$ selected at random.

$$\mathbb{P}\{y_i < \beta_0 + \sum_{k=1}^{df} \beta_k B_k(x_i)\} \approx \tau \tag{6.46}$$

The performance of the quantile regression was tested in two ways. Both for correctness when conditioned on the production level and for correctness when conditioned on location in the training set which was defined to be the whole data set.

Parameters were estimated for the 50% quantile, $\tau = 0.5$. Then 7 different test sets were create by selecting only observations where the predicted production was with in a given range, see Table 6.4. Counting how often the predicted production was above the observed production for the different test sets, showed that the quantiles estimated for the training set were correct as long the number of observations was sufficient.

Dividing the training set by periods, so that the first test set contained observations number 1-790 and the second observations number 791-1580 and so

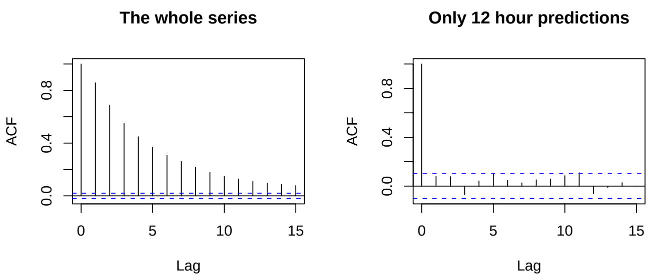


Figure 6.8: *The autocorrelation of the residuals, when the predicted production is fitted to the observed production using the model given in Eq. (6.44) with ten degrees of freedom.*

| From $[MWh]$ | To $[MWh]$ | Observed quantile | # Obs |
|---:|---:|---:|---:|
| 0 | 50 | 0.5017 | 3129 |
| 50 | 100 | 0.5014 | 1773 |
| 100 | 150 | 0.4974 | 1530 |
| 150 | 200 | 0.4892 | 1294 |
| 200 | 250 | 0.5119 | 1258 |
| 250 | 300 | 0.4708 | 480 |
| 300 | 700 | 0.8462 | 13 |

Table 6.4: *Observed performance when conditioned on predicted production.*

on, gave different results. The numbers in Table 6.5 show that the parameters estimated for the training set were only correct for 3 of the 11 test sets.

The difference between the periods is to high to be neglected. It is therefore decided to divide the data set by months and estimate $\beta_k$ for every month individually. That way the test results will not be as biased because the cases when wrong quantiles and high prices are combined will not be as many[11].

Quantiles for the whole period are shown in Figure 6.9. The quantiles shown are $\tau = 0.1, \tau = 0.2, \ldots, \tau = 0.9$.

[11] The prices change from month to month. Using a wrong quantile in high regulation cost month could have a strong negative effect on the performance of the optimal quantile bidding strategy.

| Period | Observed quantile | Period | Observed quantile |
|---|---|---|---|
| 1 | 0.4436 | 7 | 0.5741 |
| 2 | 0.5171 | 8 | 0.4956 |
| 3 | 0.4233 | 9 | 0.4132 |
| 4 | 0.5387 | 10 | 0.5817 |
| 5 | 0.6515 | 11 | 0.4487 |
| 6 | 0.5285 | | |

Table 6.5: *Observed performance when conditioned on period.*

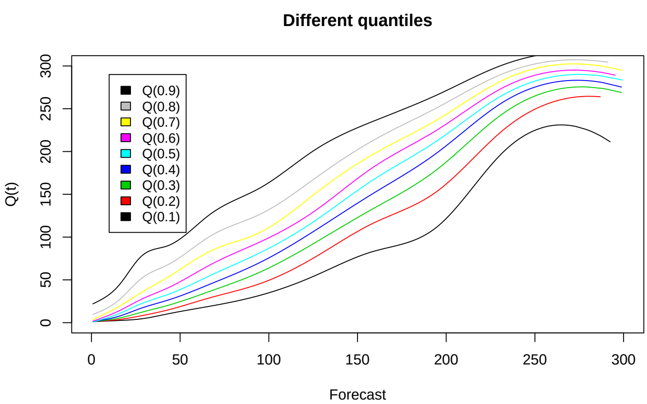


Figure 6.9: *Quantiles estimated for the whole data set. Given the a prediction of, for instance,* $50MWh$*, it can be read from the graph that the observed production will be below 100 with a 90% chance.*

### 6.2.6 Performance

In this section the performance of the method will be tested. The purpose of the tests is to see if the theoretical optimums are observed when applying the method using simulated or historical data. It will also be investigated if a higher income is obtained using the optimal quantile instead of the predicted value when bidding at the spot market. That will be tested both using artificial and observed prices. All the tests are performed in a similar way. A data set is defined, containing prices, system status, predicted and observed production for $N$ hours. For every hour, a bid is placed at the spot market following both the optimal quantile bidding strategy and the predicted value strategy. In the end the performance of both bidding strategies is compared. The performance measure is the total income using a specified bidding strategy divided by the total income when no regulation cost has to be paid[12].

The following concepts need to be clear:

- A quantile bidding strategy is defined as the act of bidding a value so that the probability of observing production below the bid is $\tau$.
- The optimal quantile is the quantile $\tau$ which returns the highest income when selling energy using the optimal quantile bidding strategy.
- The predicted value bidding strategy is defined as the act of bidding exactly the forecasted production for that hour.
- The calculated optimal quantile is a quantile calculated using the methods in the previous section.
- The observed optimal quantile is the quantile that returned the highest income in a given test.

#### 6.2.6.1 Performance when prices are known

The method requires that the regulation prices are known in advance. The prices are however not known and they are hard to predict, which is why the method will first be tested with artificial prices. That way, the properties of the method can be investigated without observing noise from price prediction errors.

[12] If there existed a prediction system which could predict the production $N$ hours ahead with out any errors no regulation would be needed. Having such a prediction system is called having "full information".

#### 6.2.6.2 System balance not taken into account

To see how the income is affected by the choice of bidding strategy and to confirm that Eq. (6.19) can be used to find the optimal quantile, bids are placed at the spot market using both the predicted value and the optimal quantile strategy. The requirement that the price must be known is solved by setting it as a predefined constant and the system balance is not taken into account Bids are placed at the spot market every hour, from the beginning of April 2004 to then end of April 2005. The configuration and the results are listed in Table 6.6, page 70, column A. The outcomes shows that going from the predicted value bidding method to the optimal quantile gives a 1% higher income. It is also confirm that the quantile posed by Eq. (6.19) is in fact the optimal quantile.

In column B the same test is performed using different regulation prices. There the regulation prices are set equal so the optimal quantile is $\tau = 0.5$. The numbers show that the difference between the bidding strategies is smaller then it was when the prices were different. Now the income is only increased by 0.6%. This is because the optimal quantile strategy uses the price difference to increase the income by placing a bid in a way so that the probability of paying low regulation cost is higher than paying high regulation cost. Note that the WPPT program used to forecast the production minimises squared error. That is why the two bidding strategies, optimal quantile and prediction, do not give same results although the regulation prices are equal. No difference in income would be expected if WPPT minimised absolute error instead of squared error.

#### 6.2.6.3 System balance taken into account as a unrelated variable

If regulation is only charged when producer adds to the total system imbalance, and the balance is unrelated to the regulation need, Eq. (6.37) can be used to calculate the optimal quantile. Test C confirms this, see Table 6.6. There the system balance is a simulated variable, uniformly distributed between -0.5 and 1. However the observed optimal quantile depended on the system balance sample although it contained 9504 samples. The standard deviation of the optimal quantile estimate after 10 estimations (for different system balance samples) was $\sigma = 0.011$.

#### 6.2.6.4 System balance taken into account as is was observed

If the observed system balance is used in the test the quantile calculated using Eq. (6.37) can not be guarantied to match with the observed optimal quantile.

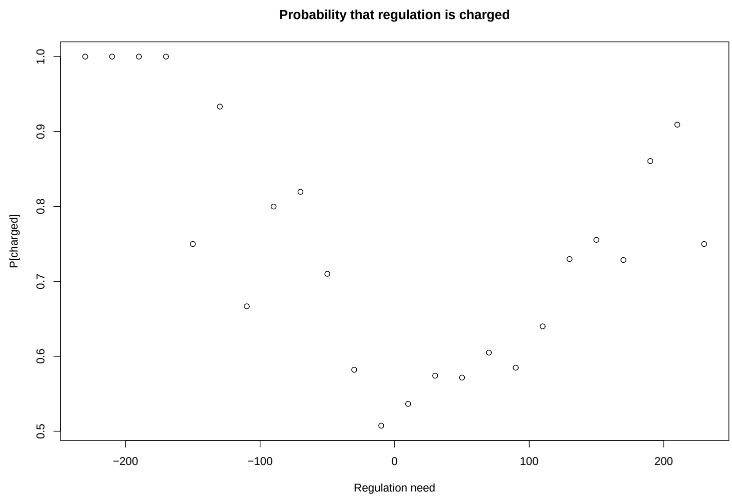


Figure 6.10: *The probability that regulation is charged depends on the regulation need, both for up and down regulation.*

This has to do with the fact that the system balance is not unrelated to the producers imbalance. If all producers follow the same bidding strategy, they do all have imbalances at the same time. So if the portion of wind is considerable in the system, wind power producers can expect that the probability of begin charged for regulation increases as the regulation need increase. This can in fact be observed in the data from East Denmark. In figure 6.10 the probability of paying regulation cost is plotted as a function of the regulation need. But although this relationship is observed, do the results not show any strange behaviour. The calculated quantile is only 0.005 from the observe optimal quantile, see column D, Table 6.6. Two possible explanations are that the symmetry of the probability curve reduces its effect or that this just happens to be this way for the given data set.

#### 6.2.6.5 Performance when prices are unknown

Replacing the predefined prices with real observed prices changes the picture. See results in Table 6.6, column E. If we assume that we know what the mean up and down regulation cost for the period in advance and select a quantile using that information the optimal quantile is calculated to be

$$\tau = 0.53 \tag{6.47}$$

However the observed optimal quantile is quite far from that, found to be $\tau^* = 0.48$. This difference can be explained by the fact that both spot and regulation prices have spikes. Placing the wrong bid when a price spike is observed can be have a observable effect on the total income and shift the optimal quantile. This is confirmed by the stability test described in the following section. Nevertheless, comparing the total income, using both bidding strategies show a small difference between the methods, the optimal quantile bidding returns a 0.2% higher income than bidding the predicted value.

#### 6.2.6.6 Stability when prices are unknown

In order to investigate how the price spikes have effect on the optimal quantile the following test was performed. The data set containing observations from April 2004 to April 2005 was repeated 150 times. The regulation prices (up and down) were then replaced by simulated prices, sampled from the originally observed prices at random. The optimal quantile for this data set was found to be extremely close to the calculated quantile, see table 6.6, column F. This supports the hypothesis that price spikes caused the high difference between calculated and observed quantile in the previous test.

#### 6.2.6.7 Performance when quantiles are not required to be correct within each month

To see how important the local correctness of the quantiles is for the method, the performance of the two bidding strategies was tested using a quantile estimated for the whole data set. In other words, the quantile was only correct when the whole period was examined but not a individual month, see Section 6.2.5.1 for further description. The results showed that the quantile strategy returned a lower income than the predicted bid in this case, see column G in Table 6.6. This indicates, that both accurate quantiles and price predictions are needed in order to gain from applying the bidding method.

#### 6.2.6.8 Local price estimates

Although no good algorithms, which could forecast the regulation prices, were developed one final test was be performed using a poor forecasting technique. For every hour, from the end of April 2004 to 2005 a quantile, calculated using Eq. (6.37), was bid. The regulation cost for the following hour was predicted to be equal to the mean regulation price in the last 720 hours. System balance

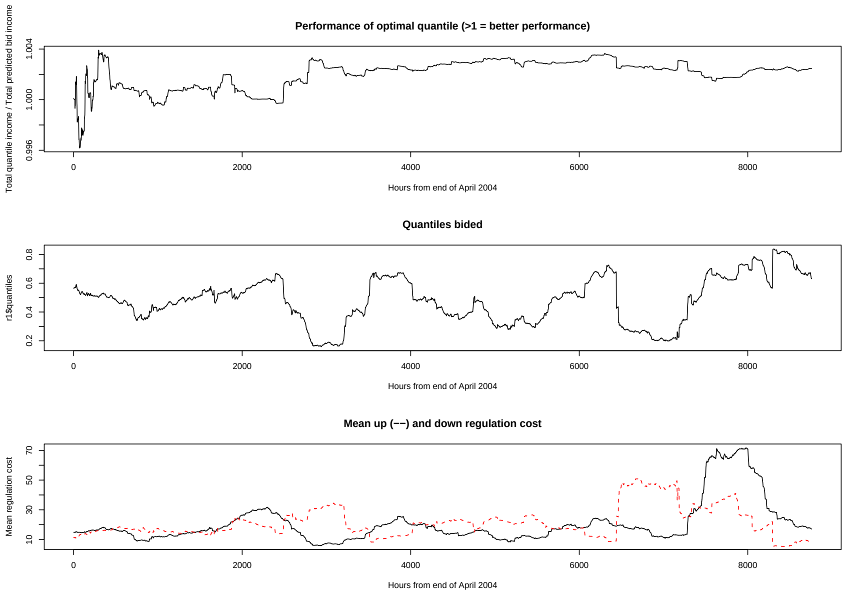


Figure 6.11: (TOP) *The performance of the two bidding methods compared. A number higher than 1 means that the optimal quantile strategy is performing better* (CENTER) *The optimal quantile changes as the local mean up and down regulation price changes.* (Bottom) *The mean up and down regulation price.*

was not taken into account. The results of the test are shown in Figure 6.11 The optimal quantile method returned a 0.25% higher income than the optimal bid. The difference is low but it interesting to see that for the data in question the optimal quantile returned constantly higher income than the predicted value method, excluding the start. It should, though, be noted that the prediction method used to predict the regulation cost is not a good method. Price spikes have a strong effect on the local mean, but the effect kicks in just after the price was observed. The prediction error is therefore high just after a spike burst and this can have and probably has a negative effect on the income.

#### 6.2.6.9 Maximum gain from bidding optimal quantiles

It would be interesting to see how much can be gained from bidding using the optimal quantile, if the prices are varying and known. It is however not possible to perform this test using historical data. In the historical data, only the down or the up regulation price is defined in each hour. Never both. For this reason,

| Test | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| Data set | A | A | B | A | A | C | D |
| Spot price | 200 | 200 | 200 | 200 | Obs. | Obs. | Obs. |
| Down reg. | 7 | 30 | 7 | 7 | Obs. | Simul. | Obs. |
| Up reg. | 30 | 30 | 30 | 30 | Obs. | Simul. | Obs. |
| System bal. | Omitt. | Omitt. | Simul. | Obs. | Obs. | Obs. | Obs. |
| Production | Obs. | Obs. | Obs. | Obs. | Obs. | Obs. | Obs. |
| Prediction | WPPT | WPPT | - | WPPT | WPPT | - | WPPT |
| Calc. quantile | 0.189 | 0.5 | 0.318 | 0.196 | 0.53 | 0.5025 | - |
| Obs. quantile | 0.189 | 0.5 | 0.321** | 0.191 | 0.475 | 0.5051 | 0.59 |
| Opt. income | 0.9849 | 0.9639 | - | 0.9897 | 0.9770 | - | 0.9743 |
| Calc. income | 0.9849 | 0.9639 | - | 0.9897 | 0.9769 | - | - |
| Pred. income | 0.9757 | 0.9583 | - | 0.9841 | 0.9749 | - | 0.9749 |

Table 6.6: *Test results from Section 6.2.6.* (*) *Optimal quantile to used when bidding.* (**) *The mean quantile found after ten simulations ($\sigma = 0.011$)* Acronyms; *Down reg: Down regulation cost, Up reg: Up regulation cost, System bal: System balance, Calc quantile: Calculated optimal quantile, Obs. quantile: Observed optimal quantile, Opt income: Income when bidding the optimal quantile, Clac. income: income when bidding the calculated optimal quantile, Pred. income: Income when bidding the predicted production.*

the maximum gain from bidding the optimal quantile can not be calculated using the information published by NordPool. One way would be to simulate the prices, but that is difficult. Both the up and the down regulation price would have to be simulated at once, conserving the relation ship between them, a relationship which is can not be observed using the published data. For now, the test using predefined prices must therefore be used as the upper bound on how much can be gained from applying the method.

Chapter 7

# Bidding at more than one market

## 7.1 About this chapter

In this chapter a new market will be introduced and added to the model. By looking at the block diagram of the problem, Figure 1.1, it becomes clear that if the market rules change (box f in figure) the mathematical model must be updated, that is done in Section 7.2.3. The new model found is too complicated to be solved analytically as before, so numerical solvers must be used instead. They, however, require that the production forecast is on a discrete format (box d). A method to transform the probabilistic forecast on to such format is explained in Section 7.2.6. In Section 7.3.2 and 7.3.3 data from the new market is analysed and the mathematical model refined according to the results of the analysis. It is also listed out how the discretisation of the production forecast is performed and how the mathematical model is prepared for a solver (box h). Finally the performance of the whole optimisation system is examined for different markets and production forecasts, see Section 7.3.4.

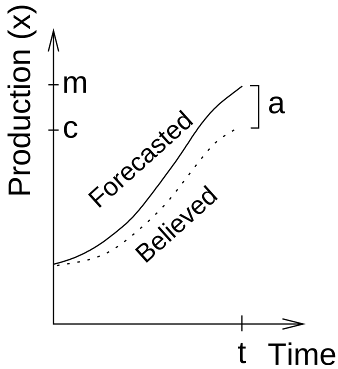


Figure 7.1: *What should a producer bids if he believes that the production will be different from the prediction? What is the prediction suggests many different possibilities?*

## 7.2 Theoretical solution

### 7.2.1 Overview

Since August 2004 producers in East Denmark have had the opportunity to sell and buy energy at a market called Elbas. The ideology of the market is to allow producers to trade imbalances after the spot market has been closed. Energy can be traded down to 1 hour before delivery. This way can a producer, who knows that the production in the following hours will not be according to plans, try to buy or sell energy at Elbas and avoid paying for regulation.

This can be explained clearly using an example. Figure 7.1, shows a production forecast, predicting that $m$ $MWh$ will be produced in hour $t$. The figure shows also what a producer believes will be produced, which is less than the prediction. If that producer can trade at Elbas, should he bid what the forecast says or what he believes. Would it not be wise to bid only the amount $c$ at the spot market and hope that any extra production ($a$ at maximum) can be sold at Elbas? What should the producer do if there are many possible production scenarios?

In this section a method to find an optimal trade plan based on numbers of different production scenarios is presented. The method is formulated as a general optimisation problem without constraints. It can therefore not be used directly in practise, such a model is introduced in Section 7.3.3. This way is chosen because a practical optimisation model can not be build without a clear understanding of when decisions are taken, what information is available when taking those decisions and how the expected income should be evaluated. Put differently, the purpose of this section is to gain information in order to make the practical model formulation more easy.

### 7.2.2 Problem definition

A wind power producer sells his energy at two markets, the spot market and Elbas. Bids at the spot market for tomorrows production must be delivered before noon today, so there are 12 to 36 hours between selling and delivering the energy. Wind power producers can deliver every energy unit they produce to the spot market and receive the spot price for it. However, regulation cost is charged if the producers do not produce exactly as much as they bidded[1]. It is therefore important to place the correct bid in order to minimise the regulation cost. At Elbas, energy can both be sold and bought with a one hour notice. Each sale or purchase has a different price. It is assumed that Elbas has a higher priority than the spot market. This means that if producers do not have enough energy to fulfill agreements on both markets, the Elbas aggrement is first fulfilled and the rest of the energy delivered to the spot market. In the unlikely situation that so little energy is available that the Elbas aggrement can not be fulfilled, up regulation cost must be paid for every unit not delivered. A detailed description of the markets is found in Section 4.2.

In order to help the producer to make decision on how much should be bidded at each market a probability density function describing the possible production levels observed $12 - 36$ hours ahead is available. It is calculated at noon ($t'$) for every hour of the following day. Prices in the markets are assumed to be known or predicted using a point forecast[2].

### 7.2.3 Finding the optimal bid

An optimal bid, is only an optimal bid with respect to the selected criteria. For instance can an optimal bid be defined as the bid causing the minimum regulation need, or the bid causing minimum regulation cost. Here the focus will be put on the latter, that is, the aim is to maximise the expected income disregarding all balance concerns. To do so a function $I_t$ is defined to measure the income in hour $t$, given the production level $x_t$ and a vector $\mathbf{u}_t$, that contains

[1] If the producer produced more than bidded, down regulation cost is charged, if he produces less, up regulation cost is charged.

[2] A forecast which does not supply probabilistic information about possible prices.

all the decisions which affect the income, for instance spot market and Elbas bids.

It is assumed that the producer has a forecasting system that can provide a probability density function $f_{X_t}^{t'}$ calculated at time $t'$ describing the production, $\{X_t\}$, at time $t$:

$$f_{X_t}^{t'}(x_t) \tag{7.1}$$

$x_{max}$ is the installed generation capacity of the wind turbines included in the prediction. A general definition of the income at time $t$ is:

$$I_t(\mathbf{u}_t, x_t) \tag{7.2}$$

where $\mathbf{u}_t$ is a vector of decisions that have effect on the income. The expected income, $z$, in hour $t$, given a set of decisions, is then:

$$z = \mathbb{E}[I_t] = \int_0^{x_{\max}} I_t(\mathbf{u}_t, x_t) f_{X_t}^{t'}(x_t)\mathrm{d}x_t \tag{7.3}$$

The maximum expected income can be gained by adjusting the decision vector $\mathbf{u}_t$ at each time $t$.

$$z^* = \max_{\mathbf{u}_t} \int_0^{x_{\max}} I_t(\mathbf{u}_t, x_t) f_{X_t}^{t'}(x_t)\mathrm{d}x \tag{7.4}$$

But what does the income function look like in reality? What does the decision vector contain and how can the problem be formulated so that it is solvable? The first step is to make a general description of a mathematical optimisation model which is not necessarily solvable. Then the model will be approximated in order to make it solvable. Three time points are of importance for the income function:

$$t' \quad : \quad \text{The time when the spot market bid must be placed (12:00)} \tag{7.5}$$

$$t-1 \quad : \quad \text{Last change to sell or buy at Elbas} \tag{7.6}$$

$$t \quad : \quad \text{The delivery hour} \tag{7.7}$$

And the following relationship must hold:

$$t' < t-1 < t \quad : \quad \text{Energy is always sold before delivery.} \tag{7.8}$$

If we assume that all trades at Elbas take place during the last hour before delivery, an income function, given the state of the markets at time $t$, the spot market bid at time $t'$ and the purchase and sales at Elbas one hour before delivery $(t-1)$, can be defined as:

$$\begin{aligned} I_t \quad = \quad & S_t^P S_t^D + E_{t-1}^{P,S} E_{t-1}^{Q,S} - E_{t-1}^{P,B} E_{t-1}^{Q,B} \\ & -R\{S_t^D - S_{t'}^Q - E_t^{ND}\} \end{aligned} \tag{7.9}$$

where the variables just introduced are described as:

$$E_{t-1}^{P,S} \quad : \quad \text{The } Price \text{ of energy } Sold \text{ at } Elbas \text{ in hour } t-1. \tag{7.10}$$

$$E_{t-1}^{Q,S} \quad : \quad \text{The } Quantity\ Sold \text{ at } Elbas \text{ in thour } t-1 \tag{7.11}$$

$$E_{t-1}^{P,B} \quad : \quad \text{The } Price \text{ of energy } Bought \text{ at } Elbas \text{ in hour } t-1. \tag{7.12}$$

$$E_{t-1}^{Q,B} \quad : \quad \text{The } Quantity\ Bought \text{ at } Elbas \text{ in thour } t-1 \tag{7.13}$$

$$S_t^D \quad : \quad \text{Energy } Delivered \text{ to the } Spot \text{ market} \tag{7.14}$$

$$E_t^{ND} \quad : \quad \text{Energy } Not\ Delivered \text{ to the } Elbas \text{ market} \tag{7.15}$$

Other variables, used in the previous chapter are explained in section 6.1.3.1, page 43. The regulation cost function $R\{e\}$ is defined in Eq. (6.14), page 43 and furthermore, $S_t^D$ is defined to be:

$$S_t^D = \begin{cases} 0 & \text{when } x_t + E_{t-1}^{Q,B} - E_{t-1}^{Q,S} < 0 \\ x_t + E_{t-1}^{Q,B} - E_{t-1}^{Q,S} & \text{when } x_t + E_{t-1}^{Q,B} - E_{t-1}^{Q,S} \geq 0 \end{cases} \tag{7.16}$$

Energy sold at Elbas is delivered unless the production is below the sold amount:

$$E_t^{ND} = \begin{cases} 0 & \text{when } x_t \geq E_{t-1}^{Q,S} \\ E_{t-1}^{Q,S} - x_t & \text{when } x_t < E_{t-1}^{Q,S} \end{cases} \tag{7.17}$$

Note that, $E_t^{ND} = 0$, in the case when enough energy is available to deliver all energy sold at Elbas. Then only regulation cost is paid by the mismatch between the spot bid and energy delivered to the spot market, that is: $R\{S_t^D - S_{t'}^Q\}$.

Examining the time marks in Eq. (7.9) shows that the decisions which affect the income are not all taken at the same time. The spot market bid is placed at time $t'$, while the Elbas bid (sale or purchase) is first placed at time $t-1$. This implies that more information about the final production (at time $t$) is available when the Elbas bid is being placed.

At that time it is assumed that the forecasting system can provide a probability density function calculated at time $t-1$, describing the production at time $t$:

$$f_{X_t}^{t-1}(x_t) \tag{7.18}$$

When a decision problem as the one being formulated here is solved, only information available before the time of the first decision can be included. But how can the Elbas decision then be included in the problem if the prediction calculated at $t-1$ is not available?

One way is to simplify the problem by assuming that the production at time $t$ is equal to the production at time $t-1$.

The expected income $z$, when applying this approximation, is given as:

$$z = \int_0^{x_{\max}} \Big[ S_t^P S_{t'}^D + E_{t-1}^{P,S} E_{t-1}^{Q,S}(x_t) - E_{t-1}^{P,B} E_{t-1}^{Q,B}(x_t) \\ -R\{S_t^D - S_{t'}^Q - E_t^{ND}\} \Big] f_{X_t}^{t'}(x_t) \mathrm{d}x_t \tag{7.19}$$

The sold and bought energy at Elbas is formulated as a function of the production because the imbalance is a function of the production and the Elbas decision is balance dependent. Finding the maximum expected income $z^*$ now involves finding the functions $E_{t-1}^{Q,S}(x_{t-1})$ and $E_{t-1}^{Q,B}(x_{t-1})$ which is a difficult or impossible task. In order to make the optimisation problem solvable $E_{t-1}^{Q,S}(x_{t-1})$ and $E_{t-1}^{Q,B}(x_{t-1})$ can both be approximated using approximation functions. That way the optimisation involves finding the parameters for the approximation functions instead of finding the functions themselves. One choice is to use the step functions defined in Eq. (7.24) and (7.25). Using them, the optimal decision can by found by selecting the correct values for the spot bid $S_{t'}^Q$, and the parameters of the approximation functions:

$$\begin{aligned}
U &: \text{Set of points which divide} && (7.20)\\
& \quad \text{the space of } x \text{ into } N \text{ intervals} && (7.21)\\
& \quad \text{between 0 and maximum production.} && (7.22)\\
U &= \{x_1, \ldots, x_{N-1}\} && (7.23)
\end{aligned}$$

$$E_{t-1}^{Q,S}(x_{t-1}, \mathbf{s}) = \begin{cases} s_1 & 0 \le x_{t-1} < x_1 \\ s_2 & x_1 \le x_{t-1} < x_2 \\ \vdots & \vdots \\ s_N & x_{N-1} \le x_{t-1} \le x_{\max} \end{cases} \tag{7.24}$$

$$E_{t-1}^{Q,B}(x_{t-1}, \mathbf{b}) = \begin{cases} b_1 & 0 \le x_{t-1} < x_1 \\ b_2 & x_1 \le x_{t-1} < x_2 \\ \vdots & \vdots \\ b_N & x_{N-1} \le x_{t-1} \le x_{\max} \end{cases} \tag{7.25}$$

$$z^* = \max_{S_{t'}^Q, \mathbf{s}, \mathbf{b}} \int_0^{x_{\max}} \Big[ S_t^P S_{t'}^D + E_{t-1}^{P,S} E_{t-1}^{Q,S}(x_t, \mathbf{s}) - E_{t-1}^{P,B} E_{t-1}^{Q,B}(x_t, \mathbf{b}) \\ -R\{S_t^D - S_{t'}^Q - E_t^{ND}\} \Big] f_{X_t}^{t'}(x_t) \mathrm{d}x_t \tag{7.26}$$

### 7.2.4 Using joint probability density functions

A better way to include the Elbas decision is by approximating the prediction used when biding at Elbas. If a joint probability density function, such as the one defined in Eq. (7.27), is available at time $t'$, the forecast for the production at time $x_{t-1}$ can be calculated exactly and the forecast available at $t-1$ approximated for a given production level $x_{t-1}$, see Eq. (7.28) and (7.29) respectively.

$$f^{t'}_{X_{t-1},X_t}(x_{t-1}, x_t) \tag{7.27}$$

$$f^{t'}_{X_{t-1}}(x_{t-1}) = \int_0^{x_{\max}} f^{t'}_{X_{t-1},X_t}(x_{t-1}, x_t)\mathrm{d}x_t \tag{7.28}$$

$$f^{t-1}_{X_t}(x_t) \approx f^{t'}_{X_t|X_{t-1}=x_{t-1}}(x_t) \tag{7.29}$$

But how can the joint probability distribution be used? If we now assume that a bid has been placed at the spot market and the prediction $f^{t-1}_{X_t}(x_t)$ is known, then the maximum expected income is gained by finding the values for $E^{Q,S}_{t-1}$ and $E^{Q,B}_{t-1}$ which maximise $z^*$. The maximum expected income $z^*$ is then is a function of the spot market bid and the prediction at time $t-1$, see Eq. (7.30).

$$\begin{aligned} z^*(S^Q_{t'}, f^{t-1}_{X_t}) &= \max_{E^{Q,S}_{t-1}, E^{Q,B}_{t-1}} \int_0^{x_{\max}} \Big[ S^P_t S^D_{t'} + E^{P,S}_{t-1} E^{Q,S}_{t-1} - E^{P,B}_{t-1} E^{Q,B}_{t-1} \\ &- R\{S^D_t - S^Q_{t'} - E^{ND}_t\} \Big] f^{t-1}_{X_t}(x_t)\mathrm{d}x_t \end{aligned} \tag{7.30}$$

Inserting the approximation defined in Eq. (7.29) into (7.30) gives $z^*$ which depends on the production level $x_{t-1}$ instead of the prediction at that time:

$$\begin{aligned} z^*(S^Q_{t'}, x_{t-1}) &= \max_{E^{Q,S}_{t-1}, E^{Q,B}_{t-1}} \int_0^{x_{\max}} \Big[ S^P_t S^D_{t'} + E^{P,S}_{t-1} E^{Q,S}_{t-1} - E^{P,B}_{t-1} E^{Q,B}_{t-1} \\ &- R\{S^D_t - S^Q_{t'} - E^{ND}_t\} \Big] f^{t'}_{X_t|X_{t-1}=x_{t-1}}(x_t)\mathrm{d}x_t \end{aligned} \tag{7.31}$$

The optimal spot market bid, can therefore be found by maximising the expected income as in Eq. (7.32).

$$z^{**} = \max_{S^Q_{t'}} \int_0^{x_{\max}} z^*(S^Q_{t'}, x_{t-1}) f^{t'}_{X_{t-1}}(x_{t-1})\mathrm{d}x_{t-1} \tag{7.32}$$

Using the following property:[37]

$$f^{t'}_{X_{t-1},X_t}(x_{t-1}, x_t) = f^{t'}_{X_{t-1}}(x_{t-1}) f^{t'}_{X_t|X_{t-1}=x_{t-1}}(x_t) \tag{7.33}$$

The optimisation problem can be rewritten as:

$$
\begin{aligned}
z^{**} &= \max_{S_{t'}^{Q},\mathbf{s},\mathbf{b}} \int_0^{x_{\max}}\int_0^{x_{\max}} \Big[S_t^P S_{t'}^D + E_{t-1}^{P,S}E_{t-1}^{Q,S}(x_{t-1},\mathbf{s}) - E_{t-1}^{P,B}E_{t-1}^{Q,B}(x_{t-1},\mathbf{b}) \\
&- R\{S_t^D - S_{t'}^Q - E_t^{ND}\}\Big] f_{X_{t-1},X_t}^{t'}(x_{t-1},x_t)\mathrm{d}x_{t-1}\mathrm{d}x_t \qquad (7.34)
\end{aligned}
$$

Again $E_{t-1}^{Q,S}(x_{t-1}), E_{t-1}^{Q,B}(x_{t-1})$ should be formulated using an approximation function in order to make the problem solvable. For instance, step functions as in Eq. (7.24) and (7.25).

### 7.2.5 Formulation using discrete variables

The production at time $t-1$ and $t$ does not necessarily have to be formulated using continuous levels for the production. The discrete random variables, $X_{t-1}$ and $X_t$, which take only values that belong to the limited sets $H_1$ and $H_2$ respectively, can be used instead.

$$
\begin{aligned}
X_{t-1} \quad &, \quad X_{t-1} \in H_1 \qquad (7.35)\\
X_t \quad &, \quad X_t \in H_2 \qquad (7.36)
\end{aligned}
$$

For these variables the joint mass function $f_{X_{t-1},X_t}^{t'}$ is defined to be [25]:

$$
f_{X_{t-1},X_t}^{t'}(x_{t-1},x_t) = \mathbb{P}\{X_{t-1} = x_{t-1}, X_t = x_t\} \qquad (7.37)
$$

If the number of possible production levels at time $t-1$ is limited then the possible optimal trade decisions at Elbas are also limited. Therefore, Elbas actions do not necessarily have to be formulated using functions. Instead vectors can be used, for instance $E_{t-1,i}^{Q,S}$ and $E_{t-1,i}^{Q,B}$, described as:

$E_{t-1,i}^{Q,S}$ : Amount sold at Elbas for production level $i$ at time $t-1$.

$E_{t-1,i}^{Q,B}$ : Amount bought at Elbas for production level $i$ at time $t-1$.

Now the optimisation problem defined in Eq. (7.38) can be rewritten, using this new formulation, on to a discrete form:

$$
\begin{aligned}
z^{D} &= \max_{S_{t'}^{Q},\mathbf{s},\mathbf{b}} \sum_{i=1}^{|H_1|}\sum_{j=1}^{|H_2|} \Big[S_t^P S_t^D + E_{t-1,i}^{P,S}E_i^{Q,S} - E_{t-1,i}^{P,B}E_i^{Q,B} \\
&+ R\{S_t^D - S_{t'}^Q - E_t^{ND}\}\Big] f_{X_{t-1},X_t}^{t'}(x_{t-1,i},x_{t,j}) \qquad (7.38)
\end{aligned}
$$

Where $x_{t-1,i}$ is the $i$'th value in $H_1$, $x_{t,j}$ is the $j$'th value in $H_2$ and $p_{i,j} = \mathbb{P}\{X_{t-1} = x_{t-1,i}, X_t = x_{t,j}\}$.

### 7.2.6 Decision Tree

Discrete joint probability mass functions $f$ are often represented graphically when problems such as the one defined in Eq. (7.38) are solved using stochastic programming. The graphical representation is called a decision tree[28] and is used to visualise possible outcomes and clarify when decisions are taken. The construction of a decision tree which fits the problem in question, will be explained through an example. In the example, it will be explained how the decision tree in Figure 7.2 was drawn.

Node number 1 is called the root node, it represents the time when the initial decision is taken and nothing is known for sure about the future production. However a discrete forecast,$f^{t'}_{X_{t-1}}(x_{t-1})$, is available and it predicts three possible production levels at time $t-1$, each with an assigned probability. To show this on the graph, three nodes are drawn in a co-ordinate system with the co-ordinates $(t-1, production)$. These are the nodes labelled 2, 3 and 4. Edges are drawn between the root and the nodes 2, 3 and 4 to show that given the available information now (the situation in the root node) only production level $x_2$ to $x_4$ can be expected. Given that the production will be observed to be $x_2$ at time $t-1$ a forecast,$f^{t'}_{X_t|X_{t-1}=x_2}(x_t)$ , is available predicting four possible production levels at time $t$. These are represented by the nodes 5, 6, 7 and 8. An edge is drawn between node 2 and 5, 6, 7 and 8 to show that it is impossible (according to the predictions) to observe the production in vertexes 5 to 8 without observing the production in vertex 2 at time $t-1$. The process of drawing a decision tree for the problem in question can therefore be described as: *Draw a root node, draw nodes for all predicted production levels at time $t-1$. For each production level at time $t-1$ find the conditional distribution for production at time $t$ and draw all possible production levels at time $t$ given that conditional distribution.* The numbers in node 1 to 4 are underlined to show that in these nodes a decision is made. It is clear that the initial decision depends on the joint probability mass function but the decisions at time $t-1$ depend on the initial decision and the one step conditional distribution.

### 7.2.7 Building a decision tree given a cumulative distribution function

A tree, as described in the previous section can not be build without the discrete probability mass functions defined in Eq. (7.39) and(7.40), or the discrete joint

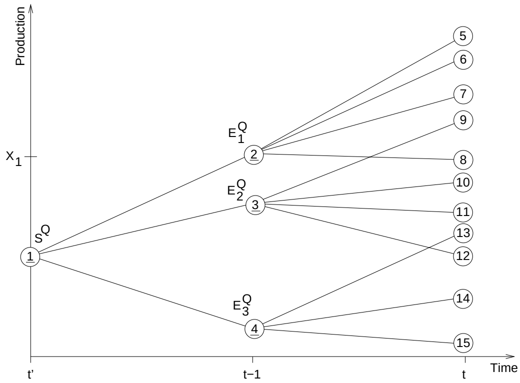


Figure 7.2: *A decision tree. At time $t'$ the initial decision, how much should be bidded at the spot market must be made. At time $t-1$ it must be decided how much is old or bought at Elbas. At time $t$ it becomes clear how much was produced and regulation cost is paid according to the difference between the bid $S^Q$ and the amount of energy delivered to the spot market.*

probability mass function defined in Eq. (7.41).

$$f^{t'}_{X_{t-1}}(x_{t-1}) \tag{7.39}$$

$$f_{X_t|X_{t-1}=x_{t-1}}(x_t) \tag{7.40}$$

$$f_{X_t,X_{t-1}}(x_{t-1}, x_t) \tag{7.41}$$

These function are however not provided directly by a normal forecasting system so here a method to approximate them, given a probabilistic forecast, is explained. It is assumed that at time $t'$ two cumulative distribution function are supplied by the forecasting system, valid at time $t-1$ and $t$ respectivelly:

$$F^{t'}_{X_{t-1}}(x_{t-1}) \tag{7.42}$$

$$F^{t'}_{X_t}(x_t) \tag{7.43}$$

These two predictions can not be used directly to calculate the joint probability mass function by multiplying there derivative, as in Eq. (7.44), because the error in the predictions is correlated.

$$\frac{\partial F^{t'}_{X_{t-1}}(x_{t-1})}{\partial x_{t-1}} \frac{\partial F^{t'}_{X_t}(x_t)}{\partial x_{t-1}} \neq f_{X_{t-1},X_t}(x_{t-1}, x_t) \tag{7.44}$$

In order to use the two predictions together a correlation structure describing the correlation of the error must exist. Here, however, another more intuitive method will be applied using the relationship defined in Eq. (7.33), page 77.

Only one of the forecasts available will be used for the long step, that is from time $t'$ to $t-1$ and then the conditional one step prediction will be approximated using historical data

**Use of the available prediction**: A cumulative mass function can be used to find a probability density functions using Eq. (7.45).

$$\frac{\partial F_X(x)}{\partial x} = f_X(x) \tag{7.45}$$

But the cumulative mass functions supplied are non parametric estimates so numerical differentiation[39] must be used instead. The result is a probability mass function on a discrete form, just as needed. First a set a set $O$ is defined, containing $N+1$ ordered points with in the range of {X}.

$$O = \{x_1, \ldots, x_{N+1}\} \tag{7.46}$$

A probability mass function, taking a non zero value in $N$ points, placed in the center between two adjoining points in $O$ can then be approximated as:

$$\hat{f}_{X_{t-1}}\left(\frac{x_{i+1}+x_i}{2}\right) = F(x_{i+1}) - F(x_i) \qquad \text{for } i \in \{1, \ldots, N\} \tag{7.47}$$

The quality of $\hat{f}$ depends both on the quality of $F$ and the selection of $O$. A poor estimate of $F$ gives a poor estimate of $\hat{f}$ but a good estimate of $F$ does not necessarily give a good estimate of $\hat{f}$. For that to be accomplished, the set $O$ must contain enough points selected in such a way that all fast changes in $F$ are caught. What is good enough depends on the problem but the following example might explain things better.

**Example** A random variable {Y} is known to take values between 0 and 10. Its probability mass function is not known and no observations are available. The only information at hand is the value of its cumulative distribution function $F$ in four points:

$$F(10) = 1 \tag{7.48}$$
$$F(9) = 0.5 \tag{7.49}$$
$$F(5) = 0.5 \tag{7.50}$$
$$F(0) = 0 \tag{7.51}$$

A discrete probability mass function can be approximated given this information, but different selection of the set $O$ gives different approximations as the selection of two different $O$'s shows. For a set $O_1 = \{0, 5, 10\}$ the probability mass function is approximated as:

$$\hat{f}_1(y) = \begin{cases} 0.5 & \text{for } y = 7.5 \\ 0.5 & \text{for } y = 2.5 \\ 0 & \text{else} \end{cases} \tag{7.52}$$

and the expected value of $Y$ estimated to be 5. For another set set $O_2 = \{0, 5, 9, 10\}$ the probability mass function in approximated as:

$$\hat{f}_2(y) = \begin{cases} 0.5 & \text{for } y = 9.5 \\ 0 & \text{for } y = 7 \\ 0.5 & \text{for } y = 2.5 \\ 0 & \text{else} \end{cases} \tag{7.53}$$

and the expected value of $Y$ estimated to be 5.5. The second estimate is closer to the true probability mass function cause the resolution of the set $O_2$ is higher than the resolution of $O_1$. The estimate can though never be better than the information provided.

If the approximated probability mass function is supposed to be a valid description of the random variable $X_{t-1}$, equations (5.3) and (5.4), page 32 must be fulfilled. If $F$ is a valid cumulative distribution function and $O$ is an ordered set, Eq. (5.4) is automatically fulfilled. However Eq. (5.3) is only fulfilled if $F$ is a valid cumulative distribution function and the boundary points, $x_1$ and $x_N$, in $O$ give $F(x_1) = 0$ and $F(x_N) = 1$.

**Historical data prediction**: Now only the probability mass function $f_{X_t|X_{t-1}=x_{t-1}}(x_t)$ is needed to build the decision tree. It can be approximated using historical observations of the production. Lets assume that a set $H$ of $N$ observations $H = \{x_1, \ldots, x_N\}$ exists. For a given production, $x_{t-1}$, the probability of possible production can be estimated by defining a set $H^s$ as:

$$H^s = \{h_i : h_i \in H \wedge x_{t-1} - \beta < h_{i-1} < x_{t-1} + \beta\} \tag{7.54}$$

$H^s$ is then a set containing observed power production given that the production in the previous hour was close to $x_{t-1}$. How close is controlled by adjusting $\beta$. In other words, the subset $H^s$ is conditioned on the production at time $t-1$.

The probability mass function

$$\hat{f}_{X_t|X_{t-1}=x_{t-1}}(x_t) \tag{7.55}$$

can be estimated by finding a histogram for the set of observations $H^s$. One way to calculate the histogram is to divide the space which the observations in $H^s$ span into $N$ equally sized intervals. One way to determine how many these intervals should be, is by using Sturges formula[40]

$$N = \lfloor ln(|H^s|) + 1 \rfloor \tag{7.56}$$

where

$$\begin{aligned} |H^s| &: \text{ Is the number of observations in } H^s & (7.57) \\ h^{UB} &: \text{ Is the highest observed power in } H^s & (7.58) \\ h^{LB} &: \text{ Is the lowest observed power in } H^s & (7.59) \end{aligned}$$

The length of each interval is calculated as:

$$l = \frac{(h^{UB} - h^{LB})}{N} \tag{7.60}$$

Interval number $i$ is then $[h^{LB} + (i-1)l, h^{LB} + (i)l)]$ for $i \in [1, \ldots, N]$. Lets define the set $H_i^s$ as a set of all the observations in $H^s$ which are within interval number $i$. Now the probability mass function can be approximated as

$$\hat{f}_{X_t|X_{t-1}=x_{t-1}}(x_t) = \begin{cases} \frac{|H_i^s|}{|H^s|} & \text{if } x = \overline{H}_i^s \\ 0 & \text{else} \end{cases} \tag{7.61}$$

Where $\overline{H}_i^s$ is the mean value of all the observations in $H_i^s$.

A decision tree can be built using the approximation methods in Eq. (7.47) and (7.61) instead of the joint probability mass function. It is though important to note that the correctness of the decision tree is highly dependent on the configuration, that is the selection of $O$ and $\beta$.

| Variable | Unit/Format | Description |
|---|---|---|
| ElbasMaxPrice | $DKK/MWh$ | The price of the highest contract during that hour. |
| ElbasAvgPrice | $DKK/MWh$ | The average price of all contracts during that hour. |
| ElbasMinPrice | $DKK/MWh$ | The price of the lowest contract during that hour. |
| ElbasBoughtDKE | $MWh$ | Energy participants in DK-E bought at Elbas. |
| ElbasSoldDKE | $MWh$ | Energy participants in DK-E sold at Elbas. |
| First observation | 2004-04-01 01 | |
| Last observation | 2005-04-30 23 | |

Table 7.1: *Additional data used to test the performance of the biding strategy. Other variables used are listed in Table 6.1.*

## 7.3 Implementing and testing the two market bidding strategy

In the previous section a method to construct a trade plan, when selling energy both at the spot market and Elbas was developed. In this section, the method will be implemented and tested using real observations. In Section 7.3.2 the market data from Elbas will be analysed in order to see what elements must be included when the method is implemented. In Section 7.3.3 the implementation is described, this involves discretisation of the prediction and formulation of a stochastic optimisation problem. Finally the implementation is tested in Section 7.3.4.

### 7.3.1 Data

The same data is used as in the previous chapter, see description in Section 6.2.1. There are additional variables, listed in Table 7.1, provided by Elkraft-System [20].

| | Minimum | Average | Maximum | Bought | Sold |
|---|---|---|---|---|---|
| Min | 4 | 7 | 9 | 0 | 0 |
| Median | 194 | 207 | 217 | 0 | 0 |
| Mean | 191 | 205 | 217 | 16 | 16 |
| Max | 670 | 733 | 793 | 640 | 543 |

Table 7.2: *Summary for the minimum, average and maximum price, and volumes traded at East Denmark, April 2004-2005.*

| Elbas net flow | Portion of time |
|---|---|
| Import | 15% |
| Export | 18% |
| Zero | 67% |

Table 7.3: *Use of Elbas in international perspective during the period. "Zero" means that the energy bought at Elbas in DK-E was equal to the energy sold at Elbas in DK-E.*

### 7.3.2 Elbas prices, demand and supply

There is not only one price defined in each hour at Elbas like at the spot market. The price is up to the participant selling the energy, therefore all contracts between sellers and buyers can have a different price during one hour. Prices of individual contracts are not stored in NordPool's data collection due to confidential agreements. Instead the lowest, the highest and the average price is listed, see statistics for theses prices in Table 7.2.

The analysis here will be focused on identifying how the Elbas price and the volume traded react to other elements in the system such as the system balance and prediction error. In order to make that possible a number of plots have been made, shown in Figure 7.3 and 7.4. The plots in Figure 7.4 are in fact kernel smoothed[3] versions of the plots in Figure 7.3.

[3] The smoothing function `LOESS`, known both in `S-Plus` and `R`, was applied with an suitable bandwidth. In should be noted that kernel smoothers tend to give biased estimate around boundaries where the number of observations is small[41]. Such places can be identified by looking at Figure 7.3.

#### 7.3.2.1 Elbas relation to the spot price

By looking at Figure 7.3.top.right it is clear that the Elbas price tends to follow the spot price, specially when the spot price behaves "normally" , that is when it is within the range of $150DKK$ and $250DKK$. However, when the spot price is not normal and spikes are observed, the Elbas price tends to be below the spot.

#### 7.3.2.2 The effect of the system imbalance

Examining the relation between the volume traded at Elbas and the total system balance, Figure 7.4.center.right, shows that when the balance is negative, energy is bought at Elbas but when the balance is positive, energy is normally sold at Elbas. In other words, Elbas seems to be used as a tool to bring the system back into balance, just as it was designed for.

#### 7.3.2.3 Elbas price versus volume

Figure 7.4.center.left shows that when more energy is sold at Elbas in East Denmark, the price tends to be below the spot price. On the other hand, are the prices above the spot price when large amounts are bought. The Elbas price has therefore the same characteristics as the price of the regulation power. When used to up regulate the price is above the spot, but when used for down regulation the price is below the spot.

#### 7.3.2.4 The effect of prediction error on the volume

Prediction error[4] is correlated with the system balance. It is therefore not surprising that the prediction error has the same relationship to the volume traded as the system balance, see Figure 7.4.bottom.left.

#### 7.3.2.5 The effect of prediction error on the price

Figure 7.4.bottom.right shows the price difference between Elbas and the spot market as a function of the prediction error. When too much wind energy is

[4] Defined as the production minus the forecast.

produced, the Elbas price falls below the spot market price and when wind energy is missing, the Elbas price rises above the spot market price.

#### 7.3.2.6 Elbas and wind power

The most important propertie of the Elbas market for a wind power producer is the behaviour of the price. Unfortunately, but naturally, does the price fall when wind power producers produce more than expected. One rational explanation might be that when one wind power producer is producing more than forecasted, other producers are most likely also producing more than forecasted. In that case the supply increases without an equal increase in demand. The opposite should hold when more is predicted than produced.

Much more detailed analysis is needed in order to give a complete description of Elbas. Such a description does though not benefit this project, cause Elbas has only been active in Denmark for a short period and the trade pattern in the market can be expected to change over the next years. The aim here is therefore only to examine which main characteristics must be included in a decision problem when searching for an optimal bid at the spot market. The conclusion is: prices must be allowed to depend on the prediction error or system balance.

### 7.3.3 Implementation

Now when a new mathematical model has be formulated and the data from Elbas examined, enough information is available to solve the actual decision problem. This will be done in three steps. First the discretisation method will be applied to the probabilistic predictions and a decision tree built. Then the mathematical models will be refined, in order to include the new information about Elbas, and finally a solver will be used to find the optimal trade strategy.

#### 7.3.3.1 Building the decision tree

Solving the problem using stochastic programming requires that the predictions have probabilistic information and that they are on a discrete form. The first step is therefore to use the quantile regression method explained in Section 6.2.5.1 to transform point predictions into probabilistic predictions. The probabilistic prediction is then discretised using the techniques explained in Section

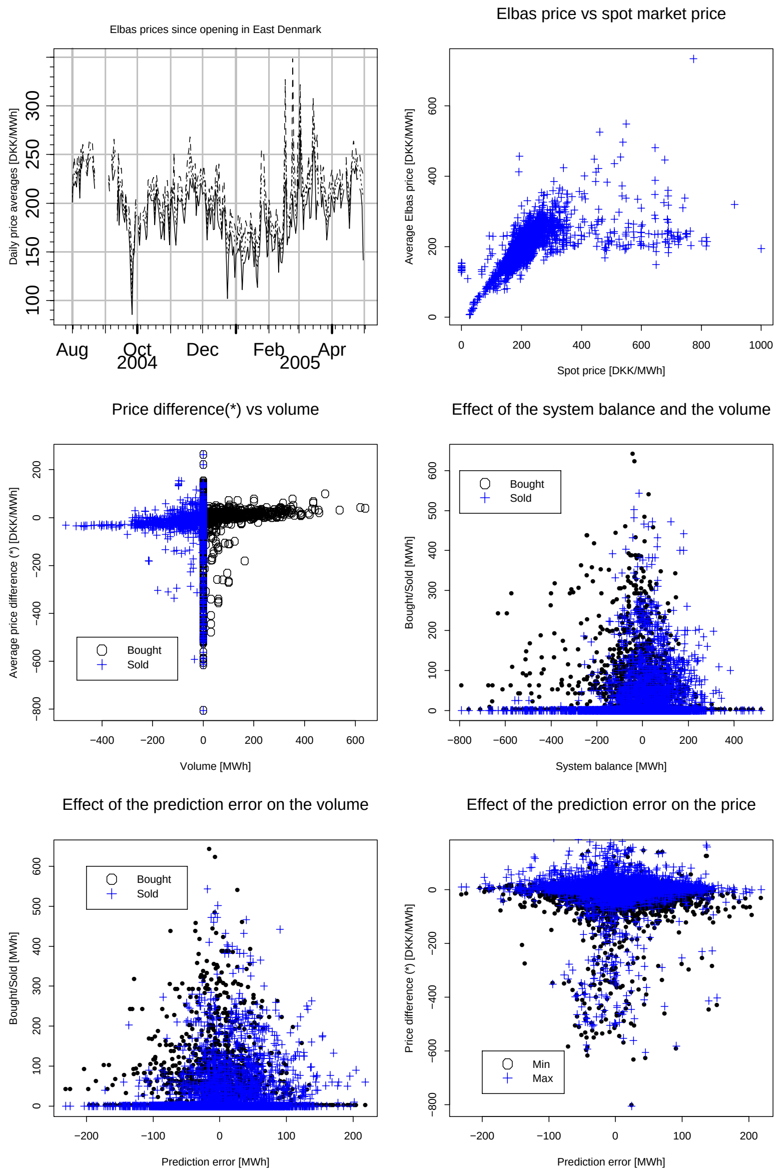


Figure 7.3: *Elbas prices and volumes, see section 7.3.2 for remarks. (*) The price difference is defined as:*$P_d = P_e - P_s$ *where* $P_e$ *is the price at Elbas and* $P_s$ *is the spot market price.*

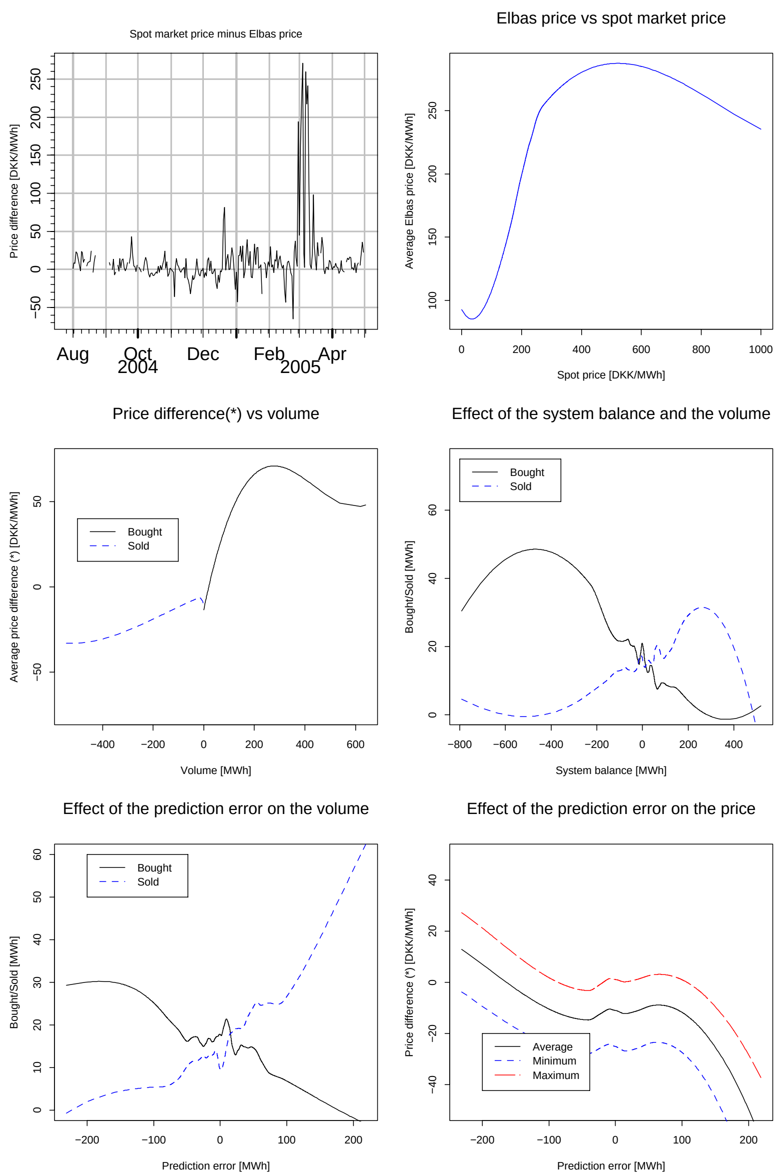


Figure 7.4: *Smoothed Elbas plots, see Section 7.3.2 for remarks.* (*) *The price difference is defined as:*$P_d = P_e - P_s$ *where* $P_e$ *is the price at Elbas and* $P_s$ *is the spot market price.*

7.2.7 page 79 and the same data as in the previous chapter. The discretisation will be performed in two steps, first the prediction for hour $t-1$ will discretised then the one step prediction for hour $t$ given the production at hour $t-1$.

**The prediction for time** $t-1$: For a given prediction, $F$, calculated at time $t'$ predicting the production at time $t-1$, a set $O$ is defined as $O = \{x_1, \ldots, x_{21}\}$ so that equation (7.62) holds.

$$\begin{aligned} F(x_1) &= 0.010 \\ F(x_2) &= 0.025 \\ F(x_3) &= 0.100 \\ F(x_4) &= 0.150 \\ \vdots &= \vdots \\ F(x_{18}) &= 0.850 \\ F(x_{19}) &= 0.900 \\ F(x_{20}) &= 0.975 \\ F(x_{21}) &= 0.990 \end{aligned} \tag{7.62}$$

The probability mass function for the production at time $t-1$, found by using Eq. (7.47) , is approximated as:

$$\hat{f}^o_{X_{t-1}}(x) = \begin{cases} F(x_{i+1}) - F(x_i) & \text{if } x = \frac{x_{i+1}+x_i}{2} \\ 0 & \text{else} \end{cases} \tag{7.63}$$

Since the end points, that is $F(0)$ and $F(x_{max})$ are not included, the total probability of all possible values of {X} is below 1, $\hat{f}^o_{X_{t-1}}$ is therefore not an valid probability mass function (see Eq. (5.3), page 32). This could be fixed by including the end points but then another problem would arise. The performance of the quantile regression is not always up to standards around the boundaries in practical problems because the number of observations there is often low. Here this boundary problem is solved by defining that no observations can be found below $x_1$ or above $x_{21}$. Using that, the probability mass function is redefined as:

$$\hat{f}_{X_{t-1}}(x) = \begin{cases} \frac{1}{0.98}\left(F(x_{i+1}) - F(x_i)\right) & \text{if } x = \frac{x_{i+1}+x_i}{2} \\ 0 & \text{else} \end{cases} \tag{7.64}$$

$\hat{f}_{X_{t-1}}$ is now a discrete probability mass function on a form accepted by the solution algorithm.

**The one step prediction for time** $t$: Now the one step prediction $f_{X_t|X_{t-1}=x_{t-1}}(x_t)$ needs to be estimated. That will be done using the histogram method outlined on page 82. For a good estimate of $f$ the correct value of the control parameter

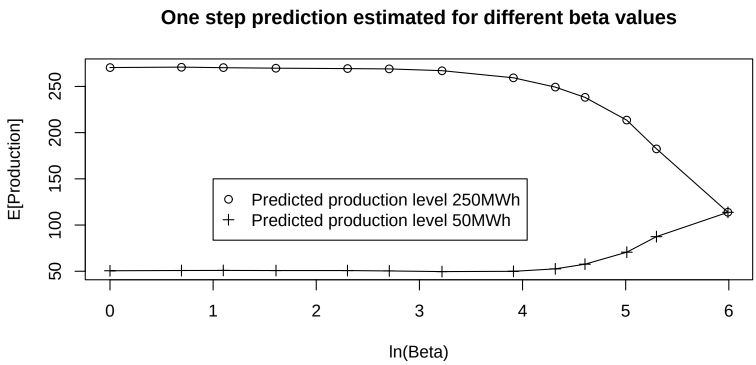


Figure 7.5: *Expected production for two different trees, one for a prediction of* $250MWh$ *and the other for* $50MWh$*, build using different values for the adjustment parameter* $\beta$*.*

$\beta$ must be selected. A large $\beta$ makes the conditional effect low but a small $\beta$ gives an estimate based on few observations. $\beta$ must therefore be selected as some value, which maximises the conditional effect while ensuring that the estimation of the distribution is based on a sufficient number of observations. The following test were performed in order to find a suitable value for $\beta$: For two different predictions, one $250MWh$ and one $50MWh$ trees were created for 13 different $\beta$ values, ranging from $1MWh$ to $400MWh$ . For each tree the expected production was calculated and plotted in Figure 7.5 (note that the x-axis is log scaled). The figure shows clearly that for an extremely large value of $\beta$ the histogram used to estimate the prediction from hour $t-1$ to $t$ is not conditioned on the production at $t-1$. Instead of estimating $f_{X_t|X_{t-1}=x_{t-1}}(x_t)$, $f_{X_t}(x_t)$ is estimated. This explains why the expected production is the same for a production forecast of $50MWh$ and $250MWh$. However, selecting $\beta$ between 1 and 10 had little effect on the expected production level.

Counting how many observations were used to calculate the probability density function gave the following results: on average 50 were used for $\beta = 1$, growing linearly to a total of 350 for $\beta = 10$. Based on this, $\beta = 4$ was selected, allowing observations to be in a $8MWh$ wide range round $x_{t-1}$.

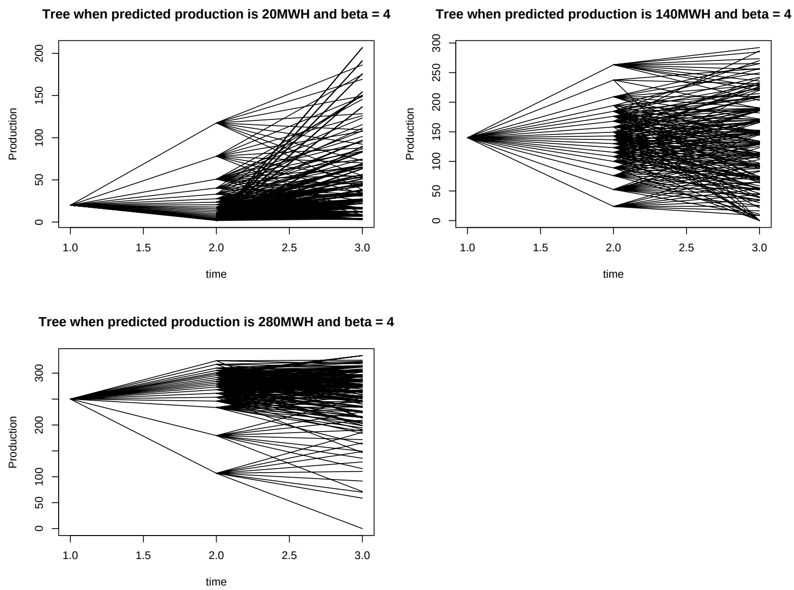


Figure 7.6: *Trees for three different production forecasts*

### 7.3.3.2 Comparison of trees and point predictions

A decision tree created at time $t'$ describing the production at time $t-1$ and $t$ can be used to find a point forecast for the production at time $t$. Each leaf node (nodes at time $t$) has a certain production level and probability assigned to it. Lets define a set of leaf nodes, $L = \{n_1, n_2, \ldots, n_N\}$, and two functions, $p$ and $g$, which take tree nodes as arguments:

$g(n_i)$ : Production (generation) in node $i$

$p(n_i)$ : The probability that the production in node $i$ is observed

The point forecast $\hat{p}$ can then be calculated as the expected production of the tree:

$$\hat{p} = \sum_{i=1}^{|L|} p(n_i)g(n_i) \tag{7.65}$$

If the expected production of a tree can be calculated, then the tree can be compared to a point prediction. By doing that, the quality of the tree can be roughly estimated[5]. This was done for the first 8000 hours in the data set

[5] Tree predictions contain more information than point predictions. All parts of the three construction can therefore not be tested by comparing it to a point prediction.

defined on page 51. For every prediction a decision tree was build and the mean squared error of it's expected production compared to the mean square error of the original point prediction. The results were that the mean square error of the tree point prediction was 4.5% higher than of the original production forecast. The tree is therefore not too far from the original production forecast although it is a worse point predictor. The difference is most likely caused by errors in the crude method used to estimate the one step prediction for the production at time $t$.

#### 7.3.3.3 Comparison of trees and probabilistic predictions

The information in the leaf-nodes of a tree can be used to estimate the cumulative distribution function for the production at time $t$. That estimate can then be compared to the cumulative distribution function supplied by the forecasting system. A Kolmogorov-Smirnov test[42][6] is an appropriate test to determine whether the two distributions are different or not. That test was used to check if the two distributions were different for the first 2242 hours in the data set (starting in April 2004). The null hypothesis *"The distributions are not different"* was rejected in 23.5% of the cases for a significance level of 0.05. However, the rejection depended on the production level predicted as the histogram in Figure 7.7.B shows. The tree prediction did not mach the cumulative distribution function of the point forecast in the cases when the production was low at time $t-1$ and falling. This is because the tree prediction is based on the point prediction for time $t-1$ and the conditional estimate of the change from one hour to another, that is, $f^{t'}_{X_t|X_{t-1}=x_{t-1}}(x_t)$, expects the production to increase or stay unchanged if the production at time $t-1$ is low. That is, in other words, $\mathbb{E}[X_t|x_{t-1}] > x_{t-1}$ for small values of $x_{t-1}$. If $f^{t'}_{X_t|X_{t-1}=x_{t-1}}$ would have been estimated, conditioned both on the prediction for time $t-1$ and $t$, then the distributions would have been expected to mach in almost all the hours. That could not be checked due to time limitations.

#### 7.3.3.4 Stochastic programming formulation

The mathematical model of the optimisation problem was created both in a continuous and discrete version in Section 7.2.3. The purpose of that formulation was to derive a correct formula for the expected income, but not necessarily to make it solvable in practise. Here the discrete formulation in Eq. (7.38), page 78, will be transformed into a solvable stochastic optimisation problem with

[6] The chapter *Are Two Distributions Different?* was available on line, free of charge (28th of July 2005).

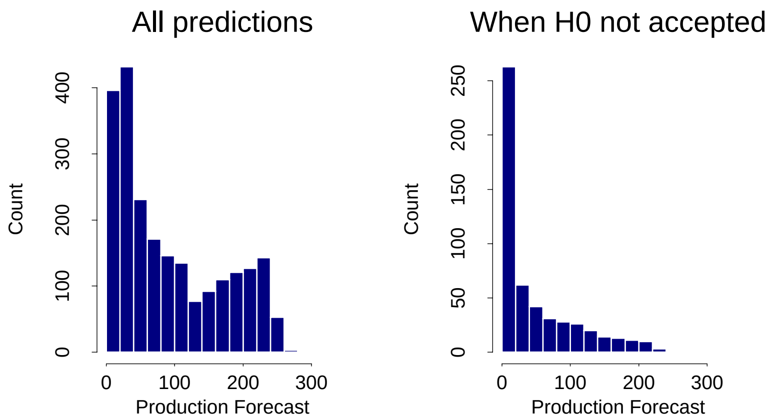


Figure 7.7: *From left: A,B. The null hypothesis "The distributions are not different" was more often rejected for low predictions than high.*

the associated constraints. The problem will be formulated with three different objective function all catching a different behaviour of the prices at Elbas. As described in Section 5.3, the formulation should contain an objective function to minimise or maximise, predefined parameters and decision variables. Decision variable which can not take any value must be bounded using linear equations. If parameters and variables are indexed, the indexes must come from a limited, predefined set. All variables, parameters and indexes used in the formulations are listed in Table 7.4.

Each realisation of the production at time $t-1$ and $t$ is called a scenario. The probability that scenario $i, j$ is observed is defined as:

$$f^{t'}_{X^D_{t-1}, X^D_t}(x_i, x_{i,j}) = p_{i,j} \tag{7.66}$$

where $f^{t'}_{X^D_{t-1}, X^D_t}$ is a joint probability mass function. $x_i$ can take N different values, $j$ can take $M_i$ different values, depending on the state $i$ at time $t-1$.

The objective is to maximise the expected income. The income function defined in Eq. (7.38), page 78 is therefore a good candidate for an objective function but some changes must be made both to make it acceptable for the solver and also to include the results from the Elbas data analysis.

Lets start with a linear version of Eq. (7.38). The only non linear term is the regulation cost function $R$ which is piecewise linear with only one brake in the point 0. In Section 5.3.4 such function is called a V-shaped function, there

| Variable | Type | Description |
|---|---|---|
| $i$ | Index | Nodes at time $t-1$ are indexed by i. |
| $j$ | Index | Nodes at time $t$ are index by $i$ and $j$. The $j$'th child of node $i$ is indexed as node $i,j$. |
| $S_{t'}^{Q}$ | Decision | The quantity bidden at spot market. |
| $S_t^D$ | Decision | Amount of energy delivered at the spot price. |
| $d_{i,j}$ | Decision | Producers down regulation need. |
| $u_{i,j}$ | Decision | Producers u regulation need. |
| $U_{i,j}$ | Decision | Up regulation when energy sold at Elbas can not be delivered, in all cases cause the production is below the Elbas bid. |
| $E_i^{Q,B}$ | Decision | Quantity bought at Elbas if the production at time $t-1$ is the production in node $i$. |
| $E_i^{Q,S}$ | Decision | Quantity sold at Elbas if the production at time $t-1$ is the production in node $i$. |
| $p_{i,j}$ | Parameter | The probability of production level $i$ at time $t-1$ and $i,j$ at time $t$ |
| $S_t^P$ | Parameter | The spot price at time t. |
| $E_i^{P,S}$ | Parameter | The Elbas selling price at time t, given that branch $i$ was observed. |
| $E_i^{P,B}$ | Parameter | The Elbas buying price at time t, given that branch $i$ was observed. |
| $R_t^U$ | Parameter | The up regulation cost at time t. |
| $R_t^D$ | Parameter | The down regulation cost at time t. |
| $P_t^U$ | Parameter | The price of not being able to deliver energy sold at Elbas. |
| $\alpha^S$ | Parameter | The initial Elbas selling price, given that the price is sale dependent. |
| $\beta^S$ | Parameter | The slope of the Elbas selling price given that the price is sale dependent. |
| $\alpha^B$ | Parameter | The initial Elbas buying price, given that the price is depends on the amount purchased. |
| $\beta^B$ | Parameter | The slope of the Elbas buying price, given that the price is depends on the amount purchased. |

Table 7.4: *Variables, parameters and indexes used in the stochastic formulation.*

it is demonstrated how it can be formulated in a linear program. Using that formulation an valid objective function with the associated constraints can be defined as:

$$\max_{S^Q_{t'}, E^{Q,B}_{\cdot}, E^{Q,S}_{\cdot}} \sum_{i=1}^{N} \sum_{j=1}^{M_i} p_{i,j} \Big[ S^P_t S^D_{i,j} + E^{P,S} E^{Q,S}_i - E^{P,B} E^{Q,B}_i - P^U_t U_{i,j} - R^U_t u_{i,j} - R^D_t d_{i,j} \Big] \tag{7.67}$$

subject to

$$\begin{aligned} - d_{i,j} &\leq S^Q_{t'} - S^D_{i,j} \text{ for all } i,j & (7.68)\\ -d_{i,j} &\leq 0 \text{ for all } i,j & (7.69)\\ S^Q_{t'} - S^D_{i,j} &\leq u_{i,j} \text{ for all } i,j & (7.70)\\ 0 &\leq u_{i,j} \text{ for all } i,j & (7.71)\\ S^D_{i,j} &\leq x_{i,j} + E^{Q,B}_i - E^{Q,S}_i + U_{i,j} \text{ for all } i,j & (7.72)\\ 0 &\leq x_{i,j} + E^{Q,B}_i - E^{Q,S}_i + U_{i,j} \text{ for all } i,j & (7.73)\\ E^{Q,B}_i &\leq \overline{E^{Q,B}} \text{ for all } i & (7.74)\\ E^{Q,S}_i &\leq \overline{E^{Q,S}} \text{ for all } i & (7.75) \end{aligned}$$

There is a new variable, $U_{i,j}$, introduced in this formulation. This is done because it is required that all Elbas bids are fulfilled at all times and only extra energy[7] can be delivered to the spot market. This can not be formulated using the following constraint:

$$\overbrace{S^D_{i,j}}^{\text{Delivered at spot market}} \leq \overbrace{x_{i,j} + E^{Q,B}_i - E^{Q,S}_i}^{\text{Energy left when Elbas bids fulfilled}} \tag{7.76}$$

In the case when $x_{i,j} + E^{Q,B}_i < E^{Q,S}_i$ negative energy would be delivered to the spot market which is infeasible. It is therefore needed to introduce a new energy source, where energy can be bought at time $t$ in order to fulfil the Elbas sale. This can be done at the regulation market, but it would make the results confusing if the normal up regulation need would be mixed with the Elbas regulation need. Therefore a new decision variable $U_{i,j}$ is introduced but it expresses how much regulation power had to be bought in order to satisfy the Elbas sale. The price of that regulation power is set to $P^U_t = S^P_t + R^U_t + L$ where $L$ is a small positive number. $L$ is only added to the price to allow the solver to detect the source of the regulation need[8]. The other constraints are quite straightforward. Constraint (7.72) ensures that all energy, when Elbas

[7] Energy that is left when all Elbas bids are satisfied.

[8] If $P^U_t = S^P_t + R^U_t$ the net effect of buying regulation power by increasing $U_{i,j}$ instead of $u_{i,j}$ is zero (this is for the objective function). Adding a number $L$ so small that the effect on the solution is not detectable by a human but detectable by a computer forces the solver to use

bids have been satisfied, is sold at the spot market. Constraint (7.73) makes sure that all Elbas bids are fulfilled. Then the constraints (7.74) and (7.75) put upper bounds on how much can be sold or bought at Elbas. The upper bound for Elbas can also be formulated so that it depends on the production at time $t-1$:

$$
\begin{aligned}
E_i^{Q,B} &\leq \overline{E_i^{Q,B}} \text{ for all } i &(7.77)\\
E_i^{Q,S} &\leq \overline{E_i^{Q,S}} \text{ for all } i &(7.78)
\end{aligned}
$$

But his model can not catch price changes at Elbas. In section (7.3.2) where the price at Elbas was analysed it was clear that the dynamics of the market had to be included in the model. It was both shown that the price was affected by the system balance and the volume traded.

If the balance of the system is related to the error in the point prediction, the price at Elbas can be scenario dependent. For the decision tree on Figure 7.2, nodes 2 to 4 could all have different prices. For instance could the price in node 4 be high cause less was produced than expected and the opposite for node 2. This behaviour can be formulated by adjusting the parameters $E_i^{P,S}$,$E_i^{P,B}$ in Eq. (7.79).

$$
\begin{aligned}
\max_{S_{t'}^Q, E_\cdot^{Q,B}, E_\cdot^{Q,S}} \sum_{i=1}^{N} \sum_{j=1}^{M_i} p_{i,j} \Big[ & S_t^P S_{i,j}^D + E_i^{P,S} E_i^{Q,S} - E_i^{P,B} E_i^{Q,B} \\
& - R_t^U u_{i,j} - R_t^D d_{i,j} - P_t^U U_{i,j} \Big] \qquad (7.79)
\end{aligned}
$$

The drawback of this formulation is that if the share of wind power is high in the system and all the producers are biding using the method presented here, the system balance will no longer depend on the point prediction error. In fact, if this was the situation, the output of the model would affect the input. Predicting how that turns out is extremely hard or even impossible. However it can be expected that the volume traded will always have an effect on the price no matter what method the producers use to plan there trade. Figure 7.3.left.center shows that currently there is a rather clear linear relation ship between the price and the volume traded. This effect can be included in the objective function, but then the formulation is no longer linear but quadratic

---

$U_{i,j}$ only in the case when Elbas sale can not be fulfilled. For instance is there no difference between the prices $100.0000 DKK/MWh$ and $100.0001 DKK/MWh$ when a rational human compares them but a computer always sees a difference if its numerical resolution is up to modern standards.

instead[9]. This formulation is given in Eq. (7.80).

$$\max_{S^Q_{t'},E^{Q,B}_{\cdot},E^{Q,S}_{\cdot}} \sum_{i=1}^{N}\sum_{j=1}^{M_i} p_{i,j}\Big[S^P_t S^D_{i,j} + (\alpha^S + \beta^S E^{Q,S}_i)E^{Q,S}_i$$
$$-(\alpha^B + \beta^B E^{Q,B}_i)E^{Q,B}_i - R^U_t u_{i,j} - R^D_t d_{i,j} - P^U_t U_{i,j}\Big] \quad (7.80)$$

### 7.3.3.5 Price scenarios

Lets assume that the following situation is at hand: There exists a price prediction for tomorrows prices. With a 30% change they will be as in column A, Table 7.5, else as in column B. This type of a situation can be formulated by assigning a price probability $p^p_k$ to each price as in the formulation given in Eq. (7.81).

$$\max_{S^Q_{t'},E^{Q,B}_{\cdot,\cdot},E^{Q,S}_{\cdot,\cdot}} \sum_{k=1}^{2} p^p_k \sum_{i=1}^{N}\sum_{j=1}^{M_i} p_{i,j}\Big[S^P_k S^D_{i,j,k} + E^{P,S}_k E^{Q,S}_{k,i} - E^{P,B}_k E^{Q,B}_{i,k}$$
$$-P^U_k U_{i,j,k} - R^U_k u_{i,j,k} - R^D_k d_{i,j,k}\Big] \quad (7.81)$$

subject to

$$\begin{aligned}
-d_{i,j,k} &\leq S^Q_{t'} - S^D_{i,j} \text{ for all } i,j,k && (7.82)\\
-d_{i,j,k} &\leq 0 \text{ for all } i,j,k && (7.83)\\
S^Q_{t'} - S^D_{i,j,k} &\leq u_{i,j,k} \text{ for all } i,j,k && (7.84)\\
0 &\leq u_{i,j,k} \text{ for all } i,j,k && (7.85)\\
S^D_{i,j,k} &\leq x_{i,j} + E^{Q,B}_{i,k} - E^{Q,S}_{i,k} + U_{i,j,k} \text{ for all } i,j,k && (7.86)\\
0 &\leq x_{i,j} + E^{Q,B}_{i,k} - E^{Q,S}_{i,k} + U_{i,j,k} \text{ for all } i,j,k && (7.87)\\
E^{Q,B}_{i,k} &\leq \overline{E^{Q,B}} \text{ for all } i,k && (7.88)\\
E^{Q,S}_{i,k} &\leq \overline{E^{Q,S}} \text{ for all } i,k && (7.89)
\end{aligned}$$

Note that the Elbas decision is now price scenario dependent because at time $t-1$ the spot price is known and, therefore, also what price scenario is being observed. This is why the expected price of the price scenarios can not be inserted directly into the model as in the previous chapter.

[9] The main difference, apart form the fact that the problem is more difficult to solve is that not all quadratic solvers can guaranty that the solution they find is optimal and those who can, can not do it for any formulation. It depends in fact on both the formulation and input data. The problem in question was solved without any troubles using the quadratic formulation and the solver `CPLEX`.

| | Price set A | Price set B |
|---|---|---|
| Probability | 30% | 70% |
| $S_k^P$ | 250 | 250 |
| $E_k^{P,S}$ | 650 | 220 |
| $E_k^{P,B}$ | 750 | 260 |
| $R_k^U$ | 800 | 270 |
| $R_k^D$ | 150 | 220 |

Table 7.5: *Price scenarios as this one can be inserted into the formulation in Eq. (7.81). Using that formulation, the spot market bid takes all price scenarios into account while the Elbas trade is price scenario dependent. Note that those are imaginary price scenarios.*

### 7.3.4 Performance

In this section the output from the model is examined. Key information is identified and a method to show the results in a graph presented. Such representation is important because the information in the output grows as the decision tree becomes more detailed. Finally the performance of the method will be tested using predefined prices. Only the quadratic, single price scenario, model (7.80) was used.

#### 7.3.4.1 The model output

When the optimisation problem has been solved a trade plan for every leaf-node[10] exists. For a tree, built as described earlier, decisions and the results for approximately 200 trajectories can be listed. Such an output list is shown in Table 7.6.

The purpose of the model is to help decision takes, but not to return results that should be bid without analysing them. It is therefore important that the output contains only the needed information, presented in such a way that an overview can easily be gained. Long detailed lists do not fit for that purpose, graphs should be used in stead.

Another important issue is how strong effect the prices have on the solution. For instance, when a producer is in doubt weather a given price prediction is accurate or not. The solution should therefore provide some kind of sensitivity analysis, showing how strong effect each input parameter has on the solution.

[10] Node at time $t$

**Graphical representation:** In Figure 7.8 results from an optimisation are shown. On the graph, the suggested bid at the spot market is drawn as a thick horizontal line, marked as line A. The coloured parts of the line represent how much, at time $t'$, is expected to be bought at Elbas and how high regulation need is expected. The expectation is calculated given that there will be need.

$$\mathbb{E}[d_{i,j}|d_{i,j} > 0] \quad : \quad \text{Length of red center line} \tag{7.90}$$
$$\mathbb{E}[E_i^{Q,S}|E_i^{Q,S} > 0] \quad : \quad \text{Length of blue center line} \tag{7.91}$$
$$\mathbb{E}[E_i^{Q,B}|E_i^{Q,B} > 0] \quad : \quad \text{Length of green center line} \tag{7.92}$$
$$\mathbb{E}[u_{i,j}|u_{i,j} > 0] \quad : \quad \text{Length of purple center line} \tag{7.93}$$

Unconditioned expectation gives biased view as it is does not show how much demand or supply is needed for the solution to be valid[11].

The line marked as Line B shows the point prediction.

All the horizontal lines which do not span the whole x-axis are decisions and results given that the production will be at a certain level at time $t-1$. For instance, if the production will be 180 $MWh$ at time $t-1$ then it is optimal to sell approximately 20 $MWh$ at Elbas (the red part) and the expected down regulation need is around 50 $MWh$ (the blue part). Line D, shows the optimal purchased amount at Elbas and what the expected up regulation need is given that the production will only by 20 $MWh$.

No information about the results for a specific production at time $t$ is given on the graph. That information is asumed to be of little importance because no decisions, that affect the income, can be taken at that time.

**Sensitivity:** Traditional OR[12] sensitivity analysis can not be applied when the objective function contains non linear terms. Sensitivity must therefore be checked by solving the problem a number of times, slightly changing the input from one run to another. That can not be done for large optimisation problems but for the problem in question, which can be solved with in one second, the calculation time does not become a problem unless the analysis are comprehensive.

[11] If the expected purchase at Elbas is calculated for all the scenarios, the scenarios where there is enough production will drag the expected value closer to zero. Lets assume that for a given solution $\mathbb{E}[E_i^{Q,B}|E_i^{Q,B} > 0] = 20\ MWh$ and $\mathbb{E}[E_i^{Q,B}] = 8\ MWh$. If it would then observed that energy is needed, 20 $MWh$ are expected have to be bought from Elbas, not 8 $MWh$. The market should therefore be able to supply 20 $MWh$ a reasonable price, not 8 $MWh$. This is way it is more important to examined the conditional expectation when evaluating weather a trade plan is reasonable or not.

[12] Operation research

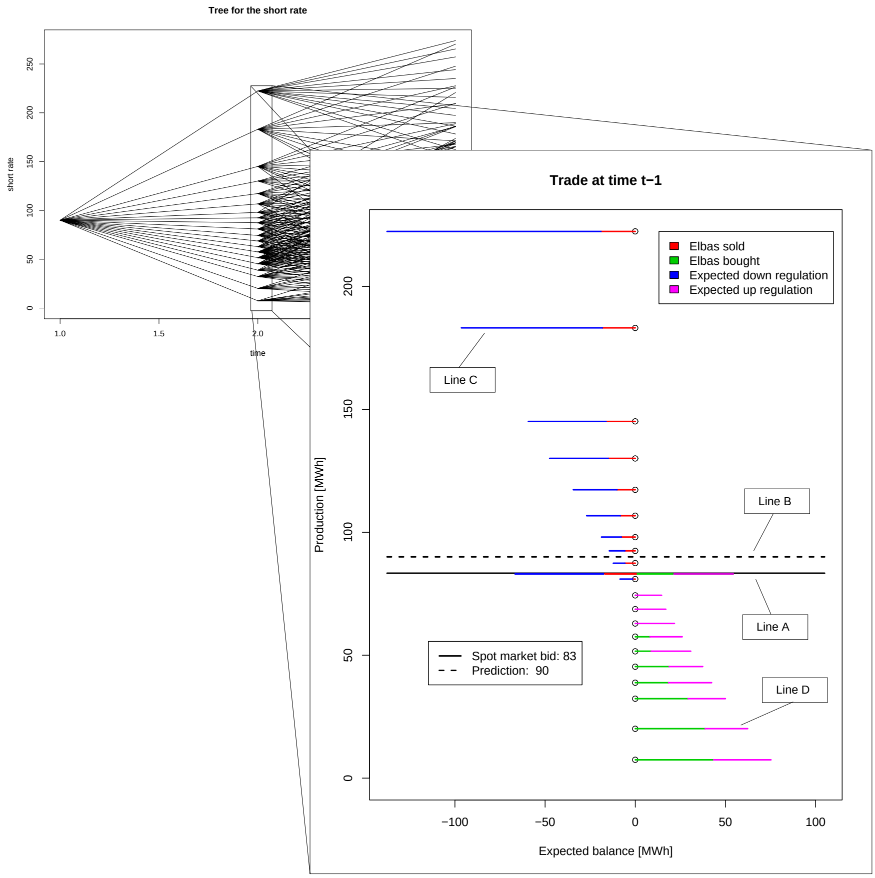


Figure 7.8: *From back to front: A,B. Plot B shows a sample output from an optimisation. Each point in that plot represents the decisions that should be made if that production is observed at time $t-1$ (see the connection to plot A).*

| $i$ | $j$ | $p_{i,j}$ | $x_{i,j}$ | $S^Q_{t'}$ | $S^D_{i,j}$ | $E^{Q,S}_i$ | $E^{Q,B}_i$ | $u_{i,j}$ | $d_{i,j}$ | $Obj$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 0.00477 | 3.76 | 83.36 | 47.23 | 0 | 43.47 | 36.14 | 0 | -448.46 |
| 1 | 2 | 0.00649 | 7.15 | 83.36 | 50.62 | 0 | 43.47 | 32.75 | 0 | 286.56 |
| 1 | 3 | 0.00270 | 12.15 | 83.36 | 55.62 | 0 | 43.47 | 27.74 | 0 | 1373.31 |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | |
| 1 | 8 | 0.00004 | 38.44 | 83.36 | 81.91 | 0 | 43.47 | 1.46 | 0 | 7076.59 |
| 2 | 1 | 0.00499 | 8.13 | 83.36 | 46.88 | 0 | 38.75 | 36.48 | 0 | 498.48 |
| 2 | 2 | 0.03395 | 15.91 | 83.36 | 54.66 | 0 | 38.75 | 28.70 | 0 | 2186.44 |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | |
| 20 | 10 | 0.00128 | 244.22 | 83.36 | 225.22 | 19.00 | 0 | 0 | 141.86 | 45807.58 |
| 20 | 11 | 0.00054 | 257.26 | 83.36 | 238.26 | 19.00 | 0 | 0 | 154.90 | 48155.32 |
| 20 | 12 | 0.00021 | 265.42 | 83.36 | 246.42 | 19.00 | 0 | 0 | 163.06 | 49623.76 |
| 20 | 13 | 0.00011 | 273.97 | 83.36 | 254.97 | 19.00 | 0 | 0 | 171.60 | 51162.22 |

Table 7.6: *The raw output from the optimisation problem lists the actions and results of every possible trajectory.*

The effect of each price was examined by solving the problem three times. Once with the price as it was predicted , then once when it was slightly increased, but other prices unchanged, and finally once when it was slightly reduced. The effect on the spot marked bid was observed. The results of such a test, for all the prices[13] except the spot price, are shown in Figure 7.9.

The selling price at Elbas was set extremely close to the spot price in order to clearly see its effect when estimating the sensitivity. The slope of the initial Elbas selling price was so steep, that changing its estimate from $S^P_{t^prime} - 1$ to $S^P_{t^prime} - 2$ lowerd the spot bid by 5 $MWh$. In this situation, the prediction for the Elbas asking price would have to be exact if the bid suggested by the method was supposed to be the optimal bid.

Using this type of sensitivity analysis traders can identify key prices and reconsider there predictions if needed.

#### 7.3.4.2 Performance test

In order to test the bidding method the following test was performed: For every hour in April 2005 energy was sold both using the point forecast, the optimal quantile and the new bidding strategy presented in this chapter. It will from now on be called "the new strategy". April was selected because there was extra transmission capacity between Sweden and East Denmark available more than 99% of the time, energy could therefore be bought form and sold to Sweden using Elbas. A longer period could not be examined because the total calculation time

[13] The configuration of the prices can be read from the graph.

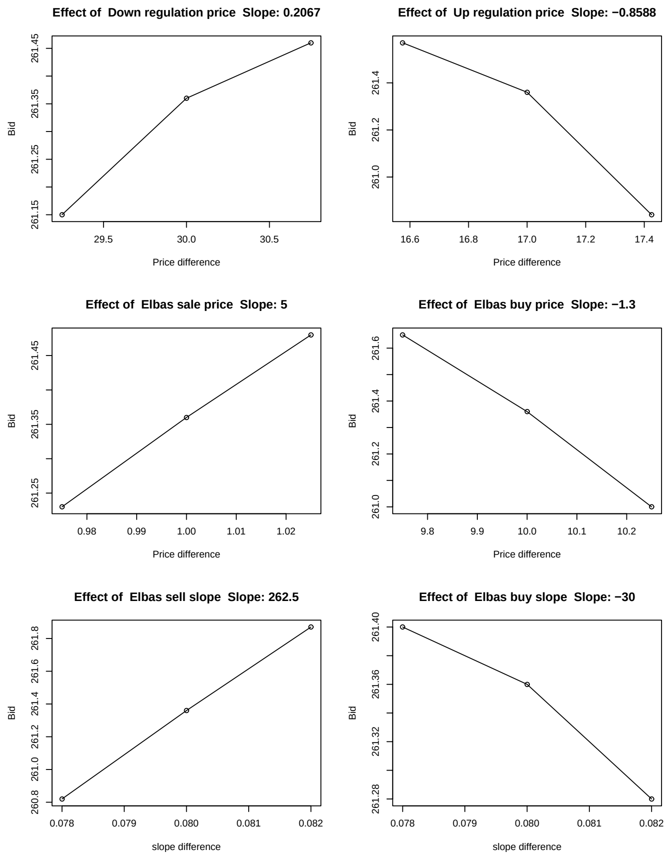


Figure 7.9: *The effect prices have on the spot bid. The figure shows that the Elbas selling price has a high effect on the solution if it is close to the spot price. The effect of each variable is written in the title.*

| Bidding method | Income |
|---|---|
| new strategy | 0.988 |
| optimal quantile | 0.985 |
| point forecast | 0.980 |

Table 7.7: *Price settings when testing the performance of the new bidding strategy.*

| Parameter | Value | Parameter | Value |
|---|---|---|---|
| $S_t^P$ | 210 | $\beta^S$ | 0.08 |
| $R_t^D$ | 17 | $\beta^B$ | 0.08 |
| $R_t^U$ | 8 | $\overline{E^{Q,B}}$ | 100 |
| $\alpha^S$ | $S_t^P$+11 | $\overline{E^{Q,B}}$ | 100 |
| $\alpha^B$ | $S_t^P$-3 | | |

Table 7.8: *Price settings when testing the performance of the new bidding strategy.*

for one hour was around 20 seconds. The calculation time makes it also difficult to test the methods using observed prices because they contain price spikes which have to high effect on the total income when a short period is examined. The new strategy was therefore only tested using predefined prices.

The prices were set as the mean value of the observed prices during the month of April. Up and down regulation cost was estimated using only values observed in hours there was active regulation and Elbas prices were estimated using only values during the hours when Elbas was active. The slope of the Elbas price was set to the value estimated in Section 7.3.2. All the prices are listed in Table 7.8.

No matter what bidding strategy was applied, energy was always traded at Elbas one hour before delivery in order to minimise the regulation cost. Put differently, given a initial bid, determined using the 3 different strategies, the trade at Elbas was decided by solving the optimisation problem defined in Eq. (7.31). Therefore could all the methods exploited the favourable buying price at Elbas. The results, listed in Table 7.7, show that the new strategy gave a income 0.8% higher than the point prediction strategy and a income 0.3% higher than the optimal quantile strategy.

Chapter 8

# Conclusion

## 8.1 Optimal quantiles

Applying the optimal quantile bidding strategy returns a higher income than the point bidding strategy, when selling wind energy at NordPool. The increase depends on the difference between the up and down regulation price and there relationship to the spot price. The regulation costs, as they are now in East Denmark, are so low that only large wind power producers can gain considerably by applying the method.

The method was not shown to increase the profit as much as it did when Bremnes tested it. The most likely explanation is that the prediction system used by E2, WPPT, performs better than the system used by Bremnes[1]

The bidding strategy, as it was originally posed by Bremenes, can be extended so that it includes the system balance. If the system balance is unrelated to the producers balance, as it might be in systems dominated by other energy sources, the results are simple and no optimisation is needed to find the bid. That does, however, not apply if the balances are correlated.

[1] Bremnes predicts production in a small area when compared to Zealand, but predictions for large areas normally perform better.

Applying the method using observed prices during thirteen month long period indicated that high precision price forecasts are needed in order to guaranty positive results. Using the mean or local regulation cost as a predictor should be avoided because regulation prices are volatile and price spikes are often observed. These spikes have a high impact on the total income if a wrong bid is placed. It was, for instance, seen that the theoretical optimal quantile could not be guarantied to be observed equal to the optimal quantile when bidding during a thirteen month long period using observed prices and the mean cost as prediction.

The quantiles are also needed to be quite correct for the method to work. Bidding using quantiles which were not accurate with in each month[2] gave actually worse results than bidding the normal way. In that case both poor price predictions and inaccurate quantiles were combined, giving bad results.

**Abridged:** The optimal quantile bidding strategy can be applied to predefined prices in order to estimate whether it is profitable to apply it or not. Preconditions to applying it successfully at a real market are correct quantile regression and quality price predictions. The results become uncertain if prices or quantiles are not of the needed precision. One of its good qualities is simplicity.

## 8.2 The new strategy

A new bidding strategy, allowing all submarkets of NordPool to be included in the planning, was developed. Applying the method when prices were predefined showed that it performed both better than the point prediction and the optimal quantile strategy. The method is formulated using a more complicated structure than the optimal quantile methods but the formulation allows more flexibility and can, therefore, be extended further.

It was observed that discritisation of the production forecast must be done carefully. The discritisation method developed, was in fact, a bit to simple and could, as a result of that, not catch all prediction scenarios.

The output from the new strategy includes more information that the output from the quantile regression method. It is therefore better suited to aid decision takes as they can plot suggested trade at different time points and evaluate whether the plan is in contrast to there believe or not.

[2] Put differently. Quantiles estimated for the whole period, were not correct when there correctness was tested taking one month at a time out of the training set.

The sensitivity to price forecasts could not be addressed due to extensive calculations but it can be expected to be analogous to the optimal quantile method. The fact that decisions are taken at two time points should, though, in certain market situations[3] make it possible to compensate for losses caused by a wrong initial decision.

**Abridged:** A new strategy was developed and it was observed to perform better then optimal quantile. The method can not be applied as easily but in return, it allows more detailed formulation than the optimal quantile method, and it its output is better suited to aid decision takers.

## 8.3 Further work

For a fully functional bidding aid system quality price predictions are needed. Such predictions are also needed in order to estimate how much can be gained by applying the bidding methods.

The construction of the decision tree should be looked at again. A better approximation is needed for the production change between time $t-1$ at $t$.

It would also be interesting to see if the simple quantile regression method can be applied when the probability of being charged for regulation is symmetric (same for up and down regulation). And if so, under what conditions.

[3] For instance must the trade at Elbas must be favourable because if it is more expensive to trade at Elbas then to use normal regulation, the new strategy returns exactly the same results as the optimal quantile strategy.

# Bibliography


[1] C. S. Nielsen, Hans F. Ravn, and Camilla Schaumburg-Müller. Two wind power prognosis criteria and regulation power costs. Workshop, Technical University of Denamrk, DTU, B.321, DK-2800 Lyngby, October 2003.

[2] John B. Bremnes. Probabilitic wind power forecasts using local quantile regression. *Wind Energy*, 7:47–54, 2004.

[3] Hannele Holttinen. *The impact of large scale wind power production on the Nordice electricity system*. PhD thesis, Helsinki University of Technology, Espoo, Finland, 2004.

[4] Haukur Eggertsson. The scandinavian electricity power market and market power. Master's thesis, Technical Universityof Denmark, DTU, Lyngby, 2003. Nr. 2003-46.

[5] U.S. Climate Change Technology Program. *Technology Options for the Near and Long Term*, November 2003. Page 35.

[6] Nord Pool ASA, Oslo - Stockholm - Helsiciki - Fredericia. *The Nordic Power Market: Electricity Power Exchange across National Borders*, 2004.

[7] Danish wind industry association. `http://www.windpower.org`, July 2005.

[8] L. Landberg, G. Giebel, H. Aa. Nielsen, T. S. Nielsen, and H. Madesn. Short-term prediction - an overview. *Wind Energy*, 3(6):273–280, June 2003.

[9] Gregor Giebel. The state-of-art in short-term preidction of wind power: A literature overview. Deliveralbe report d1.1, RisøNational Laboratory, P.O.Box 49, DK-4000 Roskilde, August 2003.

[10] Lennart Carlsson. *Internationl Power Trade - The Nordic Power Pool.* The Wold Bank Group, January 1999. Note no 171.

[11] Balmorel: A model for analyses of the electricity and chp markets in the baltic sea region. `http://www.balmorel.com`, March 2001. ISBN 87-986969-3-9.

[12] Kjell Rønningsbakk, Ole Gjerde, Kurt Lindström, Flemming Birck Pedersen, Christina Simón, and Torbjørn Sletten. *Håndtering av nettbegrensninger i elkraftsystemet.* Nordel, 2000.

[13] ElkraftSystem. Elkrafts systems markedsrapport for det østdanske elspotområde, March 2004.

[14] ElkraftSystem. Elkrafts systems markedsrapport for det østdanske elspotområde, May 2004.

[15] Nordel's homepage. `http://www.nordel.org/`, March 2005.

[16] Nordpool's homepage. `http://www.nordpool.no`, March 2005.

[17] Nord Pool SPOT AS. *Standard Terms for Tranding and Clearing in Nord Pool Spot AS' Physical Markets*, english version 2.1 edition, 2004.

[18] Nordel. *Nordic Model for Balance Pricing and Settlement*, 2003.

[19] P. Meibom, P. E. Morthors, L. H. Nielsen, C. Weber, K. Sander, D. Swider, and H. Ravn. *Power System Models, A Description of Power Markets and Outline of Market Modelling in Wilmar.* Risø, Roskilde, 2003.

[20] Elkraft's homepage. `http://www.elkraft-system.dk`, March 2005.

[21] Nord Pool ASA. *The Nordic Power Market, Electricity Power Exchange acroess National Borders*, APR 2004.

[22] Eltra's web page, section: "drift", title: "vindproduction under orkanen". `http://www.eltra.dk/`, January 2005.

[23] Eltra. *Balanceafregning og balancemarket: Daglige rutiner*, 1999. Document number: 44073.

[24] Eltra. *Balanceansvarlige daglige plan- og komminikationsrutiner*, 2000. Document number: 84073.

[25] Geoffrey Grimmet and David. Stirzaker. *Probability and Random Processes.* Oxford University Press, Greate Britian, 2001.

[26] Henrik Aalborg Nielsen, Henrik Madsen, and Torben Skov Nielsen. Using quantile regression to extend an existing wind power forecasting system with probabilistic forecasts. In *European Wind Energy Conference and Exhibition 2004, Scientific Proceedings*, pages 34–38. EWEA, 2004.

[27] Roger Koenker and Gilbert Bassett. Regression quantiles. *Econometrical*, 46(1):33–50, January 1978.

[28] John R. Birge and Francois Louveaux. *Introduction to Stochastic Programming*. Springer, 1997.

[29] ElkraftSystem. Elkrafts systems markedsrapport for det østdanske elspotområde, January 2004.

[30] ElkraftSystem. Elkrafts systems markedsrapport for det østdanske elspotområde, February 2004.

[31] ElkraftSystem. Elkrafts systems markedsrapport for det østdanske elspotområde, March 2004.

[32] ElkraftSystem. Elkrafts systems markedsrapport for det østdanske elspotområde, June 2004.

[33] ElkraftSystem. Elkrafts systems markedsrapport for det østdanske elspotområde, October 2004.

[34] ElkraftSystem. Elkrafts systems markedsrapport for det østdanske elspotområde, November 2004.

[35] ElkraftSystem. Elkrafts systems markedsrapport for det østdanske elspotområde, December 2004.

[36] P. E. Morthorst. Wind power and the conditions at a liveralized power market. *Wind Energy*, 6:297–308, 2003.

[37] Henrik Madsen. *Time Series Analysis*. DTU, Kgs. Lyngby, Denmark, 2001.

[38] Trevor Hastie, Robert Tibshirani, and J. Friedman. *The elements of statistical learning: data mining, inference, and predictiction*. Springer, 2001.

[39] David Kincaid and Ward Cheney. *Numerical Analysis: Mathematics of Scientific Computing*. SBrooks/Cole, 3 edition, 2002.

[40] Venables and Ripley. *Modern Applied Statistics with S-Plus*. Springer, New York, 1994.

[41] Henrik Madsen and Jan Holst. *Modelling Non-Linear and Non-Stationary Time Series*. DTU, Kgs. Lyngby, Denmark, 2000.

[42] William H. Press. *Numerical Recipes in C: The Art of Scientific Computing*. Cambridge University Press, Cambridge, 1992.