MODELLING AND MONITORING IN INJECTION MOLDING

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IMM

Preface

This thesis has been prepared at the Department of Mathecatical Modelling, Technical University of Denmark, and Novo Nordisk A/S in partial fulfilment of the requirements for the industrial Ph.D. degree.

The thesis is concerned with methods for quality improvement of parts produced by injection molding. The methods are illustrated with examples from the manufacturing of molded parts for a medical device at Novo Nordisk A/S.

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Last, but not least, I am grateful to my fiancée, my family and my friends for their support and patience.

Summary

This thesis is concerned with the application of statistical methods in quality improvement of injection molded parts. The methods described are illustrated with data from the manufacturing of parts for a medical device. The emphasis has been on the variation between cavities in multi-cavity molds.

From analysis of quality measurements from a longer period of manufacturing, it was found that differences in cavities was that source of variation with greatest influence on the length of the molded parts. The other large contribution to the length variation was the different machine settings. Samples taken within the same machine set-point did not cause great variation compared to the two preceding sources of variation.

A simple graphical approach is suggested for finding patterns in the cavity differences. Applying this method to data from a 16 cavity mold, a clear connection was found between a parts length and the producing cavitys position in the mold. In a designed experiment it was possible to isolate the machine parameters contributing to the variation between cavities. Thus, with a proper choice of levels for the machine variables, it was possible to reduce the variation between cavities substantially. Also an alternative model for the shrinkage of parts from a multi-cavity mold is suggested. From applying the model to data from a shrinkage study, it seemed that the observed part differences were not only due to differences in cavity dimensions.

A model for the in-control variation for a multi-cavity molding process was suggested. Based on this model, control charting procedures have been sug-

gested for monitoring the quality of the molded parts. Moreover, a capability index for multi-cavity molds has been suggested.

Furthermore an alternative method for in-line quality charting is suggested. The method is for continuous control by attributes, and it is an alternative to the batch oriented approach mostly used. The procedure is especially efficient for quality requirements of very low proportion non-conformities. For the proposed charts the ARL function is derived. It is shown that in the case where a non-conforning unit is only expected very rarely during sampling, a moving sum chart and a CUSUM chart are equivalent.

Finally, the correlation structure of 21 process variables has been studied prior to monitoring the process. It is illustrated how the process can be analysed with multivariate techniques. It was found that two principal components reflected changes in machine set-points. Thus, there seems to be great potential in monitoring the process variables using a multivariate approach.

Resumé (in Danish)

Denne afhandling omhandler brugen af statistiske metoder ved kvalitetsforbedring af sprøjtestøbte produkter. Alle metoder er illustreret med data fra produktionen af dele til et medicinsk device. Fokus har især været på variationsbidrag hidrørende fra kavitetsforskelle i multikavitetsforme.

Ved at analysere kvalitetsdata fra en længere sammenhængende produktionsperiode blev det fundet, at kavitetsforskelle var den variationskilde med den største indflydelse på længden af de støbte emner. Det andet store bidrag til variationen var maskinens forskellige indstillinger i perioden. Skudprøver taget ved samme maskinindstilling betød derimod ikke så meget sammenlignet med de to foregående.

Der er foreslået en simpel grafisk metode til at finde en struktur i kavitetsforskellene. Ved anvendelse af denne metode på data fra en form med 16 kaviteter, blev det fundet, at der var en tydelig sammenhæng mellem, hvor langt et emne var, og hvor i formen det var blevet produceret. Gennem forsøgsplanlægning var det muligt at isolere de maskininparametre, der havde indflydelse på variansen indenfor en skudprøve. Ved et passende valg af maskinindstillinger, vil det således være muligt at reducere variationen mellem kaviteterne betragteligt. Der er desuden foreslået en alternativ model til at beskrive svindforløbet af de støbte emner. Ved anvendelse af denne model på data fra et svindstudie er der ligeledes fundet forskelle på de forskellige kaviteters svindforløb, hvilket tyder på at kavitetsforskellene ikke udelukkende kan tilskrives variationer i kaviteternes fysiske dimensioner.

Der er opsillet en model til at beskrive målinger af emner fra en multikavitetsform, og på baggrund af denne model, er der foreslået kontrolkort-metoder til overvågning af kvaliteten af de støbte emner. Derudover er der foreslået mål for kapabiliteten (capability index) for multikavitetsforme.

Desuden er der foreslået en metode til godkendelseskontrol i processen for alternativ variation med kvaliteter i ppm området. Metoden er baseret på en glidende sum, og er et alternativ til den batchbaserede godekendelseskontrol, der sædvanligvis benyttes. Metoden er særlig effektiv for krav til kvaliteter med meget lav andel afvigende emner. Det bliver vist, at når et afvigende emne kun forventes at optræde uhyre sjældent, da er et moving sum kontrolkort og et CUSUM kontrolkort ækvivalente.

Sluttelig er korrelationsstrukturen mellem 21 procesvariable studeret med henblik på at overvåge processen. Det er illustreret, hvorledes processen bør analyseres ved multivariate statistiske metoder, inden der fastsættes en metode til overvågning. Det blev fundet, at to principale komponeter afspejlede skift i maskinindstillinger. Der er derfor tilsyneladende stort potentiale i at overvåge procesvariablene. Der er foreslået alternative metoder til overvågning af processen, der alle bygger på de principale komponenter for procesvariablene.

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I

Part I

Modelling and monitoring of the injection molding process.

Chapter 1

Introduction

1.1 Background

There has been a tremendous increase in the demand for injection molded products over the past several years. Early, the molding process concentrated on producing high volume products with low to moderate quality requirements. In recent years, the market has expanded to include precision molded items requiring high quality, such as medical devices or connectors for the auto industry.

The introduction of both the microprocessor and the computer controlled injection molding machine, together with the relevant instrumentation, has enabled the molding process to be suitable optimized, so as to produce plastics components comparable to machined metal parts. Proper control of the process not only ensures accurate product manufacture, but also improves the productivity of the molding process by producing components with far fewer defects occurring.

Designs for plastic parts are becoming increasingly more complex. In addition, economic constraints have led to the use of multi-cavity molds. Therefore the number of critical dimensions is increasing, not only as a result of part geometries, but it is also increasing by a factor equal to the number of cavities in the mold. Operating a process becomes more difficult when there are more critical dimensions to meet.

In general, it is important to understand that in every manufacturing process there is always some natural variation in the product - even when no external changes have been made to the process. For the injection molding process this leads to, for example, variations in dimensions or weight, and hence no two moldings are identical. Therefore, irrespective of the care taken to control the molding process, product variation is naturally inherent and inevitable.

In the present thesis the emphasis will be on studying and understanding the variation in the molding process. Especially the variation between cavities in multi-cavity molds will be studied in various ways.

A lot of research has been done in the area of modeling polymer processing processes in closed mathematical/physical models. Mold filling simulation is the most common type of process modeling. Commercial Computer Aided Engineering (CAE) software has been available since 1978. Over the years, the scope of such software has expanded beyond filling analysis to include cooling analysis, part gate location, runner sizing, weldline prediction, gas-trap prediction, warpage and residual stress analysis. Naitove and De Gaspari (1992) presented a comprehensive survey of the usage of CAE software in the molding industry. The results of the survey indicated that the majority of simulation is done during mold design and construction. However, experience has shown, that the development of process monitoring models in this kind of manufacturing environment, necessitates the use of empirically based techniques.

Quality control for injection molding is basically a two-stage process. The first stage consists of process tuning in which acceptable molded part quality is achieved. The second stage consists of some form of continuous quality monitoring and control during production. For each stage, several fundamentally different approaches have been proposed for locating and/or ensuring molded part quality.

The traditional approach to machine input selection (tuning) in the plastics industry has been based on trial and error. For this purpose samples are usually taken during start-up, and part quality attributes are measured after each sample to evaluate the quality of produced parts. A human expert then uses knowledge of the process to select the machine settings in such a way as to improve the quality of the part from cycle to cycle. This tuning exercise is repeated until the specifications for part quality are satisfied.

A more methodological approach to tuning is design of experiments (DOE), where an empirical model is formed based on data obtained from a set of designed experiments. Based on this model, the objective function of an unconstrained optimization problem is defined in terms of the part quality attributes, and the set of inputs that produce the best quality attributes are obtained as the "optimal" point of this optimization problem. Deliz and Caraballo (1995) used a fractional factorial experiment to find the processing conditions that would minimize the out-of-roundness while centering diameter and length at their nominal values. Also Xia and Mallick (1997) and Blyskal (1994) applies DOE techniques to determine optimal settings with respect to one or more dimensional measures. Beard (1999) suggests the use of DOE for process validation. His approach is to define a process window such that molding anywhere inside that process window will lead to acceptable parts.

Wortberg et al. (1997) argues that the design of experiments technique is not appropriate for building a forecast model to predict the quality of manufactured product from the actual state of the process. They criticize the DOE approach for being too expensive when a large number of factors (more than 200) should be studied, and for resulting in a break of the production. Instead they advocate the use of artificial neural networks in which the quality characteristics of the molded product are typically forecasted from the measured process data during production. A lot of ongoing research is taking place in the application of neural networks to quality prediction. See also Rewal et al. (1998) and Petrova and Kazmer (1999).

As with most manufacturing processes, statistical process control (SPC) is mostly used to chart the output of the injection molding process. Werner and Berenter (1985) suggested charting part weight to signal process problems. Operators were instructed not to adjust the process unless the control chart indicated an out of control condition. As a result of the new operating procedures, the injection molding machine was reported to run for 14 shifts without producing any defects.

One of the main problems with precision molding is the time taken for parts to acclimatize and reach their final dimensions. The need for on-line checking is vital to prevent hours of work being rejected. Hunkar (1983) applied SPC techniques to the process variables rather than the process output. Approximately 30 process variables were monitored in order to diagnosis of failure of machine systems. Wang and Wang (1991) attempted a more ambitious use of process variables for quality control. They implemented an empirically based predictive model to be used to control part thickness. The model was used to make shot to shot adjustments in the hydraulic packing pressure.

1.2 Outline of the Thesis

This thesis is organized into two parts. The first part is four chapters and contains a description of the injection molding process as well as a discussion of the results presented in the papers. The five papers are included as part II.

In Chapter 2 a general introduction to the injection molding process is presented. In the description of the process special emphasis will be given to factors that influence the repeatability of the molding process.

In Chapter 3 the results of the cases presented in the papers are summarized and discussed. Also some related general aspects not treated in the papers are discussed in this chapter. The papers should be read in connection with Chapter 3.

Finally, the conclusions are presented in Chapter 4.

Chapter 2

Introduction to injection molding

Injection molding is the most widely used polymeric fabrication process. It evolved from metal die casting, however, unlike molten metals, polymer melts have a high viscosity and can not simply be poured into a mold. Instead a large force must be used to inject the polymer into the hollow mold cavity. More melt must also be packed into the mold during solidification to avoid shrinkage in the mold. The injection molding process is primarily a sequential operation that results in the transformation of plastic pellets into a molded part. Identical parts are produced through a cyclic process involving the melting of a pellet or powder resin followed by the injection of the polymer melt into the hollow mold cavity under high pressure. The process has been described by several authors, for example Whelan and Goff (1996) and Rauwendaal (2000).

Injection molding can be used to form a wide variety of products. Complexity is virtually unlimited and sizes may range from very small to very large. Most polymers may be injection molded, including thermo plastics, fiber reinforces thermo plastics, thermosetting plastics, and elastomers.

Critical to the adoption of this high volume, low cost process technology is the ability to consistently produce quality parts. In the following description of the injection molding process, factors which influence the repeatability of the

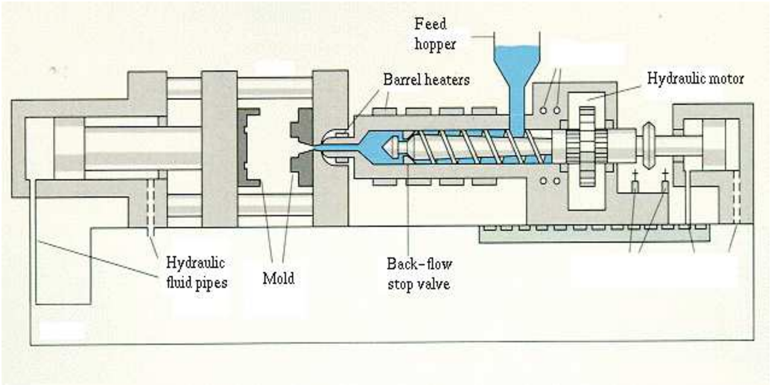

Figure 2.1: Injection molding machine.

molding process will be highlighted.

2.1 The injection molding machine

An injection molding machine is a machine which produces components by injection molding. It is most commonly a hydraulically powered, in-line screw machine although electric machines are appearing and will be more important in the market in the future.

The main units of a typical injection molding machine are the clamping unit, the plasticating unit, and the drive unit; they are shown in Figure 2.1. The clamping unit holds the injection mold. It is capable of closing, clamping, and opening the mold. Its main components are the fixed and moving plates, the tie bars, and the mechanism for opening, closing and clamping.

The injection unit or plasticating unit melts the plastic and injects it into the mold. The drive unit provides power for the plasticating unit and clamping unit.

Injection molding machines are often classified by the maximum clamp force that the machine can generate. This is the force that pushes the two mold halves together to avoid opening of the mold due to internal pressure of the plastic melt in the mold. The clamping force of typical injection molding machines

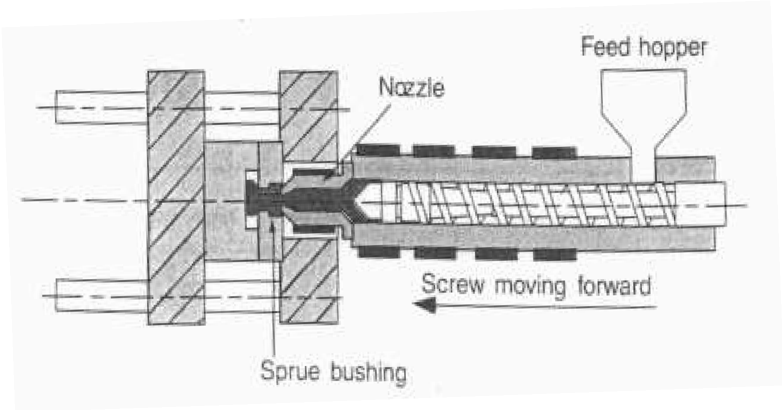

Figure 2.2: Stage 1 of the injection molding cycle: injection of the plastic melt into the mold. (From Rauwendaal (2000).)

range from 200 to 100 000 kN.

2.1.1 The injection molding cycle

There are three main stages in the injection molding cycle; stage 1, injection, followed by stage 2, holding pressure and plasticating, and finally, stage 3, ejection of the injection molded part. When stage 3 is completed, the mold closes again and the cycle starts over again.

Stage 1, Injection of the plastic melt into the mold. In stage 1, the mold is closed and the nozzle of the extruder is pushed against the sprue bushing of the mold. The screw, not rotating at this point, is pushed forward so that the plastic melt in front of the screw is forced into the mold. See Figure 2.2.

Stage 2, Holding pressure and plasticating. When the mold is completely filled, the screw remains stationary for some time to keep the plastic in the mold under pressure; this is called the “hold” time. During the hold time additional melt is injected into the mold to compensate for contraction due to cooling. Later, the gate, which is the narrow entrance into the mold, freezes. At this point the mold is isolated from the injection unit. However, the melt within the mold is still at high pressure. As the melt cools and solidifies, the pressure should be high enough to avoid sink-marks, but low enough to allow easy

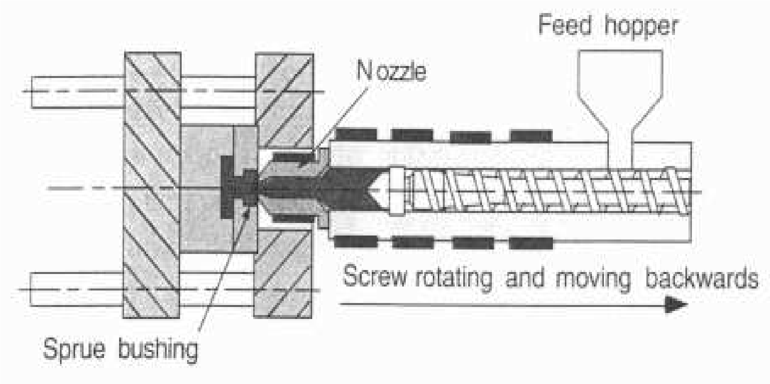

Figure 2.3: Stage 2 of the injection molding cycle: holding and screw recovery. (From Rauwendaal (2000).)

removal of the parts.

During the plastication stage, the material is pushed forward from the feed hopper through the barrel and toward the nozzle by a rotating screw. When the gate freezes, the screw rotation is started. The period of screw rotation is called screw "recovery". The rotation of the screw causes the plastic to be conveyed forward. As the plastic moves forward, heat from the electric heater bands along the barrel and shear starts to melt the plastic. At the discharge end of the screw, the plastic will be completely melted. The melt that accumulates at the end of the screw pushes the screw backward. Thus the screw is rotating and moving backward at the same time. The rate at which plastic melt accumulates in front of the screw can be controlled by the screw backpressure, that is, the hydraulic pressure exerted on the screw. This also controls the melt pressure in front of the screw.

When sufficient melt has accumulated in front of the screw, the rotation of the screw stops. During screw recovery the plastic in the mold is cooling, but typically the cooling is not finished by the end of screw recovery. As a result, the screw will remain stationary for some period until cooling is completed. This period is often referred to as "soak" time. During this time additional plastic will melt in the extruder from conductive heating. Also, the melted material will reach more thermal uniformity, although the soak time is usually too short to improve thermal homogeneity significantly.

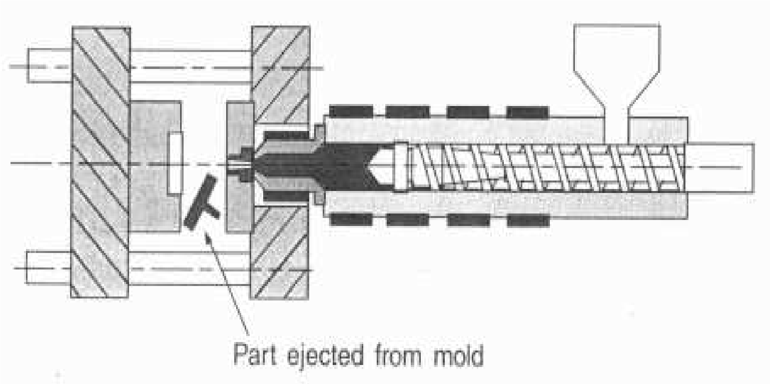

Figure 2.4: Stage 3 of the injection molding cycle: ejection of the part(s). (From Rauwendaal (2000).)

Stage 3, Ejection. When the material in the mold has cooled sufficiently to hold its shape, the mold opens and the parts are ejected from the mold. See Figure 2.4. When the molded part has been ejected, the mold closes and the cycle starts over again.

The different stages can be graphically illustrated as shown in Figure 2.5. The top bar shows the movement of the extruder screw, the second bar shows the action going on inside the mold and the third bar indicates at what times the mold is open and closed. As can be seen in Figure 2.5 the major part of the injection molding cycle is the cooling time required for the plastic in the mold to reduce to a temperature where the part can be removed without significant distortion. The main variable that determines the cooling time is the thickness of the molded part.

2.2 Plastic properties

Plastics have several properties that influence the repeatability of the molding process. First, plastics are compressible. The pressure in the mold cavity determines how much the melt is compressed. If all other variables are held constant, a higher hydraulic pressure results in a higher cavity pressure and will force more plastic into the mold cavities.

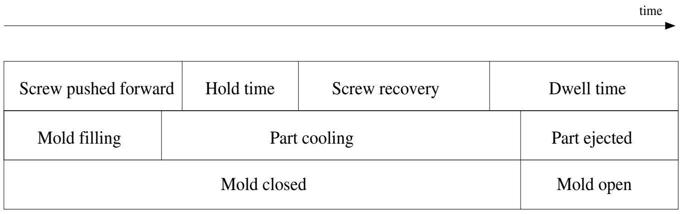

Figure 2.5: The different stages in the injection molding cycle. The top bar shows the movement of the extruder screw, the second bar shows the action going on inside the mold and the third bar indicates at what times the mold is open and closed.

Second, plastics shrink significantly when cooled. Together these properties indicate the need for the packing stage during the molding cycle. After the mold cavity is filled, continued pressure on the piston connected to the screw forces more melt into the cavity to compensate for part shrinkage due to initial cooling. Shrinkage is also influenced by the cooling rate. A faster cooling rate, i.e. colder mold temperature, results in less shrinkage. When a part is cooled very quickly, the dimensions are "frozen-in" and, therefore, the part will shrink less. A slower cooling rate gives more time for the molecules to align and, consequently, the part will exhibit greater shrinkage. Finally, shrinkage is affected by polymer orientation, alignment of the molecule and molecular segments in the direction of flow. Shrinkage is a result of two factors, a normal decrease in volume due to temperature change and relaxation of the stretching caused by carbon-carbon linkages. As there are more carbon-carbon linkages in the direction of the orientated flow, there will be greater shrinkage in this direction. Any parameter that affects the mobility of the molecular segments will affect orientation and consequently part shrinkage.

The third important property of plastic is that its viscosity is dependent on temperature and flow rate of the melt. Increases in either flow rate or temperature reduce viscosity. Higher temperatures are an indication of greater molecular motion and consequently lower viscosity. Constant viscosity is required to produce parts of consistent quality. Viscosity can affect how much the polymer is compressed in the cavity and therefore how much shrinkage will take place. Lower viscosity results in smaller pressure drops along the flow path (runner and gate) and consequently higher cavity pressure. Higher cavity pressure results in greater compressibility and consequently less shrinkage.

2.3 The mold

Each mold, sometimes referred to as a tool, is built to exact specifications of the part or parts required by the customer. The mold typically consists of two mold halves. Usually one mold half contains the cavity and forms the outer shape of the part. This part of the mold is called the cavity side. The other mold half contains a protruding shape and forms the inner shape of the part; this mold part is called the core. When the core is clamped against the cavity, the hollow space that is formed defines the shape of the part to be molded. The plastic is usually injected into the mold from the cavity side.

The mold cavities are cut to dimensions larger than the desired part dimensions to compensate for the plastic shrinkage which occurs during cooling. The cavity dimensions are equal to the part dimensions plus some shrink factor supplied by the material manufacturer. There are usually two shrink factors given, one for dimensions in the direction of the flow and one for dimensions perpendicular to the direction of the flow. Estimating shrinkage, however, is not straight forward. It is often difficult to predict the melt flow path in parts with complex geometries and therefore, not clear which shrink factor to apply. Also, as discussed earlier in the chapter, part shrinkage is influenced by the process conditions.

A sketch of a 16 cavity mold is shown in Figure 2.6. The 16 cavities are denoted A through P.

2.3.1 The runner system

A mold basically consists of properly designed sprue, runner, gate, and cavity. The sprue is the channel, cut in the stationary platen, that transports the melt from the plasticator nozzle to the runner. Once the plastic melt enters the mold, it flows through a distribution system, called the runner system, and then through the gates into the part cavities. In a so-called cold runner system, a new runner is molded in each molding cycle and the runner is ejected together with the molded parts. The plastic of the runner can often be reprocessed and molded again.

In the design of the runner system the objective is to have the plastic reach all

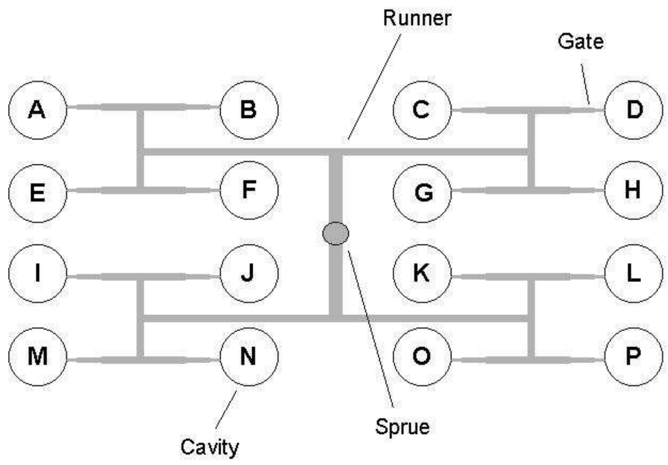

Figure 2.6: Sketch of a 16 cavity mold. Cavities are numbered A-P and connected by the runner system.

gates at the same time. This is an important issue in multi-cavity molds. In a rectangular runner system, the number of cavities is multiple of two. In a circular runner any number of cavities can be used.

The gate connects the runner to the actual part. The cross section of the gate is usually small so that the runner can be easily removed from the part and does not leave a large gate mark on the part.

2.3.2 Mold cooling

During the machine cycle, heat is first required to be put into the material and then that heat must be removed as quickly, and as consistently as possible, if the rapid production of consistent products is to be obtained. As most modern injection molding machines are screw machines, heat input is relatively easy. Heat removal from the plastics material contained in the mold is, however, difficult as plastics material contains a lot of heat and has a low thermal conductivity.

Cooling allows the plastic to solidify and become dimensionally stable before ejection. Heat that has been transferred to the mold by the molten plastic is carried away by a coolant that circulates through cored passages in the mold. Coolant temperature and flow rate determine the efficiency of heat removal.

Cooling the molded components uniformly may mean either, cooling the mold with different flow rates of cooling medium in different areas or, using the same flow rate throughout the mold but with different temperatures of cooling medium. The objective is to cool the components as quickly and uniformly as possible, while ensuring that defects such as poor surface finish and changes in physical properties are not encountered.

The design of the mold cooling passages also affects the ability to remove heat from the mold. The mold surfaces closest to the cored passages will cool first. Differences in mold temperature or mold temperature distribution will affect reproducibility of part moldings.

2.3.3 Venting

As the mold fills the air in the mold will be displaced by the advancing melt front. It is important that the mold is designed in such a way that the air displaced in the mold filling process has a chance to escape from the mold. If air does not have a chance to escape, it is compressed quite rapidly. As the air compresses, it heats up and the temperature rise can be high enough to cause burning of the plastic. Thus improper venting can not only cause incomplete filling of the molded part, but can also cause burn marks.

2.4 Process variables

Each process variable can be categorized into one of five main types such as speed, pressure, time, temperature and stroke. The relationship between the five is of an interactive nature as each variable cannot be readily isolated. This relationship can be simply demonstrated, for example, upon increasing the hydraulic back pressure, the linear retraction speed of the screw (during recovery) changes causing an increase in the screw recovery time, the melt temperature and/or homogeneity. As a result of the increase in the melt temperature further changes occur to the mold fill time, the injection pressure, the mold temperature, the product ejection temperature and the product dimensions. Hence, by increasing a pressure variable (for example, the hydraulic back pressure) three other main variable types are collectively influenced. More important, the process and subsequently the molded components are affected.

When changes to a particular process variable or machine setting do occur (which significantly affect the stability of the molding process so that reject components are produced) it is important that the correct process variable is changed so as to rectify the disturbance. For instance, the selection of the wrong hopper throat temperature can cause short moldings to be produced which then misleads the molder into altering other variables (for example the holding pressure and/or, the shot volume and/or, the mold filling speed etc.) to overcome the short molding problem. As the initial selection was incorrect, the process remains unstable but, in changing another variable type, the molder is led to believe the problem is resolved. However, in reality rejectable and/or inconsistent parts will continue to be produced throughout the production run.

The following table highlights typical process variables which need to be monitored and/or controlled each cycle. Each of the listed variables will be discussed in more detail to highlight the importance of each variable with respect to the stability of the process.

Temperat.	**Times**	**Speeds**	**Pressures**	**Strokes**
Melt	Ejection	Injection	Holding	Melt cushion
Mold surface	Mold close	Screw	Injection	Screw stroke
Barrel	Mold open	Mold close	Hyd. back	Mold open
Component	Cooling	Mold open	Ejection	Change over
Material	Cycle	Screw return	Mold safety	position
Environment	Screw recov.	Ejection	Nozzle	Decompress.
Oil	Hold press.			

Table 2.1: Process variables.

To assist in identifying which variable is causing the major interference to process stability, the variables can be divided in two types, controllable and consequential.

2.4.1 Controllable variable

This type of variable can be set to a particular value and suitably maintained at this value to within a defined tolerance band, for example, the holding pressure should be set to 60 ± 1 bar. This should be done by the control system or other mechanism on the molding machine.

A typical controllable process variable would be either the holding pressure or the holding pressure time. The holding pressure is set using the pressure control circuit of the machine's hydraulic system and the holding pressure duration by a timer or a timing device. Both of these variables can be accurately set to their respective values and are suitably controllable. The accuracy and variability of the setting value is dependent upon the effectiveness of the hydraulic and electronic control systems employed on the molding machine.

2.4.2 Consequential variable

This type of variable is one which cannot be set, as it is the ultimate result of a combination of process variables used to perform a specific function or operation, in order to produce the components to the required quality standard. Such a type of variable is much more difficult to control as it's variability is totally dependent upon the consistency and performance of other, usually controllable variables.

A typical consequential variable is the melt cushion value. The melt cushion is a variable which is the consequence of how much material is used to fill the mold and then how pressure is applied to the molten material in order to produce the component to the required dimensions. The variables associated in carrying out the above operation include the amount of injection pressure used, the stroke of the screw, the mold filling speed, the melt temperature of the material, the applied clamping force, the level of holding pressure and the holding pressure time used.

Therefore, to achieve a consistent melt cushion value to within specified limits (for example, ± 0.1 mm) it is essential that all the related variables are closely controlled so that the cumulative effect is sufficiently small to ensure that the melt cushion value does not significantly alter each cycle. As the melt cushion is the consequence of many other variables, significant importance is given to it's variation. The extent of the variability encountered is a good indicator of the level of inherent stability present within the molding process. Hence, the smaller the variation the more consistent and stable the process.

2.4.3 Speed related process variables

Mold opening and closing speeds
As it is possible to select different opening and closing speeds, and speed changes can be introduced during the opening and closing operations of the mold, these variables are regarded as controllable.

The more common system used for mold movement is the combination of the hydraulically operated closed loop volume (flow) proportional valve, or pump, and the stroke transducer.

Injection speed
The injection speed is linear speed used to fill the mold with molten material. When filling the mold, the injection speed is controlled to suit the characteristics of the product, the material and/or, the mold. The rate at which the molten material flows into the mold is dependent upon there being sufficient injection pressure available to maintain a consistent selected filling velocity. Inconsistency of the mold filling speed prevails if inadequate injection pressure is selected.

As the filling of the mold is regarded as one of the most important elements of the molding cycle, considerable investigations and technological advancements have been made by injection molding machine manufacturers to try and ensure that the selected (velocity) value corresponds to the actual value. The mold filling speed is regarded as controllable.

Screw rotational speed
Some of the heat necessary to plasticize the plastics material is created as a result of rotating the screw. The faster it is rotated the higher the temperature. It is important to ensure that the correct speed is being used otherwise process instability will occur. This means that the screw rotational speed must be capable of being accurately measured.

Screw recovery speed
Screw recovery is the return of the screw after plasticization has been performed. Screw return is obtained by rotating the screw at a predetermined speed and against a predetermined back pressure. This feature is regarded as being controllable as most modern molding machines possess a facility to adjust this speed setting.

Component retraction speed
The speed and manner in which products are removed from the mold should be regarded as controllable although the design of the mold, the configuration of the product and the processing conditions selected, dictate the speed at which the product can be extracted.

2.4.4 Pressure related process variables

Injection pressure
A correct injection pressure value is important so as to maintain a consistent mold filling velocity. The pressure value is a consequence of, for example, the melt temperature, the viscosity characteristics of the material being processed, the mold filling speed used, the surface temperature of the mold, and the distance the material has to flow in the mold cavity.

Holding pressure
Holding pressure (also called follow-up pressure) needs to be applied after the mold has been initially filled with melt, so as to compact and shape the material and thus allow the production of components which have the required dimensions and aestethic quality standards. The amount of holding pressure used is related to the dimensional and/or the visual requirements of the molded component but the amount selected should be controlled carefully for consistent product quality. Control of this variable is dependent upon the molding machine's pressure generation system. However, machine technology nowadays enables the selected pressure values to be held to within ± 1 bar. For this reason the holding pressure is regarded as controllable.

Hydraulic back pressure
When the screw is rotated, heat-softened (plasticized) material is pushed forward through the back flow valve assembly to the front of the screw. The pressure generated within, and by the molten material, forces the screw (and

back flow valve assembly) to move backwards, thus refilling the vacated volume with molten material. Hydraulic back pressure has an influential effect on the melt temperature and homogeneity. The value selected corresponds to the type of material being processed, the shot capacity of the barrel being utilized, the plasticizing capability of the screw, the rotational speed of the screw, and the quality standards of the components to be molded. The hydraulic back pressure is set either manually or electronically and the selected value should be maintained within some tolerances as the melt homogeneity is significantly influenced by the amount of back pressure selected and its consistency.

2.4.5 Time related process variables

Injection time
The period from when the screw commences its forward movement to the point where the holding pressure is applied is called the injection, or mold fill, time. The amount of material that should be pushed into the mold during this period is between 95 to 98% of the total shot volume. The time to force this amount of molten material into the mold is dependent upon important factors such as the injection speed selected, melt viscosity, screw stroke used, the dimensions of the gate.

Because of the influence from all the above factors, the injection time is often regarded as consequential.

Holding pressure time
The holding pressure time is the time, when the screw is held almost stationary in its most forward position so as to apply the necessary holding pressure to the molten material in order to pack the material into the mold cavities during the early stages of material solidification. The period of time used for the holding pressure to be applied should correspond with the time the gate takes to freeze off or, for the gate to sufficiently solidify. This holding pressure time once set is readily controlled by, for example, the process timers used in the molding machine.

Pause (dwell) time
At the end of each molding cycle, and before the commencement of the next, it is necessary to allow sufficient time for the moldings to be extracted.

Cooling time

Cooling time is necessary for the molten plastic material to cool to a temperature which will enable the moldings to be ejected from the mold without distortion. This time period is dependent upon many factors, for example, the general shape of the component, the wall thickness of the component, and the type of material being processed. The time period needed is readily set by means of electrical timers and hence is regarded as controllable.

This time period is always the longest portion of the molding cycle. During the cooling sufficient time is needed to retract the screw (sometimes called screw recovery, or dosing time) so as to refill the barrel with material.

Cycle time

The time period for product manufacture is dependent upon the accumulation of all other time increments of the molding cycle. As some of these elements are consequential, whilst others are controllable, the cycle time also has to be regarded as consequential. However, by reducing the variation of the controllable process variables to a minimum the overall cycle time variation can also be maintained within a narrow band.

2.4.6 Stroke related process variables

Melt cushion

The melt cushion is the amount of molten material left after injection. The size of the melt cushion results from selecting and controlling other process values.

Screw stroke

The screw stroke is the linear distance the screw moves or travels, from it's stationary position (after plasticization) to the selected position of holding pressure application (mold pack). A constant volume of molten material must enter the mold each cycle or the product quality will be affected. Any variation in this volume is usually related to the variation in the final (stationary) position of the screw. Low variation can be achieved by selecting the correct screw rotational speed. Upon setting the necessary parameters so as to maintain a screw stationary position the screw stroke can be regarded as controllable.

Changeover position from injection to holding pressure

The changeover from injection pressure to holding pressure is usually performed by one of three different modes. That is, by pressure dependent switch-over, time dependent switch-over and stroke (distance) dependent switch-over. The most common and preferred technique is stroke dependent switch-over, i.e. the switch-over position from injection pressure to holding pressure is initiated at a preselected distance. When the advancing screw reaches this position an electric impulse signals for a change of hydraulic mode - from injection pressure/screw velocity to holding pressure.

2.4.7 Temperature related process variables

Melt temperature
The melt temperature is not measured in the process directly. Instead the contents of the cylinder or barrel is shot directly into a cup and measured with a thermocouple. The temperature of the molten polymer inside the cylinder assembly is determined by a selection of many important process parameters and machine settings. For example, melt temperature is affected by screw speed, back pressure, cycle time, barrel temperature settings and the hopper throat temperature. Its value can be changed by simply altering one of these process variables. This variable is considered as consequential.

Mold temperature
The surface temperature of each mold half varies every cycle as the molten material enters the mold. The excess heat contained within the molten material has to be removed so that the molding can be extracted without distortion. The mold temperature is a consequence of many process and design variables, for example, the melt temperature, the cooling time, the rate at which the cooling medium is flowing through the mold and the design of the cooling circuit in the mold.

Barrel temperature
The barrel forms the outer boundary of the screw channel. For injection molding one can assume that the major portion of the heat that must be applied to the plastic is supplied by the barrel. To do this, the barrel is equipped on the outside with electrical band heaters.

Cooling water temperature
Any injection molding process is reliant upon the flow rate, the available system pressure and the temperature of the cooling medium used.

Chapter 3

Results and Discussion

In this thesis various methods for quality improvement in injection molding are suggested. The emphasis has been on studying the variation in the process. It is well-known, that in manufacturing with multi-cavity molds, the cavity differences are of major importance. The cavity-to-cavity variation has been addressed in Papers A, B, C, and E.

It was found that the largest single contribution to the variation in the finished parts was due to the difference between cavities (Paper C). This source of variation accounted for 46 % of the total variation. Consequently, any efforts in improving the quality of the product should pay special attention to the cavity-to-cavity variation.

3.1 Cavity-to-cavity variation

In Paper B output from a multi-cavity molding process is analyzed for systematic differences between the cavities in the mold. It is found that in the row-column layout of the mold, there is a systematic effect from the cavities positions. The closer to the center of the mold a part is produced, the longer it is.

Also a study of shrinkage data reported in Paper E indicated that the observed differences were not only due to differences in cavity dimensions. From the parameters in a non-linear regression model, it was found that the shrinkage rate was not the same for all cavities. Consequently, there is a potential for reducing the variation, if reasons for the differences can be found.

Furthermore a capability index taking the cavity-to-cavity variation into account has been suggested in Paper B. Using the average of the percentage non-conforming from each cavity the resulting index is equivalent to the conventional C_{pk} index with respect to the percentage of non-conforming parts.

Another paper dealing with methods to reduce the cavity-to-cavity variation is Paper A. In Paper A an 8 factor fractional factorial experiment is applied to the molding process of a part produced in a multi-cavity mold. Two factors were identified having a significant effect on the cavity-to-cavity variation. Shorter injection time results in lower variation, and longer cooling time results as well in lower variation. Thus, a proper choice of levels for the machine variables will result in more consistent quality of the molded parts.

3.2 Process variables

In Paper C the correlation of 21 process variables are analyzed. The process variables are analyzed by means of principal component analysis, which takes the multivariate nature of the variables into account. It was illustrated in an example how a retrospective multivariate analysis of production data can facilitate a deeper understanding of the correlation structure of the data, which can assist in building a proper monitoring strategy.

Two principal components were found to reflect the changes in machine set-points very well. There were clearly two levels of variation, a macro variation was the variation from set-point to set-point, and a micro variation was the variation within set-points.

The correlation between process variables and part quality was not very good. One reason for this may be that the observed process was a process in control. Again a designed experiment may provide more information on the correlation structure between the process variables, and on their correlation with part quality. The experiment could be performed by means of Evolutionary Operation (EVOP) (Box and Draper (1969)), where the experiments are carried out during production without interrupting it.

The influence of eight machine variables on part length was studied in a designed experiment. Four machine variables were found to influence the mean length significantly. In this controlled setting, the four variables explained almost all the observed variation in part length.

3.3 Productivity

Apart from part quality, another great concern for the manufacturer is the cycle time. The lower the cycle time, the more product the molder can produce. However, since cooling time is the greatest contribution to the cycle time, we have that an increase in cooling time will result in lower cavity-to-cavity variation, which will in turn lead to a greater proportion of parts within specifications. Consequently, a higher production rate leads to a higher proportion non-conforming parts. Thus, some compromise between the production rate and the quality of the parts must be sought.

To further investigate the relation between part quality and productivity, a simple cost of quality model is investigated. The hourly profit can be expressed as a difference between the gain on the conforming product, and the loss due to non-conforming product. The proportion of conforming and non-conforming product produced in an hour are both functions of the process variation, and thus functions of the cooling time. Consequently the hourly profit can be written as the following function of the cooling time, t_c,

$$U(t_c) = c_1 h(t_c) PI(t_c) - c_2 h(t_c)(1 - PI(t_c)) \tag{3.1}$$

where $h(t_c)$ is the production rate, $PI(t_c)$ is the proportion of parts produced inside the specifications, c_1 is the profit of a conforming part, and c_2 is the cost associated with a non-conforming part. In the constants c_1 and c_2 the cost of inspection, sorting and rework may be included.

To illustrate this model, it is applied to the case presented in Paper A, where a

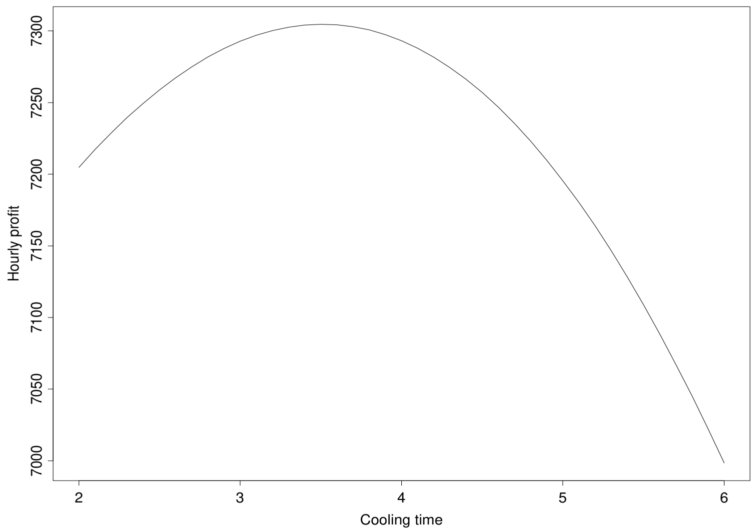

Figure 3.1: Relationship between cooling time and hourly profit.

crude relation between cooling time and cavity-to-cavity variation was found from a designed experiment. The cavity-to-cavity variation is assumed to decrease linearly with the cooling time, and a suitable model for the production rate is suggested below.

The cycle time is the sum of the cooling time and several other time components. The other time components do not depend on the cycle time, and for simplicity their sum is set to 10 seconds. The amount of parts produced per hour from the 16 cavity mold is assumed to be

$$h(t_c) = \frac{16 * 3600}{10 + ct}. \quad (3.2)$$

With c_1= 2, c_2= 3.5, $\mu = 70.13$ mm and specification limits $LSL = 70.07$ mm and $USL = 70.19$ mm respectively, the relationship between the cooling time and the hourly profit is as shown in Figure 3.1.

The hourly profit is seen to have a maximum for a cooling time of approximately 3.5 seconds. Thus, the best compromise between quality and production rate is obtained when using a cooling time of 3.5 seconds.

3.4 Monitoring and control

The objective for monitoring the process is to improve product quality, detect process changes and disturbances and increase operator awareness of the impact of process changes. Two different approaches for monitoring are discussed in this work. In Paper B charts for monitoring the quality of the molded parts are suggested, and in Paper C alternative methods for monitoring the process variables are discussed.

When monitoring the product from a multi-cavity mold, it is important to distinguish between an assignable cause that shifts the mean level of all of the cavities over time (called an overall assignable cause), and an assignable cause that changes the mean of one or more cavities relative to the remainder (called a relative assignable cause). A control charting procedure has been suggested, that is controlling the overall and the relative assignable causes separately.

In Paper A methods for control of the injection molding process were suggested. One control approach suggested, was the model based control, based on finding a suitable model for the dynamical behaviour of the process. In Paper C it was found that the sample-to-sample variation within a machine set-point was a non-stationary process, and that it was well described by an ARIMA(2,1,1) model. Since the molding process was found to be non-stationary, a control strategy is indeed needed.

In Paper D an alternative method for in-line quality charting is suggested. The method is for continuous control by attributes, and can thus be applied for example for evaluating conformance of visual characteristics.

3.5 Limitations

In the entire work presented here, attention has been given only to the length of the parts. However, other characteristics of the parts define the quality of the parts just as well. Visual characteristics and the ultimate strength of the parts are two other important quality aspects.

That the parts have the same length does not necessarily mean that the other characteristics of the parts are the same as well. However, a similar approach for dealing with other characteristics than dimensions can be taken. It is for example possible to design experiments for binary responses.

In the shrinkage study in Paper E, only one shot is measured. The effect of the molding variables is not considered. For example viscosity can affect how much the polymer is compressed in the cavity and therefore how much shrinkage will take place. One suggestion would be to study the influence of machine settings on shrinkage in a designed experiment similar to the the one in Paper A.

Chapter 4

Conclusion

The majority of the literature on control of the injection molding process is concerned with closed mathematical/physical models of subprocesses. There is, however, a lack of empirical evidence that such models provide a satisfactory description of the transfer of variation in such highly complex processes as a multi-cavity injection molding process.

In the present work the observed variation in injection molded part dimensions has been considered. In particular the variation between cavities in multi-cavity molds has been studied in various ways. The thesis suggests alternative approaches for dealing with the injection molding process. The proposed approaches are illustrated with actual production data from the manufacturing of molded parts for a medical device.

In actual production data it was found that the largest single contribution to the variation in the finished parts was due to the difference between cavities and accounting for 46 % of the total variation. Consequently, any efforts in improving the quality of the product should pay special attention to the cavity-to-cavity variation. The second greatest source of variation was the change in machine set-points accounting for 12 % of the total variation.

A simple graphical method for bringing out systematic patterns in data from a multi-cavity mold has been suggested. Upon applying the method on data from a 16 cavity mold, a clear pattern was found in the part of the variation arising from the cavities positions in the mold.

A designed experiment with eight machine variables showed that two factors had a significant influence on the cavity-to-cavity variation. Shorter injection time resulted in lower variation, and so did longer cooling time. Thus, a proper choice of levels for the machine variables would result in more consistent quality of the molded parts. Since longer cooling time affects the production rate in a negative way, it was illustrated how a compromise between quality and production rate could be found.

Furthermore a simple model for the in-control variation for a multi-cavity molding process was suggested. The model differentiates between assignable causes affecting all cavities, and assignable causes affecting only one or a few cavities. Based on the suggested process model a charting procedure has been introduced. The charting procedure is controlling the two types of assignable causes separately. Furthermore a capability index taking the cavity-to-cavity variation into account has been suggested. Using the average of the percentage non-conforming from each cavity the resulting index is equivalent to the conventional C_{pk} index with respect to the percentage of non-conforming parts.

Finally, it was found that within set-points, the quality variable (part length) was not exhibiting stationary behaviour, indicating that there is a need for monitoring and controlling the process. A multivariate analysis of 21 process variables was carried out. It was found that a change in machine set-point was reflected by two principal components. Thus, there seems to be great potential in monitoring the process variables using a multivariate approach.

II

Part II

Included papers

A

Paper A

Quality Improvement in Injection Molding through Design of Experiments

Submitted to Quality Engineering.

Abstract

Eight process factors were studied to improve the quality of a plastic part used in a medical device. The part is produced from multi-cavity injection molding. Using a fractional factorial design the factors influencing the mean length and the cavity-to-cavity variation were identified. It was found that holding pressure, injection time, holding pressure time, and back pressure all had a significant influence on the mean length of the parts. Injection time and cooling time were the only significant factors with respect to the cavity-to-cavity variation.

1 Introduction

Dimensional variation in injection molded plastic parts is a common problem in the plastics industry. In a single-cavity mold, dimensional variation is observed from run to run. In a multi-cavity mold, there may also be cavity-to-cavity variation, even though all of the cavities are filled with the same polymer melt and under the same molding conditions. In either case, variations in critical dimensions can cause problems in subsequent down-the-line assembly operations due to, for example, mismatch between mating parts.

According to Whelan and Goff (1996) many engineers tend to believe that the process variables and machine settings selected for the manufacture of a particular component, for example a car bumper, are very similar to those required for the production of syringe assemblies. In reality the process parameters which are considered important for the manufacture of one product do not have the same importance for other products. In this paper we introduce a systematic approach to identifying the key variables for a particular part. Furthermore we use the key variables to find an optimal setting and to formulate a control strategy.

Several authors have used design of experiments as a tool for finding a suitable level of the machine settings to produce parts that live up to certain quality requirements. Deliz and Caraballo (1995) used a fractional factorial experiment to find the processing conditions that would minimize the out-of-roundness while centering diameter and length at their nominal values. Also Xia and

Mallick (1997) and Blyskal (1994) applies DOE techniques to find optimal settings with respect to some dimensional measure. However, all the above focus on explaining variations in the mean, the cavity-to-cavity variation is not touched upon. Furthermore Acharya and Mahesh (1999) uses design of experiments to identify which process parameters affected the aesthetic aspects of a product, and Beard (1999) suggests using design of experiments for process validation.

Most systematic investigations of the effect of various molding parameters focus only on their effect on the mean and not on the variation.

2 The Experiment

A hollow cylindrical part used in a medical device and produced by injection molding was studied for quality improvement. The part in question is produced in a 16 cavity mold and is a part of an assembly consisting of seven molded parts. The quality characteristic that is investigated is the length of the part. The length is measured a week after the part has been produced in order to incorporate possible differences in shrinkage patterns of the individual parts.

2.1 Planned experiment

Nine factors were chosen for the experiment based on our experience and knowledge of the process. The levels were chosen such that they were clearly outside ordinary operating conditions but not further than we would still expect to get a usable product out. The nine factors included in the study are listed in Table 1 together with their levels. The barrel temperature is controlled by four heater bands, therefore the four values for each level of this factor.

Our objective for the experimentation is to find which factors and factor combinations influence mean length and cavity-to-cavity variation mostly. Using a so-called (Box et al. (1978)) 2^{9-4} fractional factorial design, an experiment was designed. The design chosen involves 32 experiments, and because the process needs some time to stabilize after each new set point it was intended to carry out the experiment over two consecutive days. This was done by in-

Variable	Low level	High level
A Mold temperature	Cooling	No cooling
B Holding pressure	53 bar	63 bar
C Injection time	0.3 sec	1.6 sec
D Fill/postfill switch	12.5 mm	16.5 mm
E Barrel temperature	(205,215,205,185)	(235,245,235,215)
F Holding pressure time	2.5 sec	4.5 sec
G Back pressure	5 bar	30 bar
H Cooling time	2 sec	6 sec
J Travel	47.5 mm	51.5 mm

Table 1: Factors and levels

troducing two blocks to the experimental design.

The chosen design was one resolution IV which means that main effects were confounded with three factor interactions and two factor interactions were confounded with other two factor interactions. The 2^{9-4} design suggested by Box et al. (1978) leads to the following defining relations

$$I = BCDEF = ACDEG = ABDEH = ABCEJ. \tag{1}$$

The block generator is chosen in such a way that it is free of main effects and two factor interactions. The block generator AEF satisfies these requirements. All experiments with $AEF = +1$ are carried out on the first day, and experiments with $AEF = -1$ are carried out on the second day.

2.2 The actual experiment

Not everything went as planned.

The particular mold was usually operated without cooling. To investigate if cooling has any effect on part quality we wanted to include cooling in our experiment. But on the day of the experiment it was found that the cooling channels were clogged. It was not possible to fix the problem on the spot and thus it was decided to continue without cooling.

The experiment was carried out on one of the production machines. On the second day of experimentation the machine could not be taken out of production as planned. Consequently only the first block of the experiment was actually performed.

As a result the design actually performed is a resolution III design instead of a resolution IV. In a resolution III design some main effects are confounded with two factor interactions. The alias structure of this design is shown up to and including two-factor interactions in Table 2. The first column is the underlying complete 2^4 factorial design for the factors B-E. The factor A (cooling) was taken out of the experiment.

B	=	EG					
C	=	EH					
BC	=	GH	=	FJ			
D	=	EJ					
BD	=	FH	=	GJ			
CD	=	FG	=	HJ			
BCD	=	EF					
E	=	BG	=	CH	=	DJ	
BE	=	G					
CE	=	H					
BCE	=	DF	=	CG	=	BH	
DE	=	J					
BDE	=	CF	=	DG	=	BJ	
CDE	=	BF	=	DH	=	CJ	
BCDE	=	F					

Table 2: Alias structure for the 2^{8-4} performed experiment.

From a physical perspective interactions with barrel temperature (factor E) are not very likely. Thus, if three-factor and higher interactions are negligible and we further assume that interactions with barrel temperature are negligible as well, this design will after all give clear estimates of all main effects except barrel temperature. Barrel temperature is confounded with two-factor interactions between holding pressure and back pressure, between injection time and cooling time, and between fill/postfill switch and travel. Especially the former two are both quite possible.

For 8 factors in 16 experiments we could have had a resolution IV design instead of the resolution III design we ended up with, but circumstances turned out otherwise.

From each of the 16 different parameter settings two shots were collected and measured. Each shot consists of 16 parts, one from each cavity. The results are displayed in experimental order in Figure 1 with the numbers 1-16 being the cavity numbers. For every experiment the letters on the x-axis denotes the experiment. When a letter is present the corresponding factor was on its high level in that experiment. It is clear from the figure that the mean as well as the standard deviation of the length measurements changes greatly from one experiment to the next. The horizontal line in the figure is the target length.

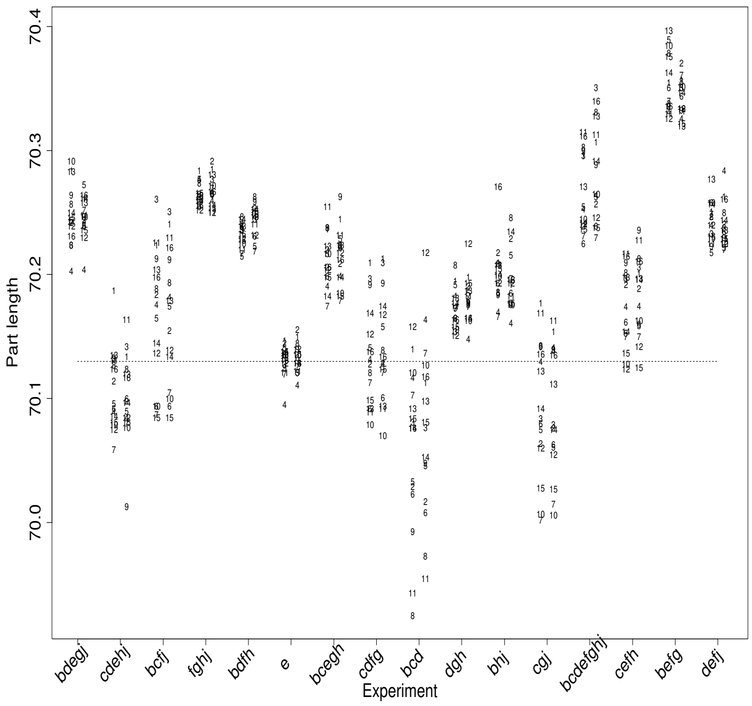

Fig. 1: The measured length of the molded parts in each of the 16 experiments (2 replicates). The numbers 1-16 are the cavity numbers.

3 Analysis

The two repetitive shots were all taken within a minute of each other. In Figure 1 we can see that the shot-to-shot variation is very limited. The two shots made under the same parameter conditions are very similar. See for example the treatment "cgj" where the six lowest measurements are two measurements from cavities 7, 10 and 15. However, the observed shot-to-shot variation may not be representative for the process dynamics in general. The two shots should not be regarded as true replicates. Raw material variations and influence from environmental factors may cause shots further apart to be much more different than two consecutive shots.

If the very limited shot-to-shot variation is used as error in the statistical analysis we will almost certainly falsely conclude that too many of our experimental factors have an effect. To avoid finding all or almost all the factors significant we decided to average the results from the two repetitions and subsequently analyze the mean over all 16 cavities and the variance between the cavities.

3.1 Analysis of mean length

Table 3 contains the effect estimates and sums of squares for the 15 effects estimated for the mean of the 16 cavities. The effects are again ordered according to the underlying complete 2^4 factorial design.

An analysis of variance leads to a model with six significant factors. Table 4 summarizes the analysis of variance for this model. The residuals from the fitted model are displayed in a normal probability plot in Figure 2. From this figure it seems that the distribution of residuals has heavier tails than the normal distribution. This indicates a violation of the analysis of variance assumption of equal variances. The response in our analysis is the mean length of the 16 cavities. And we saw from Figure 1 that the cavity-to-cavity variation varies greatly with experimental conditions. Thus the variance homogeneity assumption is probably violated. However, the identified effects are all very significant, so we still believe that they are real.

Box and Cox (1964) discussed how to deal with inhomogeneous error structures through transformations of the response variable. This method is very

Effect			Df.	Estimated effect	Sum of Squares
(Intercept)				70.1931	
B			1	0.0538	0.0116
C			1	-0.0759	0.0230
BC			1	-0.0005	<0.0001
D			1	-0.0135	0.0007
BD			1	-0.0110	0.0005
CD			1	-0.0034	<0.0001
BCD			1	0.0093	0.0003
E			1	0.0496	0.0098
BE	=	G	1	0.0532	0.0113
CE	=	H	1	0.0272	0.0029
BCE			1	-0.0040	0.0001
DE	=	J	1	0.0120	0.0006
BDE			1	-0.0075	0.0002
CDE			1	-0.0021	<0.0001
BCDE	=	F	1	0.0776	0.0241

Table 3: Estimated effects and sums of squares with mean level as response.

Source of variation	Df.	Sum of Squares	Mean Square	F_0	P-value
B (Holding pressure)	1	0.0116	0.0116	42.025	0.0001
C (Injection time)	1	0.0230	0.0230	83.597	<0.0001
E (Barrel temperature)	1	0.0098	0.0098	35.649	0.0002
F (Holding pressure time)	1	0.0241	0.0241	87.320	<0.0001
G (Back pressure)	1	0.0113	0.0113	41.091	0.0001
H (Cooling time)	1	0.0029	0.0029	10.698	0.0097
Residuals	9	0.0025	0.0003		

Table 4: Analysis of variance for the analysis of the mean length.

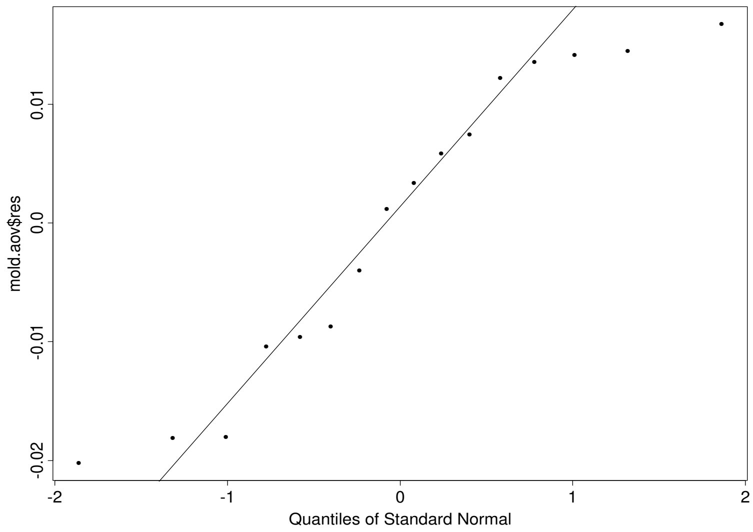

Fig. 2: Normal probability plot of residuals from analysis of mean length.

good when the variation can be reasonably described by a monotone function of the mean. However, in our case we see that low cavity-to-cavity variation occurred at several different levels of the mean. Consequently it is not possible to find a transformation that will stabilize the variance.

Instead a weighted analysis has been used, where the observations are weighted with the inverse cavity-to-cavity variance. See Draper and Smith (1998) for a more thorough introduction to weighted analyses. The residuals from this analysis are found in Figure 5. Residuals from the weighted analysis of variance model look more like normally distributed errors. Table 5 summarizes the weighted analysis of variance.

In Figure 3 the changes in the average response is displayed for the four significant factors. And the size of the significant effects are displayed in Figure 4.

The four significant factors are in good agreement with our experience with the process. Traditionally holding pressure and injection speed are believed to be among the most important factors for part dimensions, and it is usually the variables used by the molder to control dimensions.

Source of variation	Df.	Sum of Squares	Mean Square	F_0	P-value
B (Holding pressure)	1	22.3696	22.3696	13.520	0.0036
C (Injection time)	1	16.3680	16.3680	9.893	0.0093
F (Holding pressure time)	1	66.2827	66.2827	40.060	0.0001
G (Back pressure)	1	35.4379	35.4379	21.418	0.0007
Residuals	11	18.2003	1.6546		

Table 5: Weighted analysis of variance of mean length.

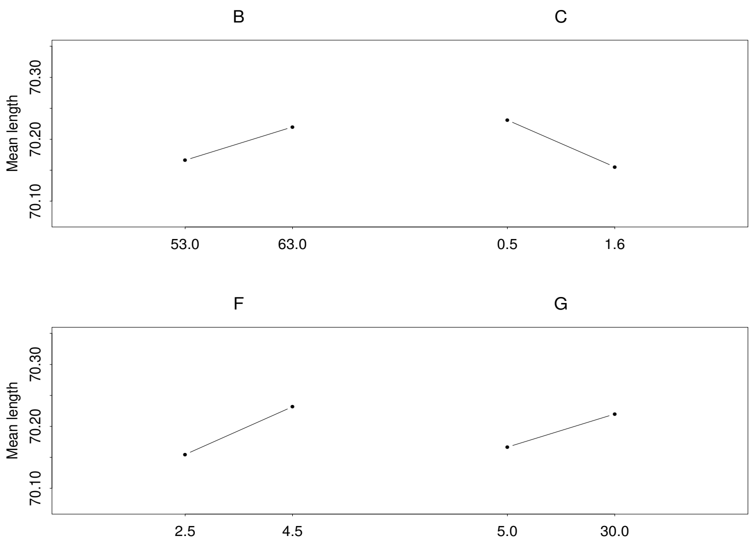

Fig. 3: Average response curves.

Increasing the holding pressure or the holding pressure time will give a better compression of the melt which will lead to less shrinkage and therefore longer parts. By increasing the injection time the injection speed is decreased which will in turn decrease the pressure transmission during the holding pressure stage. This will result in greater shrinkage which will again cause shorter parts.

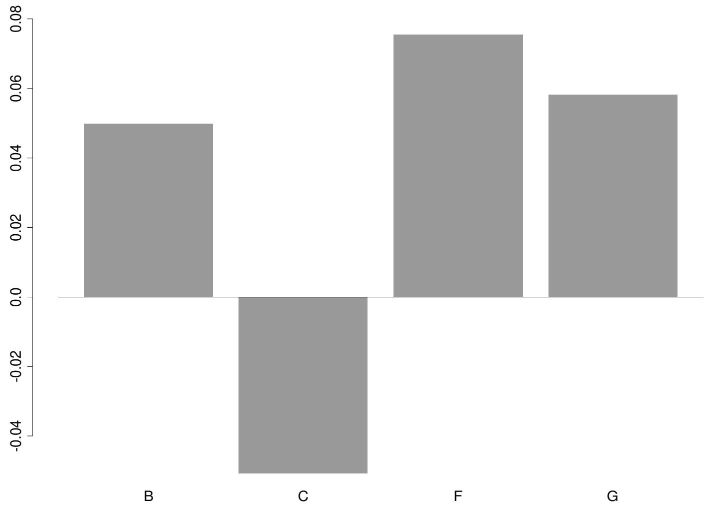

Fig. 4: Effects from the weighted analysis of the mean.

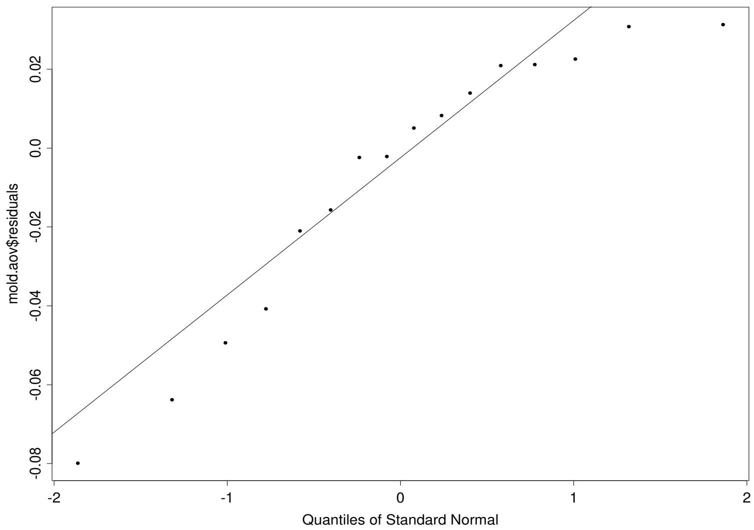

Fig. 5: Normal probability plot of residuals from weighted model.

3.2 Analysis of cavity-to-cavity variation

In most other applications of DOE to injection molding only the effect on the mean has been studied. Another great concern is how parameters affect the cavity-to-cavity variation.

According to Bartlett and Kendall (1946) a logarithmic transformation of observed variances will serve well as a basis for analysis of variance heterogeneity. Thus if we use the log-transformed cavity-to-cavity variance as response we can perform an analysis of variance to evaluate the effect of the eight factors on the cavity-to-cavity variation.

Effect			Df.	Estimated effect	Sum of Squares
B			1	0.3770	0.5684
C			1	1.8435	13.5945
BC			1	-0.2385	0.2276
D			1	0.1659	0.1101
BD			1	-0.1337	0.0715
CD			1	0.1050	0.0441
BCD			1	0.5937	1.4098
E			1	-0.4521	0.8176
BE	=	G	1	-0.1376	0.0758
CE	=	H	1	-0.6630	1.7581
BCE			1	-0.2623	0.2753
DE	=	J	1	0.2626	0.2758
BDE			1	0.1925	0.1482
CDE			1	0.0187	0.0014
BCDE	=	F	1	-0.1865	0.1391

Table 6: Effect estimates and sums of squares for analysis of cavity-to-cavity variation.

Table 6 shows the effect estimates and sums of squares for the 15 effects from this experiment. The analysis shows that main effects C (Injection time) and H (Cooling time) have significant influence on the cavity-to-cavity variation. Table 7 summarizes the analysis of variance with the logarithmic transformed cavity-to-cavity variance as response. In Figure 6 the relative size of the two significant factors is displayed.

It has wide implications for the control of the process that injection time is among the most important effects both with respect to the mean and to the cavity-to-cavity variation. Especially since injection time (or injection speed)

Source of variation	Df.	Sum of Squares	Mean Square	F_0	P-value
C (Injection time)	1	13.5945	13.5945	42.438	<0.0001
H (Cooling time)	1	1.7581	1.7581	5.488	0.0357
Residuals	13	4.1644	0.3203		

Table 7: Analysis of variance for cavity-to-cavity variation.

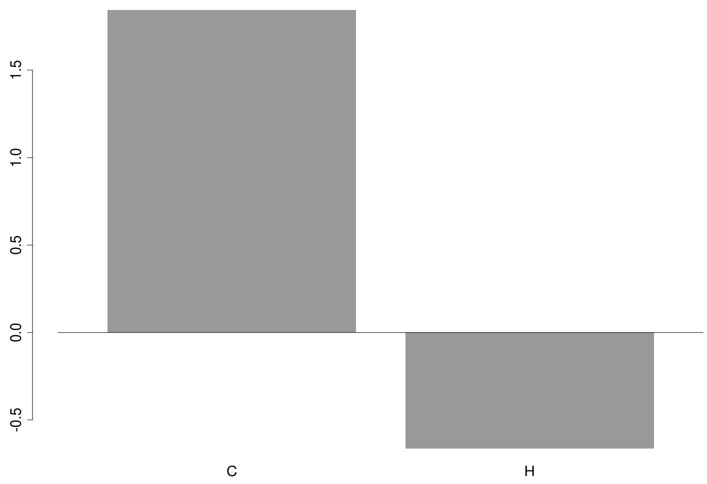

Fig. 6: Effect on cavity-to-cavity variance.

is among those variables often used for controlling the process. When using the injection time to adjust the mean level the cavity-to-cavity variation will change as well.

The residuals of the model look quite satisfactory. In Figure 7 the logarithm of the residual sum of squares is plotted against the value of lambda in a Box-Cox transformation (Box and Cox (1964)). This figure reveals that minimum of log(RSS) occurs for a lambda of zero which is the log-transformation, proving that this was indeed a suitable transformation.

Another way to analyse the observed variances could be through generalized linear models, see e.g. McCullagh and Nelder (1990). However, the results from the analysis of the log-transformed variances did not give any reason to

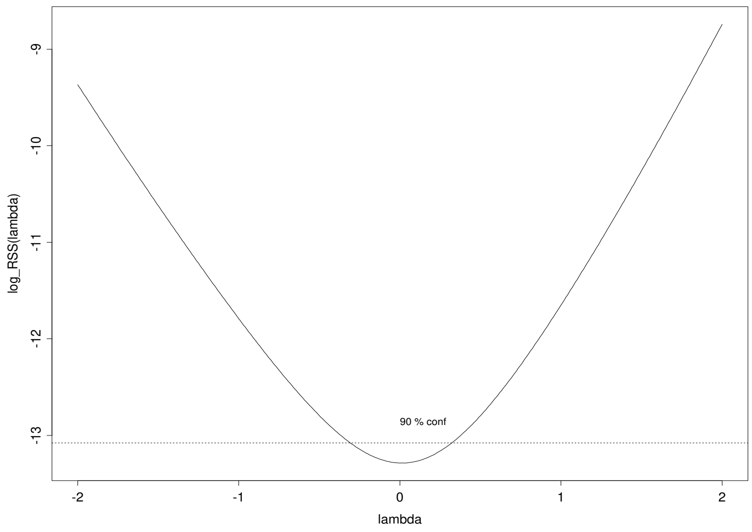

Fig. 7: Box-cox transformation for standard deviation.

do so.

4 Optimal settings

We will use what we learned about the influence of the eight factors on the mean length and on the cavity-to-cavity variation to find a suitable set point for the process. Our goal for a good set point is to be on target with minimum cavity-to-cavity variance. The target value for the length of the part is 70.13 mm.

First the cavity-to-cavity variation should be made as small as possible by keeping injection time on the low level and cooling time on the high level. However, injection time has significant influence on the mean length as well. So we will have to compensate by choosing appropriate levels of the remaining factors such that the mean level comes on target. The factor combination shown in Table 8 leads to a mean length on target.

Another important characteristic of the process is the cycle time. It must be

Variable	Value
B Holding pressure	53 bar
C Injection time	0.3 sec
F Holding pressure time	2.5 sec
G Back pressure	5 bar
H Cooling time	6 sec

Table 8: Optimum processing conditions for performed experiment.

appreciated that injection molding is a mass production process. Lower cycle times means higher productivity. So most injection molders are very concerned about keeping the molding cycle time as low as possible, and the greatest component of the cycle time is the cooling time.

Having cooling time on the high level means adding 4 seconds to every cycle. In this particular process adding 4 seconds to the cycle time is a considerable extension of the cycle time. As the influence of the injection time is even greater than that of the cooling time we should look in to the possibility of injecting even faster and then lower the cooling time from the 6 seconds. However, if the melt is injected too fast into the mold the risk of burn marks becomes a serious issue.

Some compromise between the cycle time and cavity-to-cavity variation should be sought.

5 Control

In the previous sections we learned the effect of the eight molding factors on the part dimensions. We used this knowledge to find an optimal set point for the process. However, this set point is only optimal so far the process behaves as it did during the experiment. Variations in the raw material or the outside humidity may affect the part dimensions such that the chosen set point will no longer lead to the mean length being on target.

To compensate for the systematic drift in the process mean we look into different methods for feedback adjustment. Our overall goal for a good control strategy is that we want to control the mean such that it is on target with minimum cavity-to-cavity variance. First we assume that the cavity-to-cavity variation is not affected by the process drift, i.e. injection time on the low level and cooling time on the high level will still give the lowest cavity-to-cavity variation.

A simple and robust feedback adjustment procedure is to compensate for only a proportion G of each deviation in the mean from the target. Box and Luceno (2000) has shown that repeatedly applying this simple procedure is equivalent to applying an exponentially weighted moving-average (EWMA) with a smoothing constant $1 - G$.

Another approach is the model based control introduced by Åström (1970). This approach is based on finding a suitable model for the dynamical behaviour of the process, for example in the ARMA class of models (Box et al. (1994)). The simplest method for model based control is the so-called minimum variance control where the squared deviation from target expected under the particular model is minimized, i.e.

$$\min_J \left(J = E[(\hat{y}_{t+k|t} - y_{ref})^2] \right), \tag{2}$$

where $\hat{y}_{t+k|t}$ is the k step prediction at time t under the particular process model. In the computation of this prediction the observed mean at time t is used. A potential shortcoming of the minimum variance controller is that it only considers the quality and does not take the variation of the control variable into account. If some kind of cost is associated with making adjustments these costs should be included in the cost function, as e.g. in

$$\min_J \left(J = E[(\hat{y}_{t+k|t} - y_{ref})^2] + \lambda u_t^2 \right), \tag{3}$$

where u_t is the control variable and λ is a cost associated with making adjustments. This is called LQG control.

It is sometimes inconvenient to make repeated adjustments in the above-mentioned manner. For example when the operator has to do the interventions manually. Another great concern is that when considerable measurement error is present we run the risk of making adjustments based on noise. If we adjust the process every time the observations deviate from target we will just introduce more variation to the process. However, from the two shots measured in every experiment, an estimate of the measurement error can be obtained. If the observations deviate less than the measurement error from target, there is no need to adjust the process.

Box and Luceno (2000) suggests a bounded adjustment scheme for this type of problems. Action is taken only when, at time t, a measure of present and possibly past deviations falls outside tabled limits. In this way we will only take action when we are certain that the mean has actually changed.

6 Conclusion

Using a fractional factorial design with eight factors, it has been investigated how eight process factors influenced the mean length and the cavity-to-cavity variation in the production of a hollow cylindrical part from a multi cavity mold. It has been found that holding pressure, injection time, holding pressure time, and back pressure all had a significant influence on the mean length of the parts.

Furthermore it was found that injection time and cooling time were the only significant factors with respect to the cavity-to-cavity variation. The cavity-to-cavity variation contributes well to the overall variation of the process. Consequently, this result has great practical implications for improvement of the process. A proper choice of levels for the machine variables will result in higher quality parts.

Based on these results an optimal set-point for the process is suggested. Also ways of controlling the process are briefly discussed.

Paper B

B

Quality Monitoring of Injection Molding Parts from a Multi-cavity Mold

Parts presented at the Joint Statistical Meetings, Baltimore, USA 1999

Submitted to Quality and Reliability Engineering International.

1 Introduction

A company is producing a medical device that consists of seven components all made in plastic by injection molding. The parts are produced from multi-cavity molds having between 8 and 24 cavities. The company wants to know if parts meet specifications or not. The latter can occur if the viscosity of the melted plastic is incorrect or a cavity malfunctions, possibly due to a clog somewhere. Problems with the viscosity can affect the product from all cavities, but a clogged cavity only affects the output of that particular cavity. It is important to detect both an assignable cause that results in all cavities shifting out of control and an assignable cause in which one or a few cavities shifts out of control. Typically, these two types of assignable causes have different root causes, and the distinction facilitates the identification of the problem and the solution.

To monitor the quality of molded parts from a multi-cavity mold, individual control charts for each cavity can be maintained simultaneously. Golmanavich and Nielson (1993) advocates this approach for monitoring product from a multi-cavity mold. An out-of-control condition is signaled whenever either the cavity malfunctions or the process inputs change. Rather than monitoring 16 individual charts, one would prefer to monitor the process using fewer charts. The disadvantages due to multiple comparisons are also important. As the number of charts increases, the chance of a false alarm also increases.

It is important to understand that each cavity in an injection mold will have its own performance characteristics, even when a multi-cavity mold is geometrically balanced. Combining data from several cavities can confuse the picture by creating distributions with multiple modes or produce average values which suggest that part dimensions center on the target, when the values actually straddle the target value symmetrically.

2 Multi-cavity molds

Where large numbers of components are required, particularly when they are small components, then the economic advantages and fast production capabilities of multiple cavity production are considerable. When desired part weight

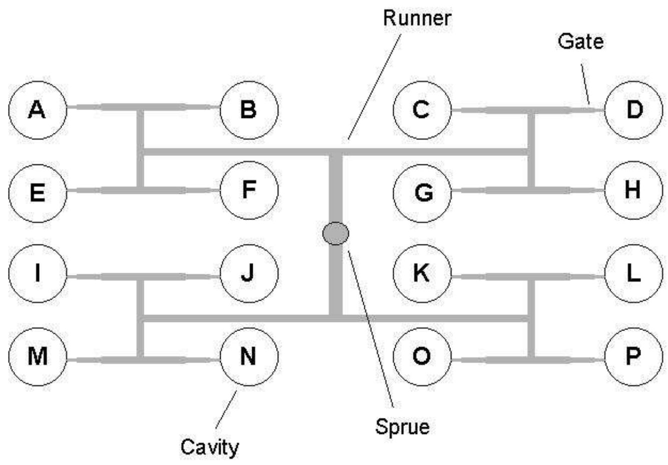

Fig. 1: Sketch of a 16 cavity mold. Cavities are numbered A-P and connected by the runner system.

is low and the quantity of parts is great, it is common to use multi-cavity molds for production of the parts. By using a multi-cavity mold the machine time is used much more efficient, because in every cycle of the process several (the number of cavities) parts are produced simultaneously, which is usually referred to as a shot. A multi-cavity mold consists of the forming cavities, a runner system for distribution of the plastic melt combined with all the necessary equipment for cooling and ejecting the parts. A sketch of a 16 cavity mold is displayed in Figure 1. The runner system connects the cavities into four clusters. There is the same flow distance to all four clusters and to all cavities in a cluster.

To provide part-to-part uniformity in a multi-cavity mold, the cavities should be cut to equal sizes, and furthermore the runner system should enable each cavity to fill at the same rate. If the flow distances from the sprue to the various cavities are equal, the runner system is balanced. As balanced runner systems contribute to melt uniformity among cavities, dimensional tolerances are easier to maintain. Unbalanced systems can cause great cavity-to-cavity variations.

In the process control a full shot is sampled and after two hours cooling and hardening time the items are measured. A full shot consists of all items produced in one production cycle, i.e. one item from each cavity. After being molded the parts from all cavities slide down the same chute into a common container. However, for traceability purposes each part has the cavity number molded in it.

An important quality characteristic of the molded parts is the part dimensions. Especially when the part is assembled with other molded parts it is crucial that the part is within the specification limits. The part dimensions are usually highly correlated such that parts longer than the target length will also have diameters greater than target diameter. Consequently usually only one critical part dimension is measured.

Figure 2 shows the part length of 7 samples of all 16 cavities. The samples are several hours apart, and they are collected as part of the process control. The cavity labels are indicated in the figure by the letters A-P, and samples from each individual cavity are connected with lines. From the figure it is seen that not all the 16 cavities are making parts with the same length, for instance parts from cavity M are generally shorter than parts from the other cavities.

In Figure 3 the average length for each cavity has been computed and plotted in various ways to investigate if there are any visible patterns in the length measurements coming from the geographical positions in the mold. If the mold had been perfectly balanced and perfectly cooled the only cavity-to-cavity variation left would be the result of dimensional variations in the steel. In this case parts from all cavities would be spread out randomly around the target length. The sketch of the mold layout in Figure 1 suggest various conceivable systematic deviations from such a purely random pattern.

Traditionally in a row-column layout it is reasonable to look for possible row- and/or column effects and interactions between rows and columns. As the cooling channels are organized in a row-column design, such effects could be ascribed to the pattern of cooling ribs.

The two graphs in the first row of Figure 3 are so-called interaction plots between rows and columns. The graph in the upper left corner of the figure illustrates the row effect and possible interactions. This graph shows the average length for all 18 samples versus the column of the mold for each cavity. The cavity numbers are indicated in the plot, and cavities in the same row of

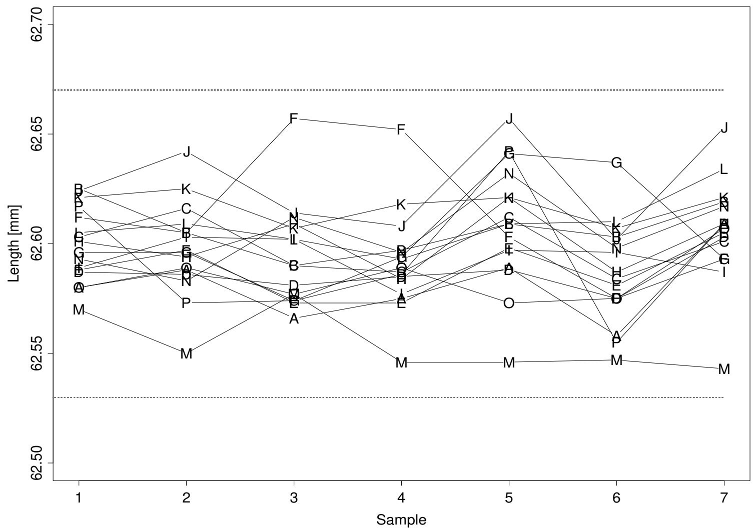

Fig. 2: Length measurements of molded parts from 16 cavity mold. The 16 cavities are labelled A-P in the plot. The two horizontal lines are the tolerance limits.

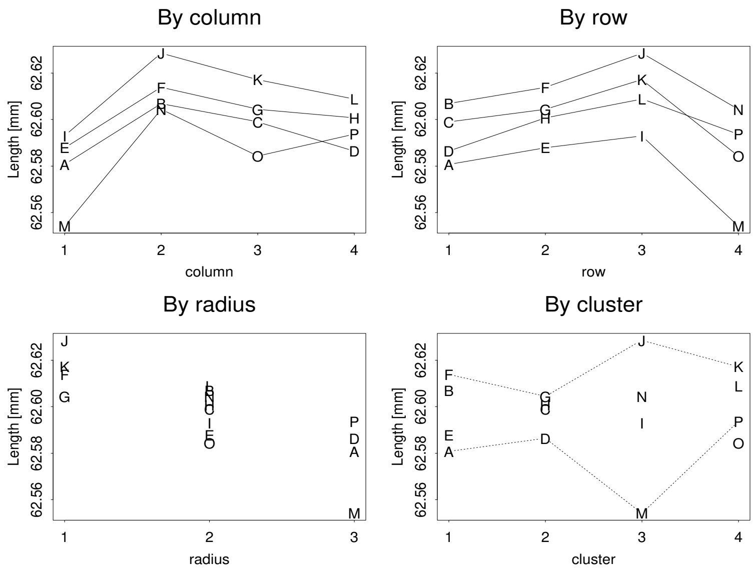

Fig. 3: Effect on the mean length from cavities positions in the mold.

the mold are connected by lines. We see a clear pattern in the observed average lengths. Parts from all rows are longer in the second column than the respective parts from the first column.

A similar pattern is observed in the plot in the upper right corner of Figure 3. Parts from all columns are shorter in the fourth row than the respective parts from the third row. The fact that most of the lines in these two plots are nearly parallel furthermore indicates that there is a systematic effect in the mean length that can be explained by the rows and columns and that any interaction between them is negligible. A possible cause of the observed patterns is that the temperature distribution of the mold is very dependent on the cooling of the mold. A non-uniform temperature distribution would mean a non-uniform pressure distribution as well, which would influence part lengths. Cooling channels would usually run in a grid of horizontal and vertical channels.

The plot in the lower left corner of Figure 3 shows the average part length versus the distance of the cavity from the center of the mold. Parts from the four cavities closest to the center of the mold, i.e. F, G, J and K, are longer than parts from the four cavities in the corners, i.e. A, D, M and P. The lengths of the remaining eight cavities falls somewhere in between. One explanation could be that the center cavities are surrounded by steel where the corner cavities have ambient air on two sides. This might cause the corner cavities to become colder which in turn would cause less cavity pressure and shorter parts.

Another effect we wanted to investigate is whether the clusters made up by the runner system had different means. If the main and secondary runners had slightly different dimensions the plastic melt would arrive at the different cavities at slightly different times. In the plot in the lower right corner of Figure 3 the length versus the cluster is depicted. The only pattern evident from this plot is that the longest part in each cluster is from a cavity in the center of the mold, and the shortest part in each cluster is from a cavity from a corner of the mold. This fact is illustrated by the two lines connecting the corner cavities and the center cavities respectively.

Figure 3 indicated a systematic pattern to the cavity-to-cavity variation. Parts from some areas in the mold had more similar average lengths than parts in general. The greatest effect was seen from the distance of the cavities from the center of the mold. If reasons are known or can be found for the suggested

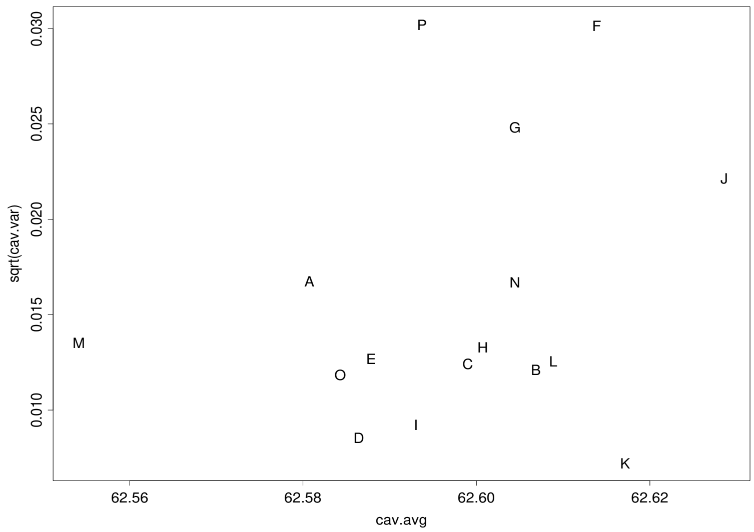

Fig. 4: Cavity standard deviation versus cavity mean.

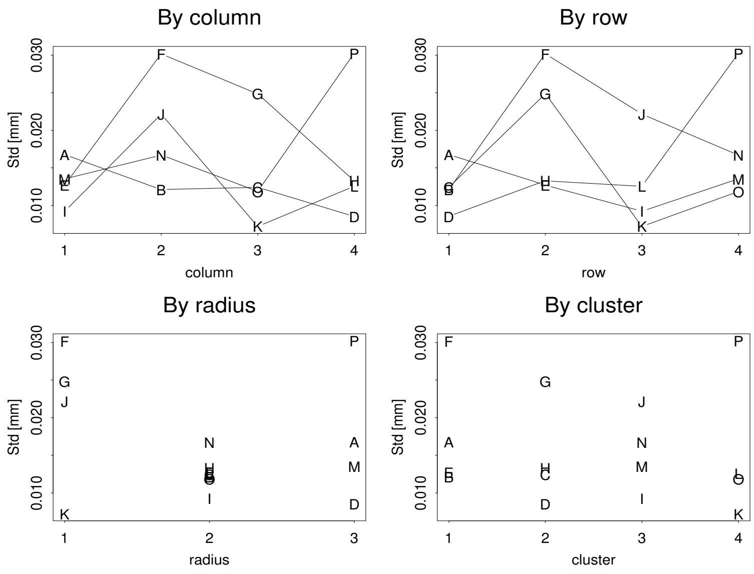

Fig. 5: Effect on the within cavity variation from cavities positions in the mold.

group behaviors, then it may be possible to adjust the causes of the differences.

Another important quality measure is the within cavity variation. In Figure 4 the standard deviation of the seven samples is shown versus the corresponding mean length for each of the 16 cavities. From this figure it seems there is no relation between the standard deviation and the mean length. Therefore, we want to investigate possible patterns in the within cavity standard deviations of the seven samples analogous to the analysis for the cavity mean. In Figure 5 the same plots are made on the within cavity standard deviation. However, from this figure no distinct patterns leap to the eye, indicating that the within cavity variation does not depend on the cavities position in the mold. Parts from cavities P and F are seen to have the greatest standard deviations, but no physical attributes of these two cavities differentiate them from the rest of the cavities.

3 Multiple stream processes

Measurements from multiple streams posses some inherent features that distinguish them from single streams. A multiple stream process consists of several identical parallel process streams. At sample time, measurements are obtained from each stream (or a subset of the streams). The measurements are in the same units and will usually have the same target value.

Other examples of multiple stream processes could be measurements from different spindles or filling heads. But also measurements of identical features on a single part such as the vanes on a compressor or impeller have the characteristics of a multiple stream process.

3.1 Multiple stream process model

From the observations made in the analysis of the data in the last section a natural process model would be a two-way analysis of variance model. However, other or no patterns at all may arise from analysis of other molds with different designs. A more general process model is needed if it shall serve a more general purpose. In the following model the process streams are modelled as fixed differences,

$$Y_{tj} = \mu_j + A_t + e_{tj} \tag{1}$$

where μ_j is the mean of the jth process stream, A_t is a normally distributed random variable with mean 0 and variance σ_A^2 that represents the variation common to all streams at time t, and e_{tj} denotes independent, normally distributed random variables with mean 0 and variance σ^2.

To detect a change in one stream relative to the others, one approach is to chart the difference of the average value of all streams and the value of each individual stream, i.e. $Y_{tj} - \hat{\mu}_j$. This is sometimes referred to as analysis of residuals (Ott and Snee (1973)). This would indeed be a suitable approach for the data in the last section. However, if the cavity-to-cavity variation cannot generally be regarded as fixed differences, an even more general process model is needed.

A more general multiple stream process model was suggested by Mortell and Runger (1995). Assume that there are s streams and let Y_{tj} denote the observation from stream j at time t. With subgroup size $n = 1$ the in-control model is

$$Y_{tj} = \mu + A_t + e_{tj} \tag{2}$$

where μ denotes the process mean, A_t is a normally distributed random variable with mean 0 and variance σ_A^2 that represents the variation common to all streams at time t, and e_{tj} denotes independent, normally distributed random variables with mean 0 and variance σ^2. The stream differences represented by μ_j in model (1) are no longer modeled separately but are now included in the e_{tj} term in model (2).

The quantity A_t can be interpreted as representing those characteristics of the product and equipment at time t that are common to all streams, such as viscosity or set injection time. And e_{tj} can be interpreted as representing deviations of the jth stream from the common characteristics caused by assignable causes such as pressure differences and clogged air vents.

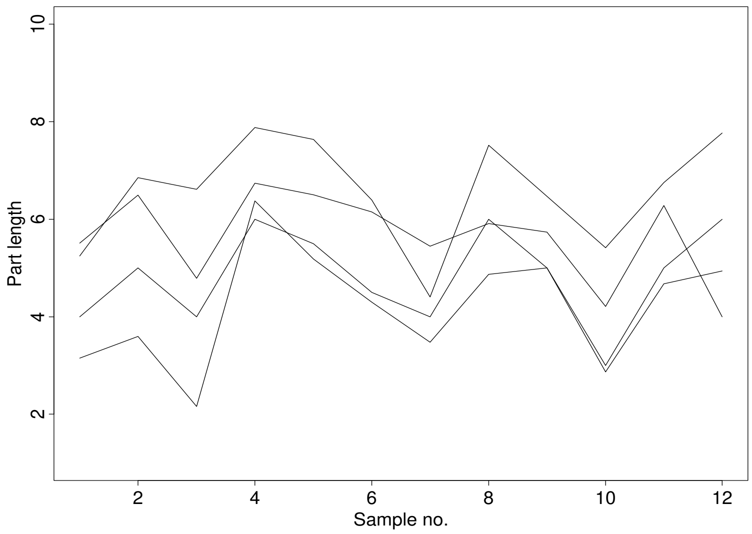

Fig. 6: A single stream deviates from the others at sample 12.

4 Monitoring multiple stream processes

When monitoring a multiple stream process, it is important to detect any assignable cause, and distinguish between assignable causes that affect all streams and assignable causes that affect only one or a few streams. Typically these two types of assignable causes have different root causes, and the distinction facilitates the identification of the problem and the solution.

A separate control chart for each stream might be used in some cases, but such charting is not sensitive to the type of assignable cause shown in Figure 6. Measurements from either stream are not unusual by themselves, but in sample 12 the observation from one of the stremas is very unusual in relation to the other process streams.

It would be desirable to have a chart that simultaneously monitors the data to detect (1) an assignable cause that shifts the mean level of all of the streams over time (called an overall assignable cause) and (2) an assignable cause that changes the mean of one or more streams relative to the remainder (called a relative assignable cause). In the process model (1) an overall assignable cause will affect the A_ts and a relative assignable cause will affect the e_{tj}s.

4.1 Traditional monitoring of multiple stream processes

The problem of conventional charting of multistream processes was discussed by Bajaria and Skog (1994). In injection molding there are numerous ways of monitoring the quality of product from a multicavity mold. Wenniges and Potente (1995) suggest tracking only the worst cavity. The worst cavity thus becomes a quality-determining or representative cavity for the entire mold. While Golmanavich and Nielson (1993) and Rauwendaal (2000) recommends keeping a separate chart for each cavity.

Charting each individual stream makes it difficult to detect any relative assignable causes. And furthermore, when σ_A^2 is large relative to σ^2, an individual control chart for each stream is ineffective in detecting a shift in a single stream of magnitude σ. This is because the control limits for Y_{tj}, for each stream j, need to include the variability of A_t, σ_A^2. The control limits for Y_{tj} are:

$$\mu \pm 3\sqrt{(\sigma_A^2 + \sigma^2/n)} \tag{3}$$

and these variance components are estimated from the available data. A shift in a single stream, that is, a shift in the mean of e_{tj} is difficult (nearly impossible) for the control chart to detect when σ_A^2 is large relative to σ^2. Consequently, in this case, the singular part of a group control chart is quite ineffective. In Figure 6 the assignable cause that occurs at time 12 is very difficult for the individual control chart of any stream to detect.

4.2 Monitoring using the group control chart

A classical approach for monitoring multiple process streams is the so-called group control chart. See Nelson (1986) and Mortell and Runger (1995) for a discussion of the group control chart.

After taking a sample from each of the streams, the maximum and minimum sample means are plotted. If both of these points are within a set of control limits, then clearly the means from all the other streams are also within these control limits. On this control chart, the control limits are based on the standard deviation of the subgroup mean from only one product stream.

If a particular stream generates the highest (or lowest) reading for r consecutive samples, then the stream is signaled to be off-target. The value selected for r is a trade-off between the risk of false alarms produced by the chart and the probability of detecting an assignable cause. Consequently, the group control chart aims at facilitating a parallel attack on both types of assignable causes, which in turn can lead to process improvements in multiple process streams.

The part of a group control chart that signals whenever a particular stream is an extreme in r consecutive samples is referred to as the runs control scheme. The average run length (ARL) of the runs control scheme for an in-control process is often calculated using the following formula from Nelson (1986):

$$ARL1 = \frac{s^r - 1}{s - 1} \tag{4}$$

where s is the number of product streams and r is the critical number of consecutive times that a particular stream is the maximum (or minimum) value. The notation "ARL1" indicates that it is a one-sided result in the sense that the ARL is for the event that a stream is the maximum (or minimum) in r consecutive samples.

However, Mortell and Runger (1995) points out some major disadvantages of the group control chart. For example, that an out of control signal is based on consecutive high (or low) readings from a single stream. Consequently, if more than one stream shifts, and if the shifts furthermore are approximately of the same magnitude, a single stream does not dominate and the detection of this assignable cause is uncertain.

4.3 Monitoring using principal components

Runger et al. (1996) suggest using principal component analysis to construct a control chart for multiple process streams. Their approach is to establish a control based on two control charts. One chart to detect disturbances from overall assignable causes (disturbances in the A_t term), and another to detect disturbances from relative assignable causes (disturbances in the e_{tj} term).

The first principal components variable (PCV) is the linear combination of the vector $\boldsymbol{Y_t}$ that has the greatest variance. The second PCV is the linear combination of $\boldsymbol{Y_t}$ that has greatest variance among linear combinations with coefficient vectors orthogonal to the first. The remaining PCVs are defined similarly. Essentially, PCA is a statistical technique for transforming a set of s random variables into a new set of s variables (PCVs) that are mutually independent, where each PCV is a linear combination of the original variables. See, for example Jackson (1991) for a deeper discussion of principal components.

Let the covariance matrix of $\boldsymbol{Y_t}$ be denoted by Σ. The coefficients of the linear combination for the ith PCV are obtainable from the eigenvector (normalized to unit length) corresponding to the ith largest eigenvalue of Σ. The eigenvector is referred to as the principal component direction.

The multiple stream process application is a special case of a multivariate process control problem in which the covariance matrix of the observed data has a special form. Given the model (2)

$$\Sigma = \sigma_A^2 \mathbf{1}\mathbf{1}' + \sigma^2 \boldsymbol{I} \tag{5}$$

where $\boldsymbol{I}$ is the $s \times s$ identity matrix and $\mathbf{1}$ is a vector of s ones. The eigenvalues of Σ are $\sigma_A^2 + \sigma^2$ (of multiplicity one) and σ^2 (of multiplicity $s-1$). The corresponding eigenvectors are $\mathbf{1}(1/\sqrt{s})$ and any set of $s-1$ orthonormal vectors that are orthogonal to $\mathbf{1}$.

Let the vector of PCVs be denoted as $\boldsymbol{U}$ and let U_1 and $\boldsymbol{U}_2$ denote the first PCV and the vector of the last $s-1$ PCVs respectively. One control chart can be based on $U_1 = \mathbf{1}'\boldsymbol{Y}/\sqrt{s} = \sum_{i=1}^{s} Y_i/\sqrt{s}$. This chart is essentially the average of all the streams at time t and is sensitive to an assignable cause that affects all streams. U_1 can be shown to be proportional to $\mathbf{1}'\Sigma^{-1}\boldsymbol{Y}$. Consequently, from a multivariate control chart perspective, U_1 is the chart recommended by Pignatiello and Runger (1990) and Hawkins (1993) for detecting an equal shift in all variables.

Let the $s \times s$ matrix $\boldsymbol{G}$ have columns equal to the eigenvector of Σ and let $\boldsymbol{G}_1$ and $\boldsymbol{G}_2$ be the $s \times 1$ and $s \times (s-1)$ matrix consisting of the first and last $s-1$ columns of $\boldsymbol{G}$, respectively. Then, $\boldsymbol{U_2} = \boldsymbol{G}_2{}'\boldsymbol{Y}$. From standard results for PCA, the covariance matrix of $\boldsymbol{U_2}$ can be shown to be $\sigma^2\boldsymbol{I}$.

A second control chart can be based on the remaining PCVs. Because the PCVs are based on orthogonal eigenvectors, an assignable cause that shifts the mean of $\boldsymbol{Y}$ along the vector $\mathbf{1}$ has no effect on a control chart created from the $s-1$ remaining PCVs.

A standard method of combining several variables in a control chart is to use the chi-square chart developed by Hotelling (1947). For $\boldsymbol{U_2}$ the chi-square chart is

$$\boldsymbol{S^2} = \boldsymbol{U_2}'(\sigma^2\boldsymbol{I})^{-1} = \boldsymbol{Y'}\boldsymbol{G}_2\boldsymbol{G}_2'\boldsymbol{Y}/\sigma^2 \tag{6}$$

The matrix $\boldsymbol{G}_2\boldsymbol{G}_2'$ is the orthogonal projection onto the subspace of the last $s-1$ principal components. From standard results in linear model analysis it follows that

$$\boldsymbol{S^2} = \boldsymbol{Y'}(\boldsymbol{I} - \boldsymbol{G}_1\boldsymbol{G}_1')\boldsymbol{Y}/\sigma^2 = \sigma^{-2}(\boldsymbol{Y'Y} - \boldsymbol{Y'}\mathbf{1}\mathbf{1}'\boldsymbol{Y}/s) \tag{7}$$

and we obtain the result, that

$$\boldsymbol{S^2} = \sigma^{-2}\sum_{i=1}^{s}(Y_{ti} - \bar{Y}_t)^2 = \sigma^{-2}\boldsymbol{H^2}. \tag{8}$$

Consequently the S^2 chart at time t is simply proportional to the variance between streams obtained from Y_t at time t. This between the streams variance is denoted $\boldsymbol{H^2}$. Clearly, S^2 is unaffected by an assignable cause that shifts the mean of all streams equally.

The approach described above separates the s measurements into two single variables, $\bar{Y}_t$ and S_t^2. These two variables are charted separately, and therefore the usual performance enhancing measures can be taken, for example CUSUM, EWMA, etc..

Furthermore, under the proposed process model $\bar{Y}_t$ and S_t^2 are independent. This makes evaluation of the in-control performance of a control based on the two charts much simpler. Furthermore the principal components monitoring meet the objective of partioning the control problem into statistics that are sensitive to the two types of assignable causes.

5 Capability Index for Multicavity Molds

Capability studies offer an excellent opportunity to quantify the potential ability with which a process can meet customer requirements. Once a process is in a good state of statistical control, its ability to meet specifications can be predicted. One popular measure for assessing this capability is called the C_{pk} index, which is defined as

$$C_{pk} = \min\left(\frac{\mu - LSL}{3\sigma}, \frac{USL - \mu}{3\sigma}\right) \tag{9}$$

where LSL is the lower specification limit and USL is the upper specification limit.

Imagine a mold with only two cavities. For the key characteristic of part length the first has a C_{pk} index of 2 and the second has a C_{pk} index of 0. This situation is illustrated in Figure 7. Because the average of the second cavity is positioned directly on the upper specification limit 50 % of the parts from this cavity will be non-conforming. A C_{pk} index of 2 indicates that the average fill volume for head 1 is at least 6σ from either specification limit. At this distance only 0.002 ppm of the parts are non-conforming. Thus, the aggregated output has an actual overall average of 25 % non-forming parts.

The arithmetic average of the two indices is 1 implying that the average part length is at least 3σ from either specification limit, leading customers to believe that the combined output of the two cavities will have a maximum of 0.27 % non-conforming parts. This does not agree with the 25 % non-conforming part

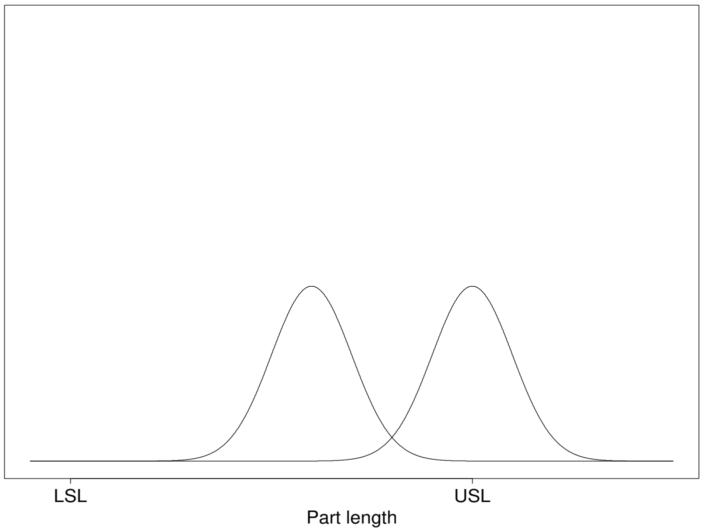

Fig. 7: Output distributions of 2 cavities.

on average that we found above. Thus assigning an average C_{pk} of 1 to the output of these two cavities would be quite misleading.

As an alternative, some quality practitioners combine measurements from all process streams to compute an overall average and standard deviation. This approach artificially inflates the overall standard deviation because the variation between streams is combined with the within-stream variation. In Figure 8 an example of this is shown. Two cavities are both producing parts well within specification limits.

The combined output of the two cavities forms a bimodal distribution. By mixing parts from the two output distributions and assuming that it is a normal distribution one risks to seriously overestimate the percentage of non-conforming parts from the process, which would penalize the manufacturer without reason.

Instead, Bothe (1999) suggests an alternative method for computing the capability index. Using an average of the percentage non-conforming from each cavity the resulting index is equivalent to the conventional C_{pk} index with respect to the percentage of non-conforming parts.

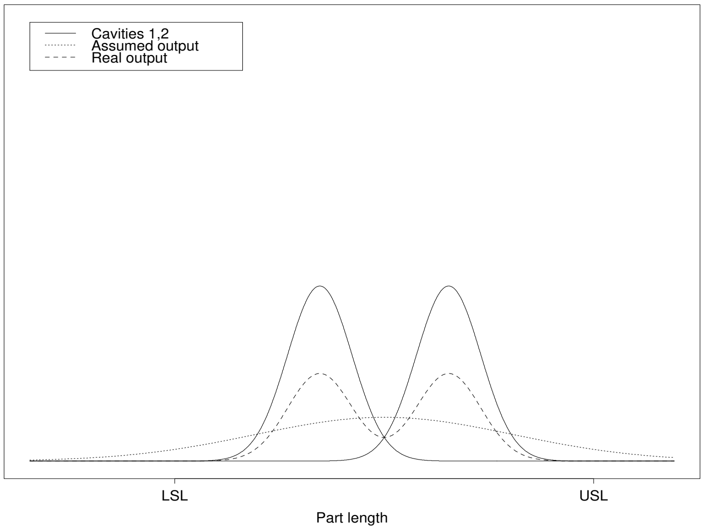

Fig. 8: Combined output distributions of 2 cavities.

To determine an overall C_{pk} of a process correctly with multiple process streams, the first step is to estimate the percentage (or ppm) of parts out of specifications for each stream. The percentage of parts above the upper specification limit is

$$\hat{p}_{USL} = \frac{\sum_{i=1}^{s} p_{a,i}}{s} \tag{10}$$

where $p_{a,i}$ is the percentage above upper specification limit for cavity i, and s is the number of cavities.

This average is an estimate of the percentage of too long parts a customer will receive and is therefore an indication of the machines ability to mold parts below the upper specification limit.

From this overall percentage, $\hat{p}_{USL}$, the corresponding quantile in the normal distribution is computed and denoted Z_{USL}. The same is done for the lower specification limit, and the minimum of these two Z values is found,

$$Z_{min} = \min(Z_{LSL}, Z_{USL}). \tag{11}$$

Then the average C_{pk} index is defined as Z_{min} divided by 3,

$$\bar{C}_{pk} = \frac{z_{min}}{3} \tag{12}$$

An advantage of this measure of aggregate capability is that it does not require the output of all process streams to have the same average, nor even have identical process spreads. That the process spreads may be unequal is relevant for mulitcavity molds because they are usually not equal. In addition, this index has no stipulation that demands the output for each stream to have a normal distribution. If one has a non-normal distribution, a distribution can be estimated and the percentage of parts outside each specification limit estimated.

6 An example

In the production of parts used in the medical device discussed in section 1 a 16 cavity mold is used. This is not the same mold analysed in section 2, but one based on the same design principles. The sampling frequency in the process control is 16 of every 9000 parts produced.

6.1 Monitoring

The monitoring scheme proposed in section 4.3 will be applied to this situation. For monitoring the mean of all cavities the average of all 16 cavities will be used. And to detect a shift in one or a few of the cavities a H^2 chart will be used. The H^2 chart is proportional to the S^2 chart as shown in (8).

Data from 206 process control samples is available. In the time interval covered by these samples, the process settings have not been changed. From these 206 samples the principal components have been computed. The loadings of the first principal component is displayed in Figure 9.

It was argued that the first principal component would be the average of all cavities. This is seen to agree very well with what was observed. The loadings of the first principal component are approximately the same size, which means

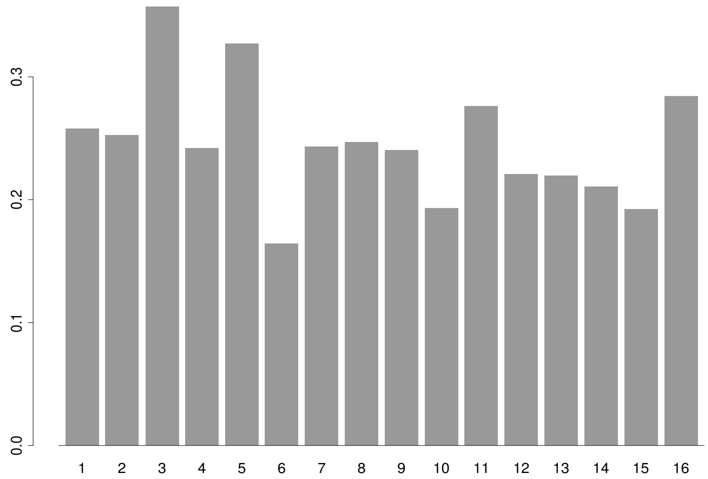

Fig. 9: First principal component loadings.

that every cavity is given the same weight. Thus, the average of the 16 cavities will be used to detect a change in the mean of the process. The successive values of the average of all cavities for all 206 samples is shown in Figure 10. The control limits for this chart should be based on variation over time, and not on the variation from stream to stream.

For detection of a shift in the output of one or a few cavities a H^2 value is computed for every sample and plotted in a chart

$$H^2 = \sum_{i=1}^{16} (Y_{ti} - \bar{Y_t})^2 \tag{13}$$

where Y_{tj} is the measurement of cavity j in sample t, and $\bar{Y_t}$ is the mean of all 16 cavities from sample t.

In Figure 11 data from the 16 cavities have been plotted in a H^2 chart. Sample number 99 is seen to give the largest H^2 value. In Figure 12 the process control measurements are displayed for samples 50-100. From this figure it can be seen that for sample 99, the parts from both cavities D and E are very long.

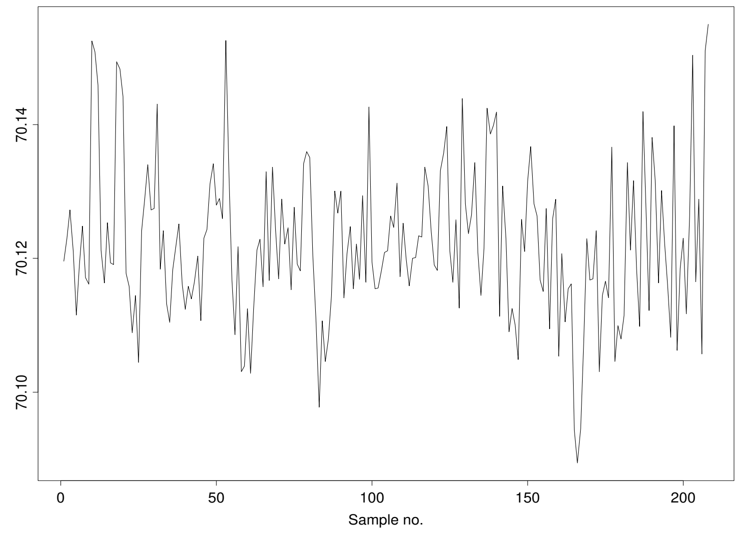

Fig. 10: Average length of parts from all cavities.

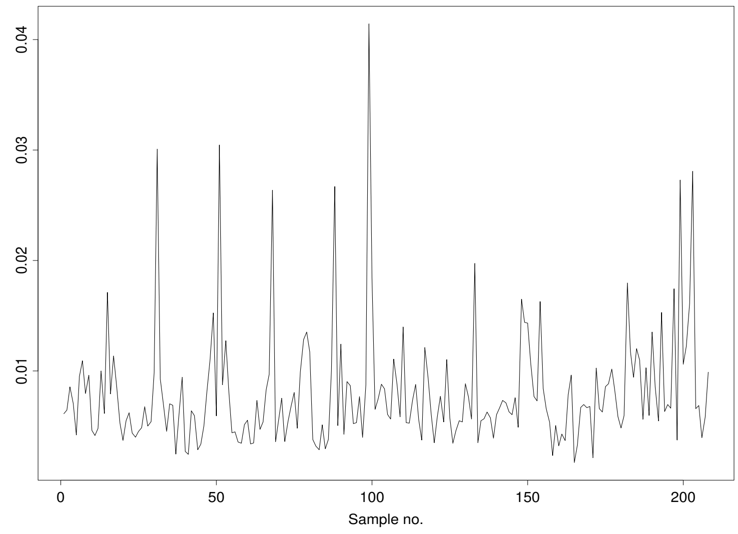

Fig. 11: S^2 chart.

Therefore it seems reasonable that this sample leads to the greatest H^2 value. However, a group chart would not detect this.

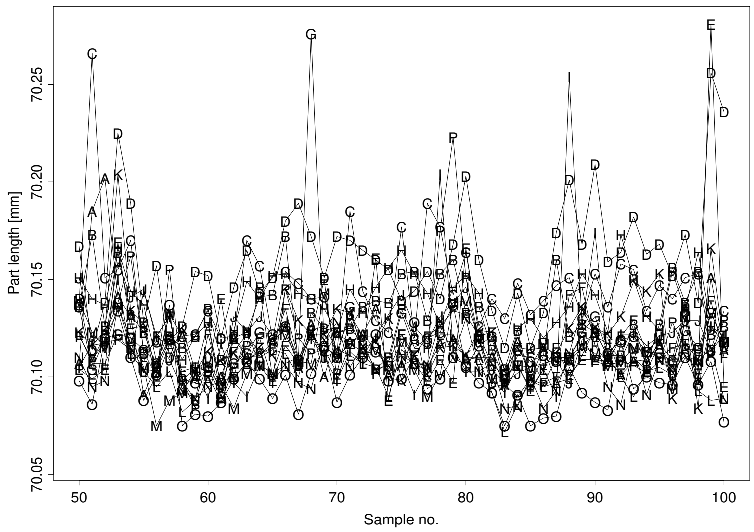

Fig. 12: Individual process control measurements.

6.2 Computation of Capability Index

To determine the overall C_{pk} index for this process, first the percentage of parts outside each specification limit is estimated for each stream. Let X_i denote the output distribution for cavity i. X_i is assumed to be normal distributed, i.e. $X_1 \in N(\mu_i, \sigma_i^2)$. Then the percentage of parts below the lower specification can be computed as

$$P\{X_i \leq LSL\} = \Phi(\frac{\mu_i - LSL}{\sigma}). \qquad (14)$$

Similarly the percentage of parts above the upper specification limit can be computed as

$$P\{X_i \geq USL\} = 1 - \Phi(\frac{USL - \mu_i}{\sigma}). \qquad (15)$$

Let us assume that for the quality data displayed in Figure 12 the specification limits are $LSL = 70.04$ mm, and $USL = 70.20$ mm respectively. We will

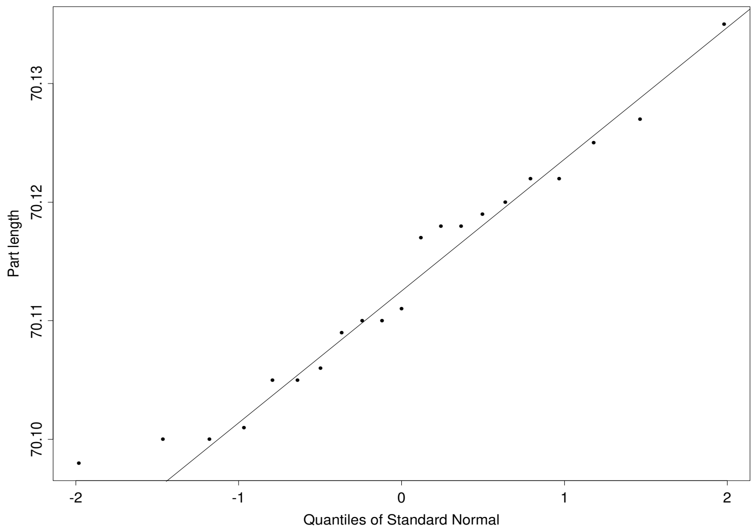

Fig. 13: Normal probability plot of length measurements from samples 60-80 for cavity 1.

only need the short-term variability to find the capability of the process. Between samples 60 and 80 the process is rather stable. Thus this portion of the quality data are used in the further computations. However, the extreme observation for cavity G at sample number 68 is not included in the computations.

For cavity A the mean and standard deviation are estimated as $\hat{\mu}_1 = 70.1132$ and $\hat{\sigma_1} = 0.0101$ respectively. To asses the normal assumption a normal probability plot of the length measurements of cavity A is displayed in Figure 13. The normal assumption seems reasonable.

Now the percentage of parts below the lower specification limit for cavity A can be computed as

$$\Phi(\frac{70.04 - 70.1132}{0.0101}), \tag{16}$$

and similarly the percentage of parts above the upper specification limit can be computed as

$$\Phi(\frac{70.1132 - 70.2}{0.0101}). \tag{17}$$

The same is done for the other 15 cavities, and the total and average percentage of parts below and above the corresponding specification limits can be computed. Table 1 displays these results for all 16 cavities. The overall percentage of parts below the lower specification limit is an estimate of the true process percentage non-conforming. This average is labeled $\hat{p}_{LSL}$ and is an indication of the process' ability to mold parts with length above the LSL.

Cavity	Percentage below LSL	Percentage above USL
A	0.00	0.00
B	0.00	0.00
C	0.00	1.15
D	0.00	1.44
E	0.02	0.00
F	0.00	0.00
G	0.00	0.00
H	0.00	0.01
I	0.15	0.01
J	0.00	0.00
K	0.00	0.00
L	0.00	0.00
M	0.00	0.00
N	0.00	0.00
O	0.00	0.00
P	0.28	0.70
% "out" for all cavities	0.03	0.20
Avg. % "out" for process	0.002	0.013

Table 1: Averaging the percentage non-conforming from the 16 cavities.

From these overall percentages, $\hat{p}_{LSL}$ and $\hat{p}_{USL}$, the corresponding quantiles in the normal distribution are computed. The Z_{LSL} value corresponding to a

$\hat{p}_{LSL}$ of 0.002 % is 4.11. And the Z_{USL} value corresponding to a $\hat{p}_{USL}$ of 0.013 % is 3.65. Now the average C_{pk} index is defined as the smaller of these two estimated Z values divided by 3:

$$\text{Average}C_{pk} = \frac{\min(4.11, 3.65)}{3} = 1.21. \tag{18}$$

If this is not an acceptable quality level, then the process must be improved. From examining Table 1 improvement should first concentrate on moving cavities C and D to the middle of the tolerance.

7 Conclusion

In this paper it was found that a major contribution to the variation in the quality characteristic of parts produced by a multi-cavity mold was the cavity-to-cavity variation. Moreover a clear pattern was found in this part of the variation arising from the cavities positions in the mold. If reasons are known or can be found for the suggested group behaviors, then it may be possible to adjust the causes of the differences.

It was argued that parts from a multi-cavity mold has a very special variation structure due to the cavity-to-cavity differences that should be accounted for. A simple model for the in-control variation for a multiple stream process reflecting the overall and relative variation was introduced. Based on the suggested process model a charting procedure has been introduced. The charting procedure is controlling the overall and the relative assignable causes separately.

Furthermore a capability index taking the cavity-to-cavity variation into account has been suggested. Using the average of the percentage non-conforming from each cavity the resulting index is equivalent to the conventional C_{pk} index with respect to the percentage of non-conforming parts.

Paper C

Multivariate Monitoring of the Injection Molding Process

C

Abstract

Some methods for multivariate monitoring the injection molding process variables are suggested. It has been illustrated that the multivariate analysis of process data is extremely valuable prior to monitoring the process variables. In actual process data, two principal components were found to reflect the changes in set-points very well. Furthermore two distinct levels of variation were identified. The macro variation was the variation from set-point to set-point. And the micro variation was the variation within set-points. Moreover it has been found that the variation caused by cavity differences was the greatest source of variation in the observed variation in the length measurements. Seven different set-points were included in the analysis. The variation caused by the different set point was the second greatest contribution to the overall variation.

1 Introduction

One of the main reasons for developing effective process monitoring is that the injection molding process is particularly vulnerable with respect to quality control. In some instances, stringent quality requirements force the manufacturer to test one hundred per cent of the produced item. This approach is totally uneconomic and in many instances handling of the molded components could contaminate the product and could even cause damage.

Another major problem with monitoring only the finished product is the time span between production and measurement of the samples. After being molded the parts need to become dimensionally stable before measured. This delay could cause hours of production to be outside specifications before changes to the process are being made.

The availability of cheaper and more advanced sensor technology has resulted in injection molding machines being better equipped for monitoring purposes. If focus was on monitoring the process rather than the product, this delay would not be an issue. Multivariate statistical process control is increasingly being recoginized as a valuable tool for providing early warning of process changes and also for achieving a better understanding of the process.

Another approach for better control of the process is that of predictive modeling where the quality of the molded parts are predicted from the values of the process variables in each cycle. An empirical process model is build from either a designed experiment or from online measurements, and this model is used for prediction. A lot of ongoing research is taking place in the application of neural networks to quality prediction. See for example Wortberg et al. (1997) and Petrova and Kazmer (1999).

Injection molding is subject to high frequency disturbances that often affect only one or two shots and then disappear without a trace. A typical example is localized contamination of raw materials that may affect one shot without affecting any of the shots before or after it. Random samples and measurements of parts are not well suited to detect this type of variation. A sufficiently accurate model, however, will be able to detect cycles which will lead to product of an inferior quality. Injection molding machines are often equipped with part diverters that allow monitoring computers to direct potentially defective parts to the scrap bin. This is what Berkery (1993) refers to as the exception catching strategy.

2 Injection molding process data

A hollow cylindrical part used in a medical device is produced in large quantities by means of injection molding.

For every cycle of the injection molding process 21 key process variables are recorded through sensors in the injection molding machine. The recorded variables are listed in Table 1 below. The process variables are categorized into four main types, stroke, time, temperature, and pressure.

Each variable can be grouped into one of two types, controllable or consequential. A controllable variable is set to a particular and controlled at this value within a defined tolerance band. And a consequential variable is one which cannot be set directly. It is the ultimate result of several controllable variables. The consequential variables are marked with a ’*’ in Table 1.

For every cycle of the process these 21 variables are recorded, and for every box of molded parts produced the average and standard deviation is calculated

Strokes	Times
1 Screw stroke	4 Charging time
2 Melt cushion *	5 Dwell time
3 Change over position	6 Injection time
	7 Cycle time *
	8 Cooling time
Temperatures	**Pressures**
9 Nozzle temp	18 Injection pressure, peak *
10 Barrel zone 1 temp	19 Hydr. pressure at point
11 Barrel zone 2 temp	of change over *
12 Barrel zone 3 temp	20 Back pressure, peak *
13 Mold temperature	21 Holding pressure, peak *
14 Crosstemperature	
15 Heater temp 1	
16 Heater temp 2	
17 Oil temp	

Table 1: The 21 process variables monitored in each cycle.

and stored for each variable. A box consists of all parts produced from the 12 cavity mold in approximately 100 cycles. In Figure 1 the average for all recorded process variables are shown for each of 2300 consecutive boxes produced.

3 Multivariate statistical process control

Traditionally only the quality variables are used to evaluate whether the molding process is in a state of control. However, when the process variables are monitored as well they are usually monitored in individual control charts. Beard (1999) suggests to define limits for some key variables such that molding anywhere inside the process window always will lead to an acceptable part.

However, the process performance is a multivariate property and must also be treated as such. By this it is meant that a process in control must simultaneously have the right combination of all the individual process variables. Each

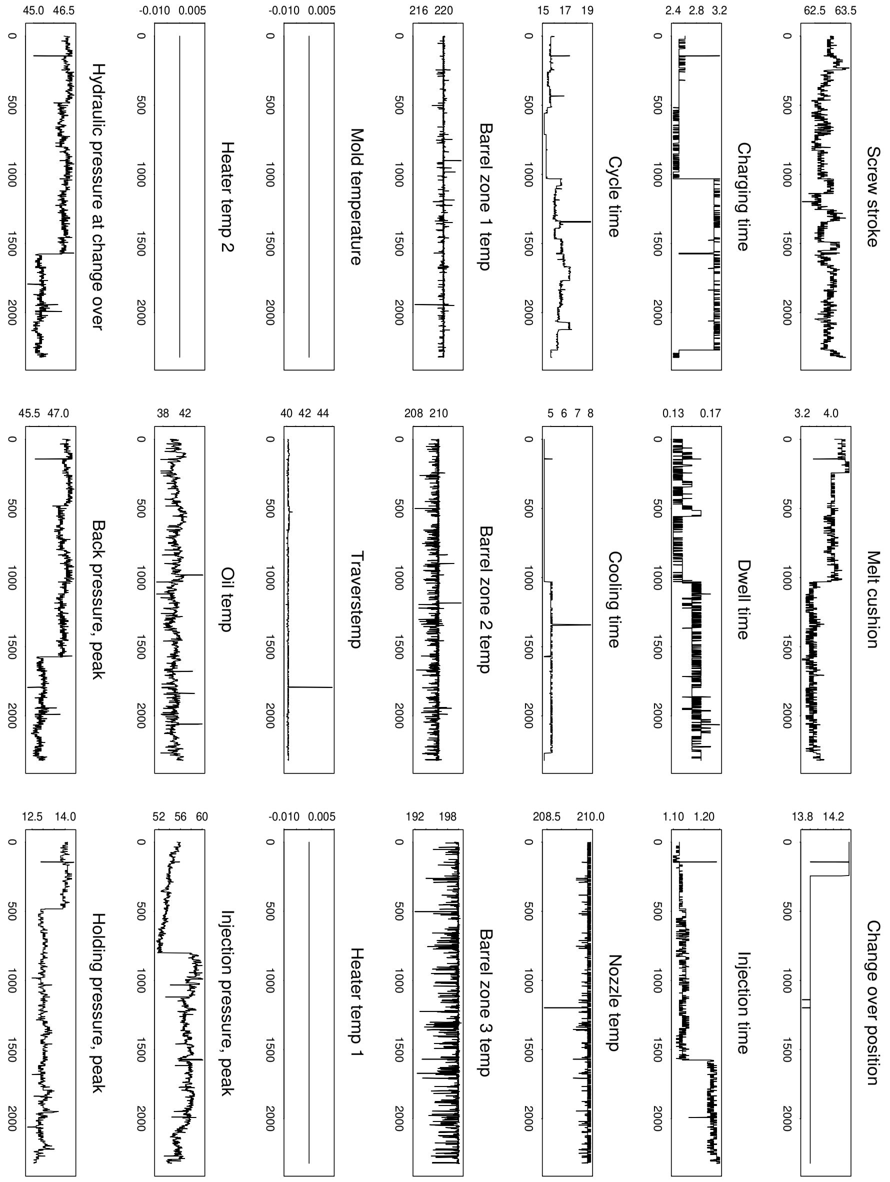

Fig. 1: Averaged process variables for 2300 consecutive boxes produced. Each box represents approximately 100 cycles on which the average is applied.

individual process variable has little meaning by itself.

In such situations monitoring univariate control charts on each of the process variables can be very misleading and hard to interpret. This is illustrated in Figure 2 with two correlated variables. The point indicated by the '$+$' is well within the control limits for each of the two variables independently. However, the indicated point is clearly unusual when the relationship between the two variables is taken into consideration.

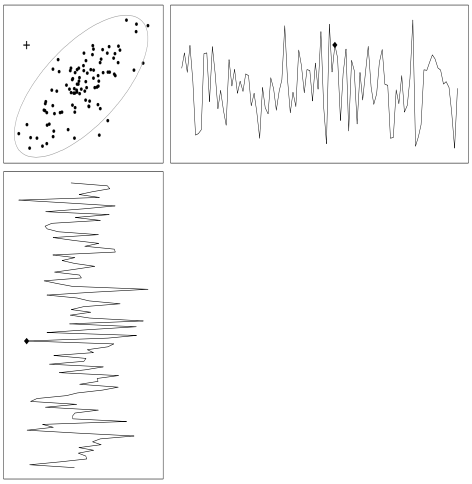

Fig. 2: Univariate and multivariate data plots.

The argument for using proper multivariate techniques for monitoring the process rather than separate univariate control charts is the same as that of using designed experiments (DOE) rather than performing experiments on one variable at a time. The presence of variable interactions in DOE leads to the same difficulties in interpreting the results of one factor at a time experimentation as does the presence of correlation among variables in interpreting univariate SPC charts. Also see MacGregor (1997) for a discussion on the multivariate nature of quality.

3.1 Principal component analysis

Often the highly correlated variables that are recorded in a manufacturing process are the result or manifestation of a relatively small number of physical characteristics of the process. Consequently, recorded measurements of the k variables often fall in a lower-dimensional subregion of k-dimensional space.

A principal component analysis is concerned with explaining the variance-covariance structure of a set of variables through a few linear combinations of these variables.

Consider a n by k data matrix, $\boldsymbol{X}$, representing n observations on each of k variables. The principal components are then defined as:

$$\boldsymbol{T} = \boldsymbol{X}\boldsymbol{P}$$

where $\boldsymbol{X}$ is the normalised data matrix, $\boldsymbol{P}$ is the matrix of coefficients (loadings) which show the relative importance of each variable to the corresponding principal component. $\boldsymbol{T}$ is the matrix of principal component scores which act as surrogates for the observations. The loadings for the individual principal components are the eigenvectors of the sample correlation matrix ($\boldsymbol{X'X}$), while the eigenvalues (λ_i) of the sample correlation matrix are a measure of the amount of variation explained by each individual principal component. For a more thorough description of principal component analysis see for example Johnson and Wichern (1998).

3.2 Traditional multivariate control chart techniques

The original multivariate monitoring chart introduced by Hotelling (1947), the T^2 chart, can be applied to a large number of variables. Unlike the ellipse format, it is not limited to two variables. Moreover, the points are displayed in time order rather than as a scatter plot, and this makes patterns and trends visible.

For the jth point, the T^2 statistic is defined as

$$T^2 = (x_j - \bar{x})'\boldsymbol{S}^{-1}(x_j - \bar{x}), \tag{1}$$

where $\boldsymbol{S}$ is an estimate of the covariance matrix, Σ.

A great advantage of the T^2 chart is that it offers a single method for indicating a general shift in the mean vector of the process. However, the T^2 statistic is not immediately interpretable and once it signals, more work is needed to diagnose which variable or variables have shifted. To cope with interpretablility problem most authors recommend use of other charts in addition to the T^2 chart.

The T^2 in (1) can also be expressed in terms of the sum of the principal component scores divided by the eigenvalue of the corresponding principal component

$$T^2 = \sum_{i=1}^{A} \frac{t_i^2}{\lambda_i} \tag{2}$$

where t_i is the principal component score for the ith principal component and λ_i is the corresponding eigenvalue. See for example Jackson (1991) for more details.

The first stage in applying PCA for process monitoring is to collect a reference data set when the process is operating under normal operating conditions and is producing good quality product. PCA is then applied to this reference set and the principal component scores of observed future process measurements are then computed using the loadings from the reference data set.

3.3 Other multivariate control charts

Other multivariate control charts are mainly variations of the T^2 statistic that Hotelling developed. However, Hotellings T^2 is based entirely on the most recent observations of the process variables and consequently the procedure signals only when a relatively large shift in the mean vector occurs. To make the monitoring more sensitive to smaller shifts in the mean application of EWMA and CUSUM methods have been proposed.

3.4 Monitoring with moving PCA

Moving PCA (MPCA), proposed by Kano et al. (2000), is based on the idea that a change of operating conditions can be detected by monitoring directions of principal components.

In order to detect a change of PCs, the reference PCs representing a normal operating condition should be defined, and the angles between the reference PCs and the PCs representing a current operating condition should be used as a basis for monitoring. The index A_i can be used for evaluating the change of PCs,

$$A_i(t) = 1 - |w_i(t)^T w_{i0}|$$

where $w_i(t)$ denotes the reference of ith PC. Both w_i and w_{i0} are unit vectors.

For applying MPCA, reference PCs and control limits must be determined. The following procedure is adopted:

1. Acquire time-series data when a process is operated under a normal condition. Normalize each column (variable) of the data matrix, i.e. adjust it to zero mean and unit variance.

2. Apply PCA to the data matrix, and define the reference PCs, w_{i0}.

3. Determine the size of time-window, w. Generate data sets with w samples from the data by moving the time-window. Apply PCA to the data sets, and calculate PCs, w_i.

4. Calculate the index A_i, and determine the control limits.

For on-line monitoring, the data matrix representing a current operating condition is updated by moving the time-window step by step, and it is scaled by using the mean and the variance obtained at step 1. Then, PCA is applied to the data, and the index A_i is calculated at each step. For updating PCs step by step, recursive PCA algorithm can be used (Li et al. (2000)) instead of using the time window.

3.5 Multivariate monitoring for autocorrelated data

Recently several methods of applying control charts to autocorrelated data are found in the literature. Alwan and Robert (1988) and Montgomery and Mastrangelo (1991) all present a similar approach to the problem of autocorrelated data. To fit an appropriate time series model to the observations and apply control charts to the stream of residuals from this model.

For multivariate process control with autocorrelated data the literature is still very sparse. Runger (1996) suggest taking a a state-space approach to the modelling.

4 Application to injection molding process

To study the multivariate nature of the process variables displayed in Figure 1, a principal component analysis is applied. It is found that the first four principal components explain 72 % of the total variation in the data.

The loadings of the first four principal components are shown in Figure 3. The numbers below the bars in the figure are the variable numbers from Table 1. These loadings show in what directions of the 21 dimensional space that the variation mainly takes place. The first principal component is the difference between on the one hand side the average of the duration of different stages of the process, and the average of the cushion and the various pressures on the other. The second principal component is the average of the four temperatures of the molding machine.

The scores of the four principal components at time t are computed as the product of the loadings from Figure 3 and the vector of process variables at time t. Time series plots of the scores of the first four principal components are shown in Figure 4. It is noticed that principal components one and four go through changes of their levels. Both principal components one and four can be interpreted as a difference between the average of some controllable variables, and the average of some consequential variables.

That the process operates in regimes is even more clear in the scatter plots of

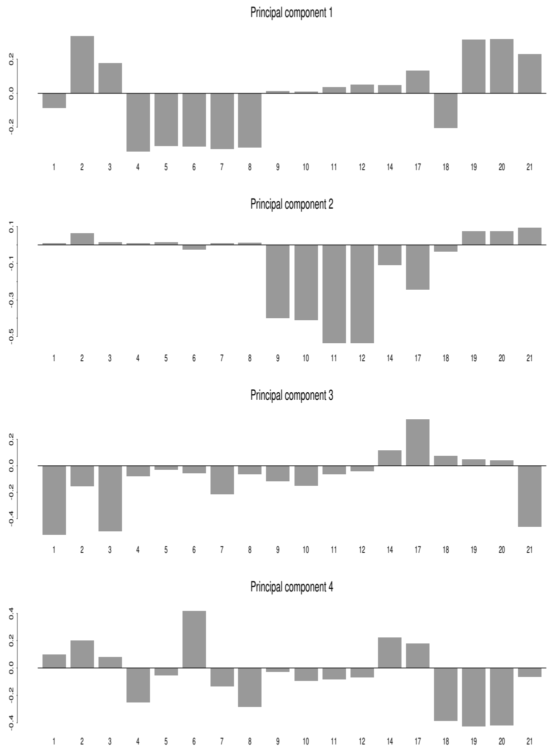

Fig. 3: Loadings of first four principal components. Variable numbers refer to numbers in Table 1.

scores from the first four principal components shown in Figure 5. Especially principal components one and four versus each other seems to give an interesting pattern. In Figure 6 the scores of the fourth principal component are shown versus the scores of the first principal component. The numbers in the figure are the corresponding box numbers, where each box is the average of 100 cycles. In Figure 6 seven distinct clusters can be identified, and it is clear from this figure that the variation of the process takes place in more than one level. The macro or cluster-to-cluster variation is caught by principal components one and four. And the more high frequency micro or box-to-box variation is the variation within clusters.

Taking a closer look into the operator log book for the molding machine, it is found that these clusters coincide with changes in the machine settings. Thus the cluster-to-cluster variation is due to manual changes in the operating con-

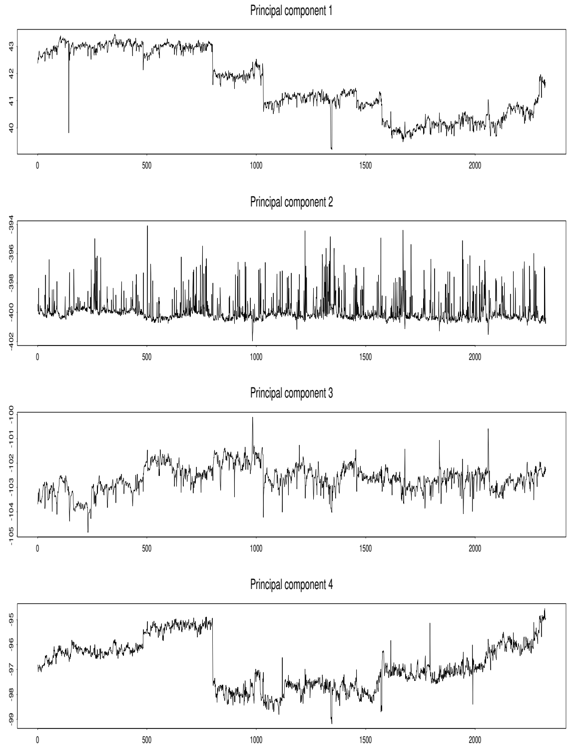

Fig. 4: Principal component scores for the first four principal components.

ditions of the molding machine. Consequently the box-to-box variation can be assigned to local variations in raw material etc. A similar pattern in the principal components due to set-point changes is reported by Weighell et al. (2000) in the study of a polymer film manufacturing process.

4.1 Monitoring within clusters

The methods proposed in the previous section could all be applied to the injection molding process. The first stage in applying PCA for process monitoring is to collect a reference data set when the process is operating under normal operating conditions and is producing good quality product. PCA is then applied to this reference set and the principal component scores of observed future process measurements are then computed using the loadings from the reference

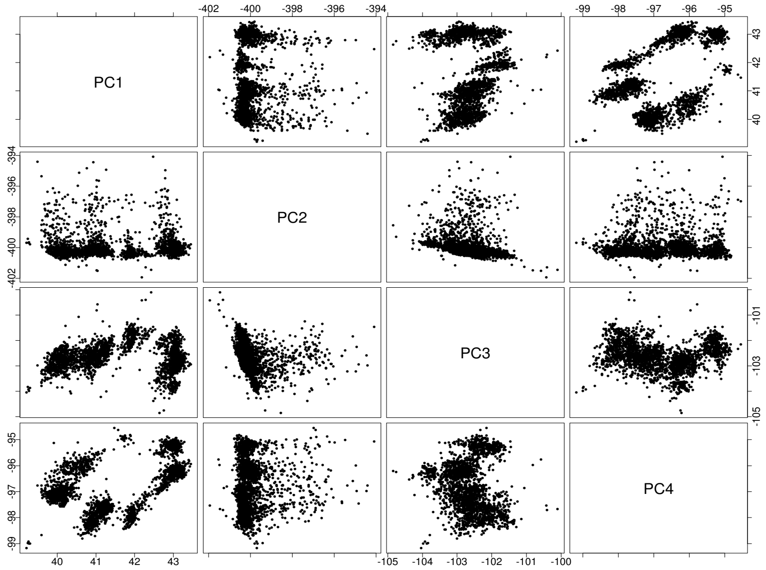

Fig. 5: Scatterplot of principal component scores for the first four principal components for all process variables in Figure 1.

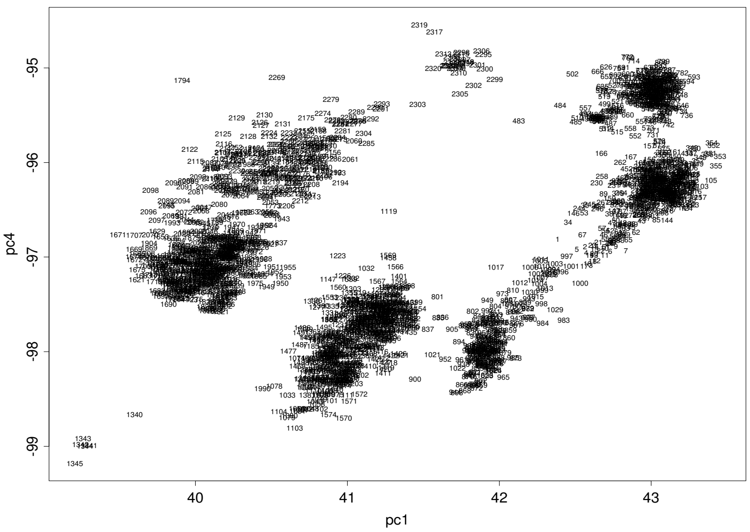

Fig. 6: Fourth principal component scores versus first principal component scores. The numbers correspond to the box number.

data set.

As reference data set the cluster in the north-east corner of Figure 6 (high scores on both PC1 and PC4) will be used. If the process is stable within set-points, such that the measured characteristics are influenced only by variations in common causes, then the principal components will exhibit similar scores for all seven set-points. The process scores are computed for all process measurements using the loadings from the reference data set. In Figure 7 the scores are shown in scatter-plots for the first four principal components.

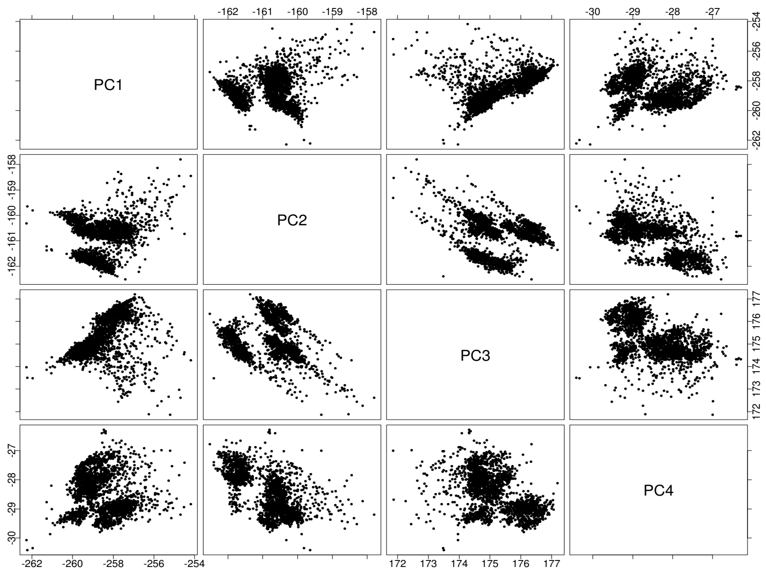

Fig. 7: Scatterplot of principal component scores for all process variable measurebents with principal components based on only one set-point.

Again some clusters related to the set-points can be identified in Figure 7. This means that within set-points the correlation between the process variables is not the same. Consequently extreme care should be applied in choosing reference data set and control limits.

5 Relationship to quality measurements

For every two boxes produced one shot of all 12 cavities is sampled and measured at the QC lab. In Figure 8 the average and standard deviation of the length measurements from the 12 cavities are displayed. All numbers are in mm. Just after box number 1000 the mean length is noticed to increase about 0.04 mm. This change coincides with a change between the two clusters with clusters with the lowest scores on the fourth principal component in Figure 6. The very high standard deviation approximately in sample 1400 is due to only one very long part that is probably a bad measurement. Therefore this observation is removed in further analysis.

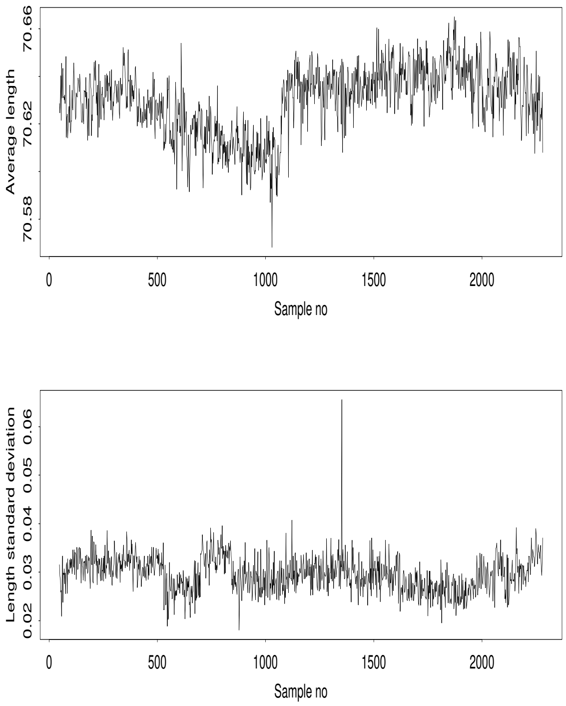

Fig. 8: Average and standard deviation of QC measurements.

With a sufficiently accurate model, the quality of the molded parts can be predicted on-line. However, because the process variables, with the exception of the mold temperature, are measured on the machine as opposed to the mold, there are limitations to the predictions they can provide. Also the difference in aggregation levels of the quality and the process variables respectively has an impact on the possibility of modeling the relationship between the process and the quality variables. The quality measurements are only one sample from more than 200 cycles whereas the process control variables are average values computed from more than 100 cycles.

Furthermore one should generally be very careful when building models based on in-control data. In Box et al. (1978) some hazards of fitting regression equations to happenstance data are discussed. Instead of building a model to be used for predicting the quality from the process variables, we will rather build models with the objective of analyzing the variation between and within set-points.

When building a model to predict the mean quality of the parts it is important to keep in mind that the variation in the process variables takes place in at least two levels. Since we are not trying to build an operational model but rather just study the variation, we will build two separate models to see what process variables are important to the macro variation and to the micro variation respectively.

First we will try to find a suitable model for the micro variation. For this purpose the cluster in the West corner of Figure 6 has been chosen. As we do not know the exact value of the process variables of the measured shot we will use the averages from the box the shot belongs to instead. As response we use the average of the 12 cavities.

A model with four significant terms gives a R^2 of 0.23. The four significant parameters are:

Screw stroke

Melt cushion

Barrel zone 2 temperature

Holding pressure, peak.

To find a model for the macro variation we choose to use only every 10th QC sample in the entire data set. However, the influence of the micro varitaion is not eliminated in this way, only reduced.

A model with four significant parameters is estimated. This model gives a R^2 value of 0.49. The four significant parameters are:

Screw stroke

Charging time

Oil temperature

Holding pressure, peak.

It may be dangerous to rely too much on the R^2 values. However, they do give a rough indication of how well the variation is explained by the explanatory variables. In the case above it is clear that the variation in the length measurement is poorly explained by variations in the process variables within the same set-point. The variation in length measurements between set-points is more correlated with the changes in process variables from set-point to set-point.

5.1 Time series model for quality data

From the analysis above it was found that the variation within set-points was not well described by the variation in the process variables. Another way of describing this micro variation is by means of times series analysis. In each of the seven set-points a times series model describing the sample-to-sample variation was found.

Figure 9 shows the standardized residuals from fitting this model, as well as autocorrelation function and partial autocorrelation function. Neither of these contradicts the model.

It is not possible to fit a reasonable ARMA model to the quality data from either of the set-points. However, an ARIMA model was found to describe the data

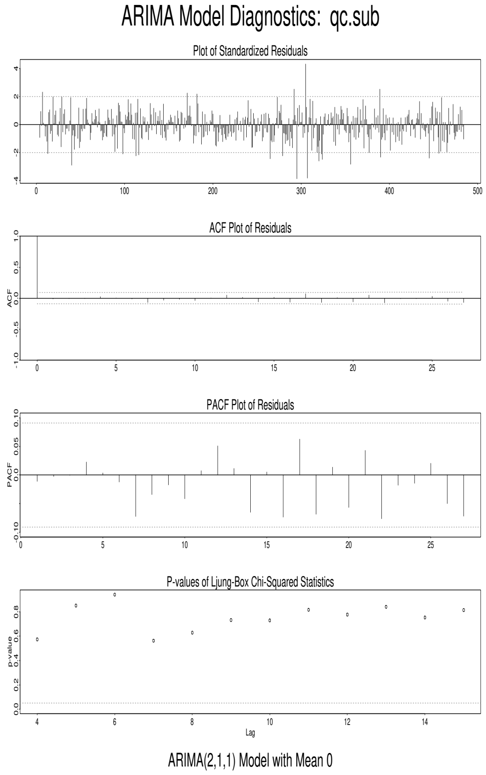

Fig. 9: Diagnostic plots for ARIMA models fitted to observations from single cluster.

in each cluster well. This indicates that the observed quality averages is not a stationary process. The correlation structure within each cluster is described well by an ARIMA(2,1,1) with very similar parameters.

6 Separation of the variation

It is of great interest to compare the different sources of variation in the part quality measurements. From the data discussed previously it is possible to identify three major sources of variation. The first is the impact from changes in the machine settings on the product quality. The second is the variation from box to box within a set-point. Finally the variation between cavities is known to be an important factor in the overall variation of the finished parts.

To assess the sources of variation the following mixed effects model is considered

$$L_{ijk} = S_i + R(S)_{j(i)} + c_k + e_{ijk} \tag{3}$$

where S_i is set-point i, $R(S)_{j(i)}$ is the jth sample within the ith set-point, and c_k is the effect of cavity k. S_i and $R(S)_{j(i)}$ are interpreted as random effects. In Table 2 the resulting analysis of variance table is displayed.

Source of variation	df	Sum of squares	Mean square	F value
Set-point	6	1.4531	0.2422	608.53
Sample within set-point	1106	1.1469	2.61	
Cavities	6.0345	0.5486	1378.46	
Residual	12232	4.8680	0.0004	

Table 2: Analysis of variance table.

To find the variance components associated with each of these sources of variation the expected mean squares must be determined. Table 3 shows the expected mean squares for model (3).

Source of variation	Expected Mean Squares
Set-point	$\sigma^2 + 12\sigma^2_{R(S)} + 1806.8\sigma^2_S$
Sample within set-point	$\sigma^2 + 12\sigma^2_{R(S)}$
Cavities	$\sigma^2 + 1113(\sum c_k^2)/12$
Residual	σ^2

Table 3: Expected mean squares table.

σ^2_S is the variation due to changes in set-point, $\sigma^2_{R(S)}$ is the variation due to samples within set-point, and $(\sum c_k^2)/12$ is the mean square for the cavity effect.

Now the variance components associated with each source of variation can be determined. The standard deviations due to set-points, samples within set-points, and cavities are shown below

$$\begin{aligned} \sqrt{(\sum c_k^2)/12} &= 0.0222 \\ \sigma_{R(S)} &= 0.0073 \\ \sigma_S &= 0.0116 \end{aligned} \tag{4}$$

The two greatest sources of variation are seen to be the variation between set-points and the variation between cavities. Samples within a set-point do not contribute much to the overall variation, indicating that compared to the changes in set-points the process is very stable. The greatest contribution to the observed overall variation is the variation between cavities. The cavity-to-cavity standard deviation is twice as big as the standard deviation of the set-point variance component

The proportion of the total variance explained by cavity differences is 46 %. For set-points and samples within set-points, the corresponding proportions are 12 %, and 5 % respectively. The residual variation accounted for the remaining 37 %.

7 Conclusion

Some methods for monitoring the process variables of the injection molding process have been suggested. The methods proposed are based on principal component analysis and take the multivariate nature of the variables into account. It is illustrated in an example how a retrospective multivariate analysis of production can facilitate a deeper understanding of the correlation structure of that data which can assist in building a proper monitoring strategy.

Two principal components were found to catch the changes in set-points very well. There was clearly two levels of variation. The macro variation was the variation from set-point to set-point. And the micro variation was the variation within set-points. Regression analysis showed that the mean length variation between set-points was better described by changes in the process variables than the variation within set-points. However, the variation within set-points was described well by a non-stationary times series model.

Based on a mixed effects model it has been found that the variation caused by cavity differences was the greatest source of variation in the observed variation in the length measurements, explaining 46 % of the total variance. Seven different set-points were included in the analysis. The variation caused by the different set-points was the second greatest contribution to the overall variation, explaining 12 %.

Paper D

Continuous Attributes Sampling based upon Moving Sums

To be submitted.

D

Abstract

This paper describes a new application of moving sum charts to continuous acceptance sampling by attributes. The procedure is especially efficient for quality requirements of very low proportion non-conformities. For the proposed charts the ARL function is derived. It is shown that in the case where a nonconforning unit is only expected very rarely during sampling, a moving sum chart and a CUSUM chart are equivalent. Furthermore is introduced a measure for the average number of units produced before a shift is noticed. This new measure is an effective tool for comparison of quality control procedures with different sampling frequencies. The moving sum method is compared to lot based acceptance plans and it is found that with a lower proportion of product inspected the moving sum plan will give the same or better assurance of quality.

1 Introduction

Acceptance sampling is being de-emphasized as a quality control tool in favor of up front process improvements. A major reason is that the essential focus of acceptance sampling is on decisions regarding the product and generally not on the process. However, there are situations encountered in manufacturing where acceptance sampling is still useful. One such situation is where authorities require acceptance procedures regarding the product. There is a need methods to assure that the quality is in compliance with the desired level. In the pharmaceutical industry this is often the situation.

For measurements on a continuous scale the so-called acceptance control charts introduced by Freund (1957) serve the dual purpose of monitoring the process and providing acceptance rules for product. However, for measurement by attributes such procedures are not readily available and often management resorts to the use of standard lot acceptance sampling plans.

In this paper we discuss various techniques that combine procedures for product acceptance with a process-oriented approach and we propose a sampling procedure aimed at both product acceptance and process control. The goal is to prevent foreseeable defects as quickly as possible, such that corrective action can be taken before the defect is built into a large amount of parts. The method proposed is a continuous sampling scheme based upon moving sums. The method is illustrated by an industrial example.

2 Continuous Sampling

Today many manufacturing operations do not result in the natural formation of lots. However, often the product is accumulated into lots for the purpose of inspection. Continuous sampling is an inspection technique to be used for in process acceptance as an element of a line producing a continuous flow of product. There is no accumulation of product and no interruption of the flow of product. In the following a brief review is given of the most important methods for performing continuous sampling.

2.1 Lot Acceptance

One common way of dealing with acceptance sampling of product from a continuous process is to group the product artificially into lots and perform ordinary acceptance sampling to sentence each lot. In this way there is no possibility of catching the error early. Usually the plan used would be taken from a standard collection of acceptance sampling.

One obvious and major disadvantage of the lot based approach is the time lag between production of the product and the inspection results. Quality information is often received too late to have any effect on the quality of the present production. Another disadvantage can be that accumulation of product into lots requires additional storage space and logistics efforts.

2.2 CSP-1

The most widely studied continuous sampling plan is the CSP-1 introduced by Dodge (1943). These sampling plans consist of alternating sequences of sampling inspection and screening (i.e. 100 % inspection) if a non-conforming unit is found during sampling. The zero-defect rule appears to be in harmony with contemporary philosophy and with quality requirements in the ppm range. The sampling plan is fully determined by the sampling frequency and the screening interval.

The CSP-1 sampling plans are indexed by their Average Outgoing Quality Limits (AOQLs). For plans with very low AOQL's, sampling and screening intervals imply intensive inspection. This is an inherent feature of the AOQL approach. Often in acceptance sampling the AOQL falls close to the point of inflection on the OC curve thus making it a rather pessimistic measure of the expected quality. Furthermore, in practice the cause of a certain defect should be caught and corrected long before the quality becomes as bad as AOQL. Because of the pessimistic nature of AOQL it is difficult to choose the appropriate plan to match specific requirements to the product quality which will often be formulated in terms of AQL.

Several modifications and extensions of the CSP-1 procedure have been proposed, see e.g. Dodge (1970). This work includes a relaxation of the zero defect rule. More recently attempts have been made to match the CSP-1 plans with lot based sampling plans, see e.g. Wasserman (1990).

The psychological aspect of the CSP-1 procedure is not to be underestimated. The fact that every time a non-conforming unit is found there is a "punishment" in form of 100 % inspection demonstrates to everyone involved in the production that quality is taken seriously and that essentially only zero defects is acceptable.

In general, the requirement of these plans to switch back and forth from 100 % inspection to sampling inspection is both costly and difficult to schedule. For manufacturing plants there are seldomly extra resources available to install on-line, 100 % inspection at arbitrary times for arbitrary periods. So resource constraints may require that product continues to be sampled with fixed frequency.

2.3 Beattie plans

A convenient inspection procedure would be to base the decision upon a modified Shewhart control chart for the number of defectives. A procedure like this would be similar to the acceptance control chart developed by Freund (1957) only for attributes data. The flow of product is accepted as long as points are within the limits of the control chart and rejected if the last point is beyond the limits. The main drawback of a procedure like this is that it would require very large sample sizes if protection against low proportion non-conformities is wanted. A plan that accumulates evidence gathered from previous samples may be preferred to one that makes a decision based only on the last sample.

Beattie (1962) proposed an acceptance sampling procedure based upon a CUSUM procedure. Beattie's procedure combines two CUSUM procedures, one for use when the product is in a zone of acceptability, and a second when the product is not acceptable. In a CUSUM procedure a sum, S, of successive values of $(d-k)$ is being accumulated, where d is the number of defective items in the sample and k is a constant for the actual CUSUM chart. The product is accepted as long as the sum of defects, S, is less than h (the "decision interval") and every time the sum becomes negative it is reset to zero. Where the Beattie plan differs from the traditional CUSUM procedure is that if S becomes greater than h the product is rejected and the cumulation restarted at $h + h'$, where h and h' are positive constants. The sampling continues and the product is rejected until S is less than h again.

A major advantage of Beattie plans over CSP-1 plans is that they do not require periods of 100 % inspection. In periods of rejection the sampling is continued until there is evidence that the quality has improved again. Still the Beattie plans are not very well known. They are for instance hardly covered in any introductory texts to statistical quality control.

3 Moving Sums used for quality control

When dealing with very low proportions of non-conformities and reasonable sample sizes we will rarely expect to get a non-conforming unit in our sample. In this case the decision interval, h, of the CUSUM will be less than one but greater than $1 - k$. This means that a single non-conforming unit in the cumulative sum will not cause the chart to signal. However, if two non-conforming units appear within a time window of h/k samples the chart signals. Due to the reset procedure the memory of the CUSUM is limited to the last h/k samples. This is illustrated in Figure 1. Because of this limited memory the CUSUM procedure is equivalent to considering the sum of the last h/k samples, and hence the CUSUM procedure is equivalent to the moving sum procedure.

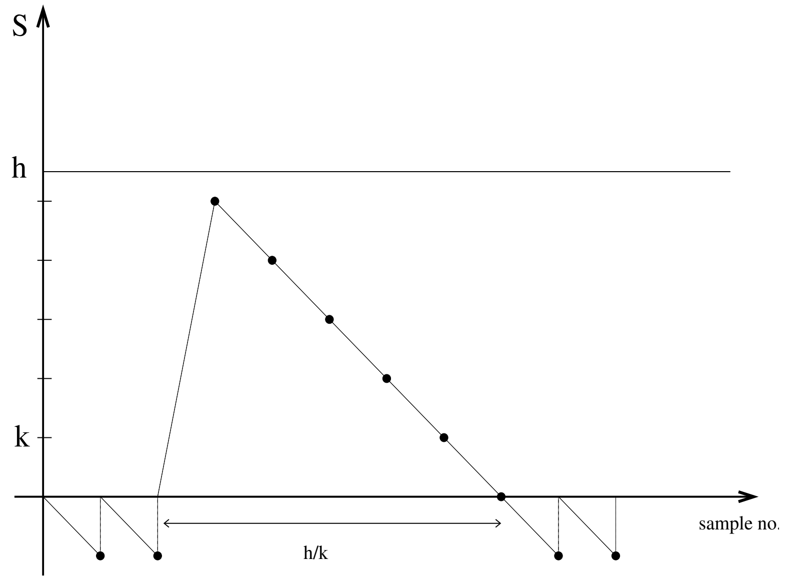

Fig. 1: Illustration of the CUSUM for the following series of non-conformities: 0,0,1,0,0,0,0,0,0,0.

The moving sum procedure is basically the same as the moving average control chart. In the moving sum the total sum of non-conforming units in r successive samples of sample size m are used for making a decision about the product that these r samples cover. If more than one non-conforming unit is present in the r samples, the product covered by these r samples is rejected. When a new sample is taken the oldest sample is no longer part of the sum.

One important aspect of the moving sum procedure is its simplicity. It is much easier for the practitioner to understand a decision rule based on the sum of the number of non-conforming units in the last r samples compared to the rule based on the CUSUM. In addition it may be more difficult to relate the value of the CUSUM to how the process is performing compared to the status of a moving sum.

3.1 ARL for the Moving Sum

In order to derive expressions for the ARL function of the proposed moving sum procedure a Markov chain representation of the procedure is introduced. The ARL function enables a comparison of the characteristics of this procedure with those of other procedures. Consider a moving sum where only one defect is allowed in the case of $r = 6$ samples. In Figure 2 an illustration of the seven acceptable states of the Markov chain is shown. Each box represents one of the last six samples, and the number written inside is the number of non-conforming units in that sample.

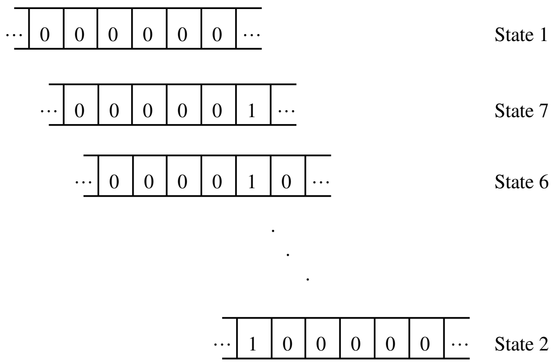

Fig. 2: Acceptable states for the Markov chain.

In general the number of acceptable states for the Markov chain will be $r + 1$, because one non-conforming item can be in any r of the samples or all samples can be conforming. The non acceptable states are all the states with two or more defects in the sum. These states are merged into one absorbing state (State 8) in the transition probability matrix shown below, where transition probabilities not shown are zero.

$i \backslash i+1$	1	2	3	4	5	6	7	8
1	p_0						p_1	$1-(p_0+p_1)$
2	p_0						p_1	$1-(p_0+p_1)$
3		p_0						$1-p_0$
4			p_0					$1-p_0$
5				p_0				$1-p_0$
6					p_0			$1-p_0$
7						p_0		$1-p_0$
8								1

With p as the non-conformance probability we have that p_0=$(1-p)^m$ is the probability of zero non-conforming units in the new sample of size m, and p_1=$mp(1-p)^{m-1}$ is the probability of one non-conforming unit in the sample.

This matrix of transitional probabilities is a so-called phase type distribution transtition matrix. From Neuts (1975) we have that more generally we can write the matrix of transition probabilities for phase type distributions as

$$\boldsymbol{P} = \begin{bmatrix} \boldsymbol{T} & \boldsymbol{t}^0 \\ \boldsymbol{0} & 1 \end{bmatrix}. \tag{1}$$

From the same source we find that the probability density of the time until absorption for a Markov chain given by the matrix (1) is

$$P\{X=n\} = \boldsymbol{\alpha}'\boldsymbol{T}^{n-1}\boldsymbol{t}^{0\prime}\boldsymbol{e} \; n = 1, 2, \ldots, \inf$$

where $\boldsymbol{e}$ is a vector of ones and $\boldsymbol{\alpha}$ the initial probability vector. The probability generating function is given by

$$R(z) = z\boldsymbol{\alpha}'(\boldsymbol{I} - z\boldsymbol{T})^{-1}\boldsymbol{t}^0, \tag{2}$$

From the probability generating function $R(z)$ the expected number of samples till absorption is found by differentiating with respect to z and putting z equal to 1. By utilizing that the row sum is equal to one, i.e. $\boldsymbol{Te} + \boldsymbol{t}^0 = \boldsymbol{e}$, we obtain

$$E\{X\} = \boldsymbol{\alpha}'(\boldsymbol{I} - \boldsymbol{T})^{-1}\boldsymbol{e}. \tag{3}$$

The expected waiting time till absorbtion is the average run length (ARL). The expression (3) may be reduced using symbol manipulating software like Mathematica. With initial probability vector $\boldsymbol{\alpha} = (1, 0, \ldots, 0)$, i.e. state 1 as initial condition, and for r samples in the moving sum the reduced expression for the ARL is

$$\begin{aligned} ARL_{ms}(p) &= -\frac{1 + (1 + \sum_{i=1}^{r-2} p_0^i)p_1}{-1 + p_0 + p_0^{r-1}p_1} \\ &= \frac{-(1 + (m(1 + \sum_{i=1}^{r-2}(1-p)^{im})(1-p)^m - 1)p)}{(1-p)^{1+m} + p + m(1-p)^{rm}p - 1}. \end{aligned} \tag{4}$$

Now for every combination of r and m it is easy to find the ARL-function with an ordinary spreadsheet. This makes it possible for the quality assurance personnel to design the plan rather easily. For a procedure based on CUSUM charts it is often necessary to make tedious simulations of the ARL-function which requires considerable more computer skills.

The ARL function has been calculated for combinations of r equal to 4 and 12, and m equal to 10 and 30. The results are shown in Figure 3. It is noticed that for sample size $m = 10$ the number of samples included is of greater importance compared to the case $m = 30$.

Next the ARL has been calculated for all combinations of r between 2 and 14 and m between 10 and 30. In Figure 4 contour plots are shown for $p = 0.01$. The contour lines naturally coincides with the r*m hyperbole. It is noticed that when more than four samples are included in the sum the contour lines becomes very flat. So including more samples in the moving sum means less compared to increasing the sample size.

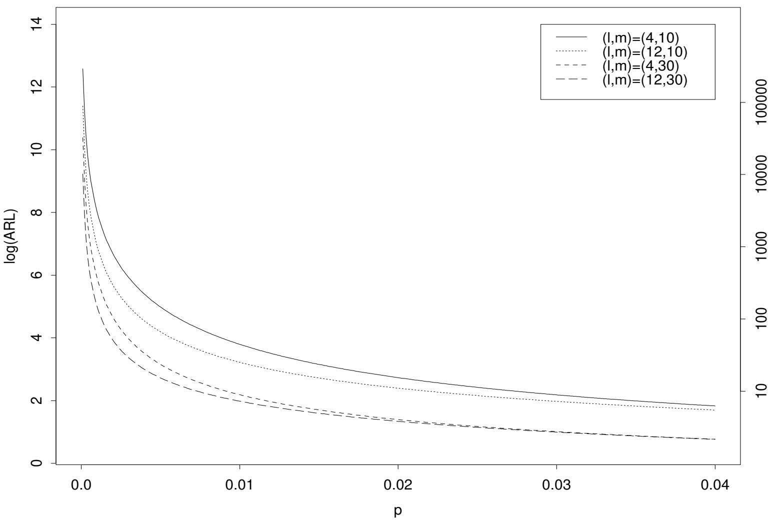

Fig. 3: Average Run Length for the moving sum.

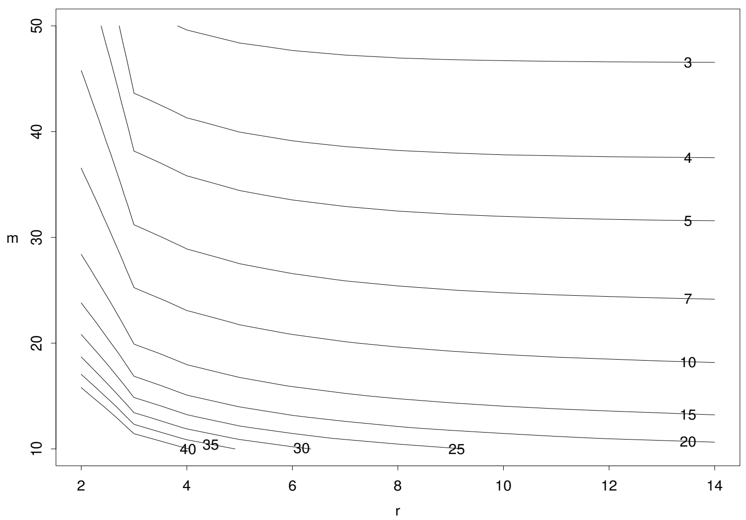

Fig. 4: Contour lines of ARL for the moving sum for p=0.01.

3.2 Delimiting the defect

When there is evidence that the process mean is greater than desired, action should be taken to adjust the process. When there is convincing evidence that the process mean is back where required, the product should be accepted again. Another problem is dealing with non-conformities in what was already produced. We believe that the solution to this problem should be dictated by the practical circumstances. One idea is to simply go back r samples from the alarm. Another method could be to adopt the philosophy of the Beattie plans and go backwards with another moving sum.

3.3 Generalizing the results

We have shown that in the case where the new sample is either zero or one the CUSUM is identical to a moving sum. This is a very important case for practical purposes. However, it is possible to write the Markov chain for any fixed number of allowed non-conforming units in the sum. When the number increases so does the complexity of the Markov chain. More work is needed on investigating the difference of a CUSUM and a moving sum procedure in the case where two or more non-conforming unit are accepted in the moving sum.

4 Example

We want to compare the moving sum procedure with a lot based acceptance plan.

A medical device is produced by means of injection molding in a continuous process. Previously the quality control has been carried out by lot sampling procedures. A particular visual non-conformance has an $AQL = 0.4\%$. The product is artificially grouped into lots of size N=10 000. From ISO 2859-1 we find sample size and acceptance number (n, Ac) = (200,2).

In this case inspection is far more costly than the cost of product. Therefore the long screening intervals needed by a CSP-1 plan are simply not realistic in this case. Instead we choose to compare the above lot based acceptance plan with a moving sum consisting of $r = 3$ samples each of size $m = 20$ with respect to how efficient they are at detecting a shift. The same proportion of product is sampled, i.e.

$$\frac{n}{N} = \frac{r * m}{M},$$

where M is the total amount of product covered by the r samples. See Figure 5 for an illustration of the symbols.

Since the sampling frequency of the two procedures are very different the usual ARL measure is not the most appropriate one to compare. Snow et al. (1992) introduced the concept of average unit run length (AURL) as a way to compare ARLs when the sample sizes differed. We choose to use the term AURL to describe a slightly different concept, viz. the average number of product produced before a shift is detected. Basically the ARL is the average number of samples taken before a sample indicates that the process mean has shifted. The ARL is defined as

$$ARL_{lot}(p) = \frac{1}{P\{\text{Rejection}|p\}}$$

where $P\{\text{Rejection}|p\}$ is the probability of rejection of independent samples. ARL_{lot} can be calculated as

$$ARL_{lot}(p) = \frac{1}{1 - OC(p)}$$

and hence for the lot based sampling the AURL becomes:

$$AURL_{lot}(p) = N * ARL_{lot}(p)$$

For the moving sum the AURL can similarly be calculated as

$$AURL_{ms}(p) = M/r * ARL_{ms}(p) = m * N/n * ARL_{ms}(p)$$

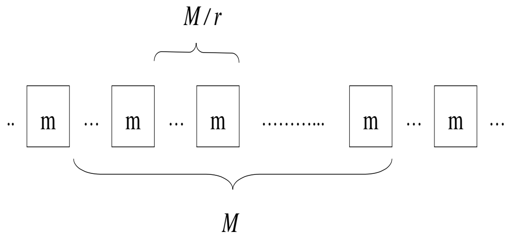

Fig. 5: M is the number of product units covered by the r samples in the moving sum, i.e. M/r is the number of units covered by one sample.

In Figure 6 the logarithm of the AURL is shown for each of the two strategies. We notice that the AURL-curve of the moving sum is completely below that of the lot based strategy. This means that for any given quality the moving sum has a lower AURL than the lot based sampling procedure. For a quality $p = 0.01$ the difference in AURL is 16357. That is in average 16357 more parts would have been produced before detection when using the lot based strategy. The price we have to pay for an earlier detection is a higher rate of false alarms.

Another strategy would be to match the AURL close to AQL of the moving sum to the AURL of the lot based strategy. By sampling less frequently the AURL of the moving sum is shifted upwards. This is demonstrated in Figure 7 where the proportion of product sampled is now 5 % smaller than it was in Figure 6. Now we find that for qualities close to 1 % we get approximately the same AURL. And for qualities greater than 1 % the moving sum will signal earlier than the lot based plan. Thus with less inspection we obtain a better assurrance when using the moving sum method compared to the standard lot based method.

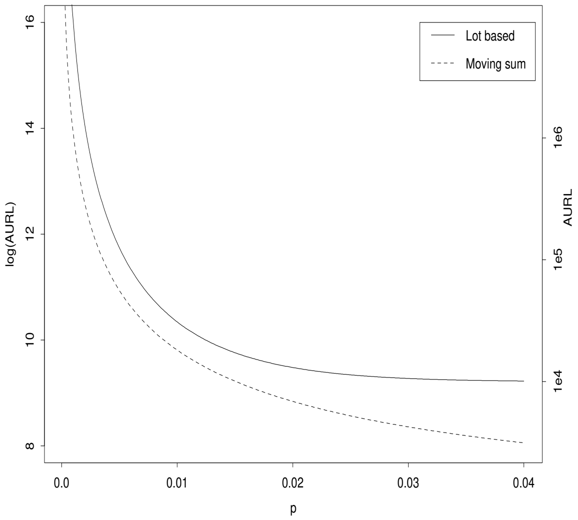

Fig. 6: AURL for same proportion sampled.

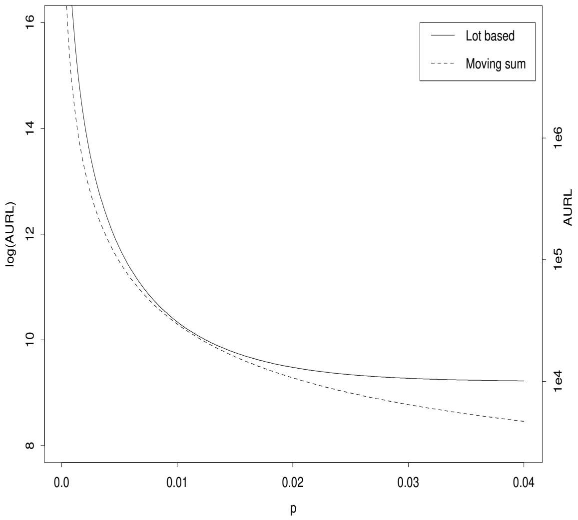

Fig. 7: AURL for moving sum with smaller proportion sampled than the lot based approach.

5 Conclusion

This paper has presented a new continuous sampling procedure based on moving sums. It was shown that in the case of very low proportion non-conformities and reasonable sample sizes the moving sum and the CUSUM procedure are exactly the same. An expression for the ARL of the moving sum was found thus making it possible for the users to design their own plans with an ordinary spreadsheet.

It was demonstrated that with less inspection effort a better quality assurance was achieved in the sense that bad qualities were detected much faster. Furthermore it was found that for comparisons between procedures with different sampling frequencies the suggested $AURL$ measure was appropriate.

We are convinced that the moving sum also more generally will perform very close to the CUSUM procedure. In the attributes case it is possible to set up Markov chains and thus to obtain expressions for the ARL with any number of non-conforming units in the sum. Also more work could be done in the area of delimiting the cause of the defect in what was already produced.

Paper E

Modeling Shrinkage with a Non-Linear Regression Model

Technical report

E

1 Introduction

The reduction in volume, that occurs when a molded part is ejected from a mold and allowed to cool, is known as shrinkage. Because of shrinkage most plastics components are smaller than the mold used to produce them. Shrinkage is a result of two factors - a normal decrease in volume due to temperature and pressure, and relaxation of the stretching caused by carbon-carbon linkages. As there are more carbon-carbon linkages in the direction of the orientated flow, there will be more shrinkage in this direction.

In multi-cavity molds, each cavity of the mold fills at a different time. This is the result of minor differences in flow paths, temperature, and gating/rheological effects. The pressure exerted on the plastic at the end of the filling process is time dependent, which means that, since each cavity fills at a different time, each final pressure will be different. Since plastic shrinkage is a linear function of cavity pressure, each cavity will shrink differently depending on when it filled.

According to most sources the majority of the shrinkage occurs during the first 20 days. However, it is impractical to wait till 20 days after the part is produced before it can be evaluated if the part meet specifications. If the part is too short and the process has not been adjusted it is likely that a great proportion of the parts produced during those 20 days are too short as well.

2 Shrinkage data

A hollow cylindrical part used in a medical device is studied in a shrinkage study. The part is produced in a multi-cavity mold with 16 cavities.

A sample of parts from all 16 cavities of a mold is measured 24 times over the first 20 days after the parts are produced. The parts are measured 14 times over the first 24 hours after production, and then once a day for the 8 succeeding days. And finally the last two measurements are made after 15 and 20 days.

The data are displayed in Figure 1. The shrinkage curves have similar shape but differ between cavities. One of the objectives of the analysis is to investigate

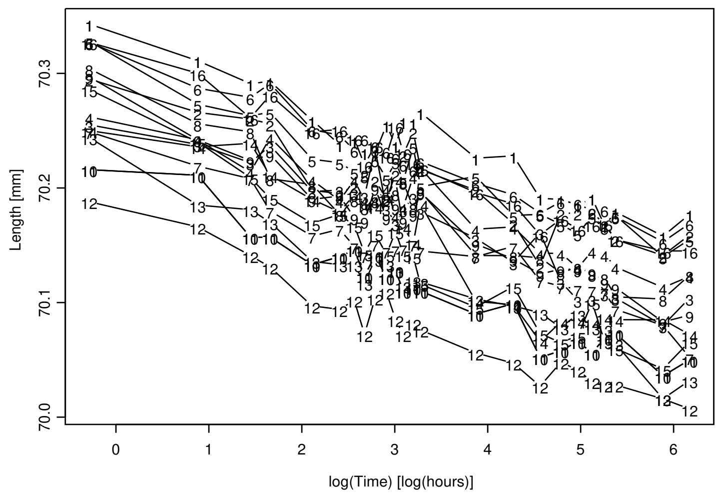

Fig. 1: Shrinkage of the length of 16 parts - one part from each cavity. Each part is measured 24 times during the first 20 days after produced.

if there are any patterns in the difference in the shrinkage curves. This result is interesting because it will indicate if the variation observed between parts from different cavities could be due to difference in shrinkage patterns.

3 Models for the shrinkage

A major reason for the interest in modelling shrinkage is to determine sensible control limits based on on a model for the shrinkage. Say, every part must meet specifications after d days. Tracing back from there using a valid model for the shrinkage will give control limits for every control point wanted. This is illustrated in Figure 2.

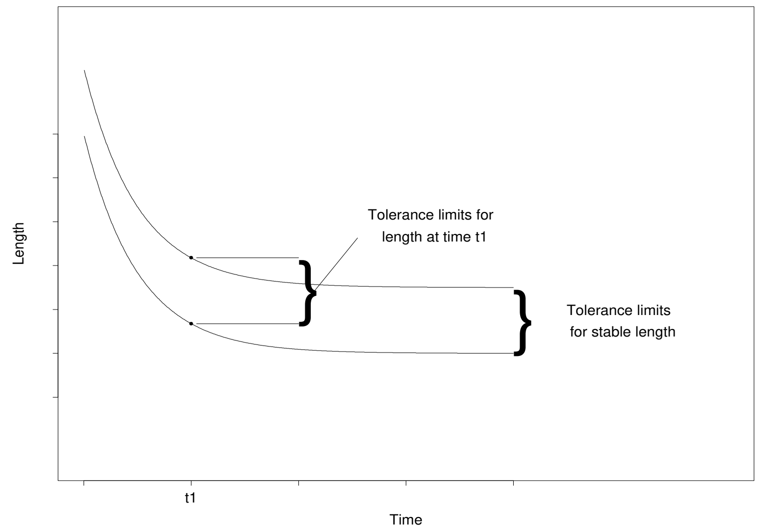

Fig. 2: Control limits based on model for the shrinkage. The dotted lines are the tolerance limits after d days.

3.1 Traditional model

Traditionally the shrinkage is described by an exponential decay model. In a previous analysis of the data it was found that the function below gave a good description of the shrinkage.

The part length at time t, $Y(t)$, can be expressed as

$$Y(t) = At^{-b} = Ae^{-b\log(t)} \tag{1}$$

where A is the length of the part at time 0 and b is the decay rate.

This function can be written as an ordinary linear regression model

$$\log Y = \log A - b\log(t) + \varepsilon$$

or in the more general notation of a general linear model

$$Y' = \beta_0 + \beta_1 X + \varepsilon$$

with $Y' = \log Y$, $\beta_0 = \log A$, $\beta_1 = -b$, and $X = \log(t)$.

A shortcoming of model (1) is the position of its asymptote

$$\lim_{t\to\infty} Ae^{-b\log t} = 0,$$

insinuating that the plastic parts eventually will shrink to length zero. This differs somewhat from both experience and our understanding of what is going on. In our case experience tells us that after approximately 14 days no more shrinkage can be observed.

3.2 Suggested model

Even though model (1) seemed to fit the data well it did not agree with our understanding of what is physically taking place. Consequently an alternative model for the shrinkage is suggested. A crucial shortcoming of model (1) is the position of the asymptote. Since it is not possible to model an asymptote with a linear model we will look in the class of nonlinear models for an appropriate model.

If we ignore the grouping of the length measurements into cavities and fit a common model to all the cavities, the length of the part, y_{ij}, at time t_j produced in cavity i is modeled as

$$Y_{ij} = c_0 + A_1 e^{-b_1 t_j} + \varepsilon_{ij} \tag{2}$$

where the error terms ε_{ij} are assumed to be independently distributed as $N(0, \sigma^2)$. The interpretation of the parameters is as follows:

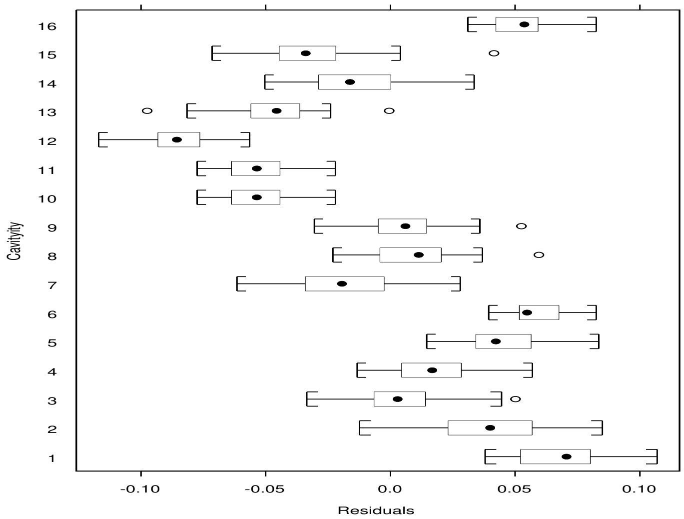

Fig. 3: Boxplots of residuals from a common model for all cavities. Each boxplot represents residuals from one cavity.

c_0 is the value of the asymptote

A_1 is the total length lost in shrinkage

b_1 is a rate constant of the exponential decay.

An exponential model like model (2) can arise as the solution of a first order differential equation and is often encountered in engineering applications.

The model is estimated using non-linear regression in S-plus. The least squares method used in the S software is described by Bates and Watts (1992). Also in Bates and Watts (1988) a more thorough treatment of non-linear regression can be found.

In Figure 3 the boxplots of the residuals by cavity tend to be mostly negative for some cavities and mostly positive for others. Because a single shrinkage curve is used for all subjects, the individual differences noticed in Figure 1 are incorporated in the residuals, thus inflating the residual standard error.

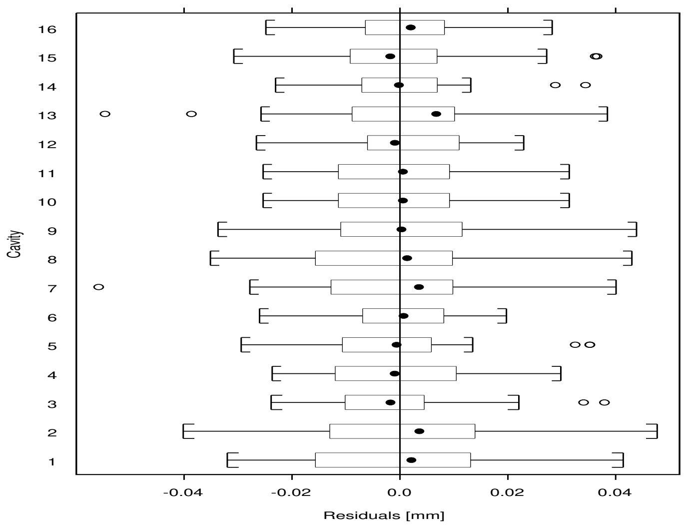

Fig. 4: Boxplots of residuals from a model with individual parameters for each cavity. Each boxplot represents residuals from one cavity.

To fit a separate exponential model to each cavity, thus allowing the cavity effects to be incorporated in the parameter estimates, we express the model as

$$Y_{ij} = c_{0i} + A_{1i}e^{-b_{1i}t_j} + \varepsilon_{ij} \tag{3}$$

where, as before, the ε_{ij} are independent $N(0, \sigma^2)$ errors.

The boxplots of the residuals by cavity, shown in Figure 4, indicate that the cavity effects have been accounted for in the fitted model.

The plot of the individual 95% confidence intervals for the coefficients in model (3), shown in Figure 5, gives an idea about their variability among cavities.

The constants A_{1i} do not seem to vary substantially among cavities, but the remaining parameters do. A new model with common A_1 parameter and individual c_0 and b_1 is consequently fitted to the data. This model is written as

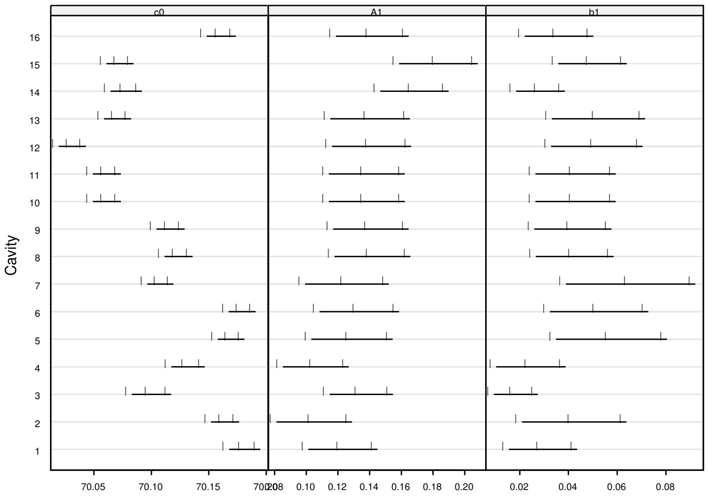

Fig. 5: Parameter confidence intervals from fitting model (3) to data.

$$Y_{ij} = c_{0i} + A_1 e^{-b_{1i} t_j} + \varepsilon_{ij}. \tag{4}$$

Source	df	SS	MS
Model (3) (Full)	336	5.957	0.0177
Model (4) (Sub)	352	6.4375	0.0183

Table 1: Residual sum of squares for models (3) and (4).

The effect of the A_1 parameter can be determined from $MS_{A_1} = MS_{sub} - MS_{full}$ with degrees of freedom, $df_{A_1} = 15$. Thus the effect of A_1 can be tested using the test statistic

$$Z = \frac{MS_{A_1}}{MS_{sub}}$$

which is distributed as $F(df_{A_1}, df_{full})$. With the present data we get $Z = 1.694$ which corresponds to the 95.4 % quantile in a $F(16, 336)$ distribution.

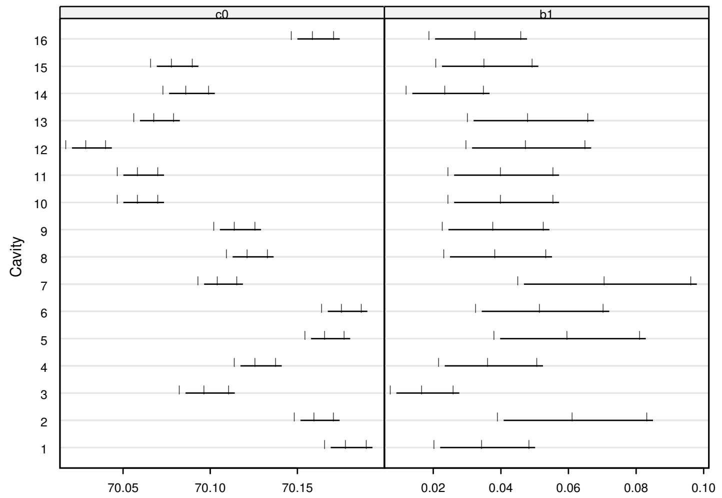

Fig. 6: Parameter confidence intervals from fitting model (4) to data.

Thus the conclusion is that the A_1 parameters are not significantly different. This means that the total shrinkage of a part is the same no matter what cavity the part comes from.

The estimated parameters of model (4) are displayed in Figure 6. It seems that parts from cavity 1-8 are having greater c_0 parameter than the last 8.

The plot of the fitted curve with the observations overlayed, presented in Figure 7, shows that the general trend in the data is captured well by the model. However, there is a common tendency in the deviations from the curves. This tendency is further displayed in Figure 8 where the residuals are plotted in time sequence for each cavity.

When performing the same test on the b_{1i} parameters it is concluded that they are not all the same. This means that parts from the different cavities shrink with different rates. However, usually the same shrinkage curve would be applied to all cavities. If calculations of the mean and standard deviations in the process control are based on values predicted by the model, the estimates could be seriously disturbed.

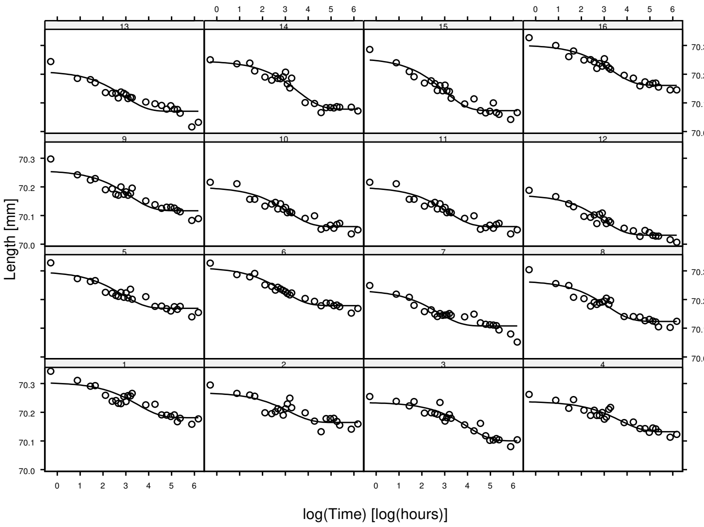

Fig. 7: Fitted curve with observations.

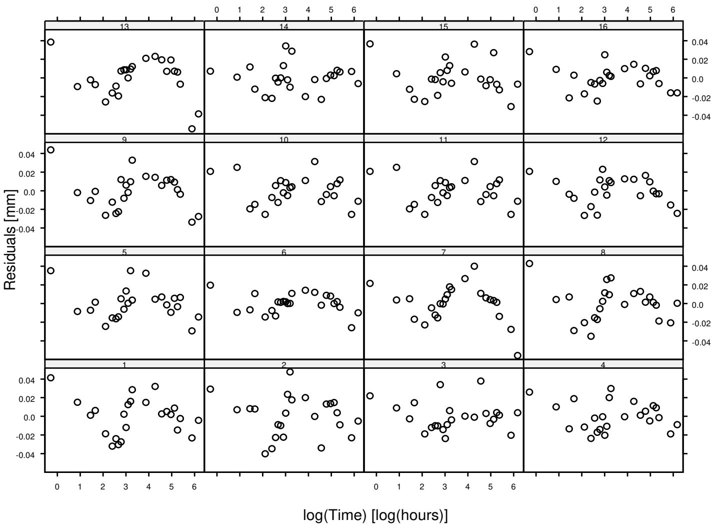

Fig. 8: Residuals in time sequence.

The difference in shrinkage rates indicate that any observed differences in part dimensions are not only due to differences in cavity steel dimensions.

4 Conclusion

In this paper shrinkage curves of parts produced from a multi-cavity mold have been analysed using non-linear regression techniques. It has been found that the total shrinkage of a part is the same, no matter what cavity produced that part. However, the rate of the shrinkage is not the same for all cavities. Thus values of the stable length predicted from 2 hour measurements risk being upset by this fact. Furthermore the difference in shrinkage rates indicate that any observed differences in part dimensions are not only due to differences in cavity steel dimensions.

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