Elastic wave propagation in anisotropic, inhomogeneous materials

–

application to ultrasonic NDT

Søren Halkjær

First edition

This thesis is submitted as a partial fulfillment of the requirements for obtaining the Ph.D. degree. The study has taken place as an industrial research Ph.D. project administered by the Danish Academy of Sciences (ATV).

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Second edition

The first edition was successfully defended at a Ph.D. defense in Februrary 2000. In this second edition, a small number of typos occuring in the first edition have been corrected.

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Simulation and implementation of ultrasound based inspection methods for materials and material combinations difficult to inspect.

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Abstract

This thesis is concerned with mathematical modelling of elastic wave propagation in anisotropic, inhomogeneous materials in relation to ultrasonic non-destructive testing (NDT). The aim of the project is the application of mathematical modelling to the challenging and difficult task of optimizing inspection procedures as well as to the study of received transducer signals from included material defects.

Due mainly to the inhomogeneous nature of the materials considered, a numerical finite-difference approach (EFIT), being explicit in the time-domain, has been chosen for implementation in a simulation program. The theory is developed for the general 3D case, but for computational reasons, only 2D problems are considered in actual simulations. Any type of elastic anisotropy and inhomogeneity distribution may be considered in a simulation as well as circular - and rectangular defects with stress-free surfaces (cracks) of arbitrary orientation and size.

The analytic solution to the 2D problem of a line source on the free surface of an anisotropic homogeneous medium (Lamb's problem) is presented and has been implemented for comparison with the numerical solution. In general, there's is a very good agreement between the results of the two methods. The analytic solution has been used for calculating the sound field from a transducer in 2D (and, intentionally, later on in 3D as a generalization). However, it turned out that the numerical method was much faster in doing this in 2D (for time durations relevant to ultrasonic NDT, i.e. microseconds).

Of main interest in ultrasonic NDT are the recorded transducer signals (A-scans) due to scattering of the elastic waves at material defects. These signals are subjected to analysis and interpretation by a qualified NDT operator. Typically, only relative signals (with respect to the reference signal from some standard defect type) are of interest, and therefore a relatively simple and computationally fast model of the transducer signal is considered as opposed to a full model of a (piezoelectric) transducer. This model is compared with another fundamentally different receiver signal model based on electromechanical reciprocity relations. The two models produce similar results and thus support the validity of the simpler approach. The numerical method EFIT together with the two A-scan models has also been compared with full 3D analytic solutions. The shapes of the A-scans are similar, although some discrepancies between the signal amplitudes exist.

An important and difficult problem in ultrasonic NDT is the inspection of anisotropic, inhomogeneous austenitic welds. A mathematical description of the anisotropic structure of such a weld is presented and has been implemented in the simulation program. Several simulations have been carried out to study the effect of the weld on the elastic wave propagation. In general it is found, that vertically polarized shear waves are heavily influenced by the weld, while horizontally polarized shear waves and longitudinal waves are much less influenced. This is in agreement with experimental observations.

Finally, as a case study of a real ultrasonic inspection situation, a material component containing an austenitic weld together with an anisotropic cladding and buttering from a power plant is considered. A material model is presented, and through a parameter study the best transducer position (and to some extent frequency) for inspection of a specified rectangular crack is determined. Such modelling activities may be of great importance in the future.

Resumé (Abstract in Danish)

Denne afhandling omhandler matematisk modellering af elastisk bølgeudbredelse i anisotrope, inhomogene materialer med henblik på ultralydbaseret ikke-destruktiv prøvning (NDT). Målet med projektet er anvendelsen af matematisk modellering til den krævende og udfordrende opgave at optimere inspektionsprocedurer samt til studiet af modtagne transducer signaler fra indlejrede materialefejl.

Hovedsaligt på grund af den inhomogene natur af de betragtede materialer er det blevet besluttet at studere og implementere en numerisk endelig-differens metode (EFIT), der er eksplicit i tiden, i et simularingsprogram. Teorien bag metoden udvikles for det generelle 3D tilfælde, men af beregningsmæssige årsager betragtes der kun 2D problemer i selve simuleringerne. Vilkårlig elastisk anisotropi samt inhomogenitet i materialet kan betragtes såvel som indlejrede cirkulære - og rektangulære fejl med trykfrie overflader (cracks) og med en vilkårlig orientering og størrelse.

Den analytiske løsning til 2D problemet omhandlende en liniekilde på den trykfrie overflade af et anisotropt, homogent materiale (Lambs problem) præsenteres. Den er endvidere blevet implementeret i et program med det formål at sammenligne dennes resultater med den numeriske metodes ditto. Generelt er der rigtig god overensstemmelse mellem de to metoder. Den analytiske løsning blev endvidere implementeret med det formål at være en hurtig og effektiv metode til beregning af lydfeltet fra en transducer i 2D (og senere i 3D). Det viste sig dog, at den numeriske metode beregnede de samme resultater langt hurtigere i 2D (for tidspunkter, der er relevant i ultralyd NDT, det vil sige mikrosekunder).

Af stor interesse for ultralyd NDT er de optagne transducer signaler (A-scans) fra spredning af lydfeltet på indlejrede materialefejl. Disse signaler er normalt genstand for analyse og fortolkning af en kvalificeret ultralydoperarør. Normalt er det kun relative signaler (i forhold til reference signalet fra en kendt standard defekt), der er af interesse. Derfor betragtes der i denne afhandling kun en relativ simpel og beregningsmæssig hurtig model af transducer signalet i stedet en omfattende model af en (piezoelektrisk) transducer. Denne model sammenlignes med en anden fundamental forskellig model baseret på eletromekaniske sammenhænge. De to modeller giver stort set identiske resulater, der understøtter rimeligheden i anvendelsen af den simple beregningsmodel. Den numeriske metode EFIT samt de to beskrevne metoder til beregning af A-scan sammenlignes derefter med fulde 3D analytiske løsninger. Formene af de to signaler udviser de samme tendenser, men der er forskelle i de beregnede amplituder.

Et vigtigt og udfordrende problem i ultralyd NDT er inspektionen af austenitiske svejsninger, der er anisotrope og inhomogene af natur. I et forsøg på at studere dette problem nærmere, præsenteres der en matematisk model af den anisotrope struktur i en sådan svejsning. Denne model er endvidere implementeret i simuleringsprogrammet. Der foretages adskillige simuleringer af lydudbredelsen i sådan en svejsning for forskellige svejsekonfigurationer for at studere svejsningens indflydelse på lydudbredelsen. Generelt kan det konkluderes, at vertikalt polariseret tværbølger påvirkes i udtalt grad, mens horisontalt polariseret tværbølger samt longitudinale bølger kun påvirkes i mindre grad. Dette er i overensstemmelse med praktiske erfaringer.

Endeligt betragtes et studie af en virkelig ultralyd NDT inspektion, der involverer en materialekompoenent med en austenitisk svejsning samt et anisotropt beskyttelseslag og overgangsmateriale fra et kraftværk. Der opstilles en materiale model for denne komponent, og igennem et parameter studie bestemmes den optimale transducer position (og tildels frekvens) til inspektion af en specificeret rektangulær fejl. Sådanne modelleringsaktiviteter forventes at være af stor vigtighed i fremtiden.

Preface

This Ph.D. project has been carried out as an industrial research Ph.D. project as administered by the Danish Academy of Technical Sciences (ATV) and financially supported by the Danish Agency for Trade and Industry under the Ministry of Business and Industry. The project has taken place in the period 1/1/1997 – 12/31/1999 and has been carried out with the following partners

- Company partner: FORCE Institute
- University partner: Department of Mathematical Modelling (IMM) at The Technical University of Denmark (DTU)
- Third party: Andrex

The following people forming the steering committee has been supervisors in the project

- Technical Director, Willy Damgaard Kristensen (FORCE Institute)
- Associate Professor, Mads Peter Sørensen (IMM, DTU)
- Director, Jens Ulrik Madsen (Andrex)

During the project the following two refereed contributions to conference proceedings have been authored

- S. Halkjær, M. P. Sørensen and W. D. Kristensen. Sound propagation in an anisotropic medium due to a line source: A comparison. L. Arkeryd, J. Bergh, P. Brenner and R. Pettersson, editors, *Progress in Industrial Mathematics at ECMI 98*, European Consortium for Mathematics in Industry (ECMI), p. 62–69, 1998.
- S. Halkjær, M. P. Sørensen and W. D. Kristensen. The propagation of ultrasound in an austenitic weld. Accepted for publication in *Ultrasonics*.

Furthermore, an internal business report analyzing industrial aspects of the project has been authored

- S. Halkjær. *Ultralydmodellering som nøgleteknologi til strategisk positionering* (in Danish). FORCE Institute.

Acknowledgments

The outcome of the project has been influenced by a number of people through valuable and inspiring interactions. In the following, the most important people together with their contributions are listed.

First, I would like to thank my advisor at IMM, Mads Peter Sørensen for always taking the time for discussions of results and answering questions in relation to the project. My advisor at FORCE Institute, Willy Damgaard Kristensen, has provided me with the necessary resources to focus entirely on the project. For this, I'm very grateful.

Industrial Postdoc at FORCE Institute, Steen Arnfred Nielsen, has offered to proofread the manuscript, and has carried out this task superbly. He has contributed with a number of valuable corrections and suggestions, and for this I am very grateful. Likewise, my supervisor at IMM, Mads Peter Sørensen is thanked for his proofreading of the thesis.

During a period of 4 months in autumn 1997 I visited Dr. Ing. E. Neumann and his group at Bundesanstalt für Materialforschung und -prüfung (BAM) in Berlin. The theoretical activities of the group are centered around plane wave propagation in anisotropic, inhomogeneous materials (ray-tracing). Much of my knowledge of elastic plane wave propagation in these materials was formed during this stay. I'm grateful to E. Neumann for making this visit possible, and especially to Dipl. Ing. Vijayendra Munikoti, whom I worked closely together with resulting in many fruitful discussions.

In March, 1999 I had the opportunity to visit Prof. A. Boström and his group at Department of Mechanics, Chalmers Technical University, Göteborg. One research activity in this group is analytic modelling of a complete ultrasonic testing situation in (an)isotropic, homogeneous materials. The so-called electromechanical reciprocity relations are used to model the relation between the elastodynamic state of an elastic medium due to a material defect and the corresponding (change of) transducer signal. During the stay I worked on adapting this relation to the numerical modelling approach used in this thesis. I'm grateful to A. Boström for giving me this opportunity and for always being ready to answer my questions. There was a nice atmosphere with the other PhD. students and it was very informative to learn about other modelling approaches to the same problem considered in this project.

At FORCE Institute, I have been working together with Peter Krarup for studying experimentally the wave propagation through an austenitic weld. I've had many fruitful discussions with Hans Ole Olsen and Kurt Viggo Rasmussen on ultrasonic inspection techniques and their point of view of the role of mathematical modelling concerning these. They have suggested a number of practical ultrasonic inspections situations which could benefit from mathematical modelling. It has been very inspiring to discuss results from the performed simulations with them. I would like to thank them for a nice working atmosphere and for always taking the time for discussions or answering my questions.

Symbols and conventions

A number of abbreviations, symbols and conventions are used throughout thesis. The definitions for the most important of these are described below.

Tensors may either be written in roman with indices indicating the order of the tensor or in bold notation. There is however a single exception to this rule. The 6×6 elastic stiffness matrix $\boldsymbol{c}$ together with the inverse matrix $\boldsymbol{s}$, called the elastic compliance matrix, are ordinary matrixes but not tensors. The 4'th order elastic tensor is always denoted c_{ijkl} and the 4'th order compliance tensor is denoted s_{klij}.

Einstein summation convention: The convention is used that whenever an index appears twice in an expression, a summation over the index is implied, i.e. $a_i b_i \equiv \sum_i a_i b_i$.

Differentiation: If a comma (,) is present in the index, a differentiation with respect to the variable following the comma is implied, i.e. $u_{,j} \equiv \partial u / \partial x_j$.

$\boldsymbol{c}$	:	6×6 elastic stiffness matrix (not a tensor)
c_{ijkl}	:	Fourth order elastic stiffness tensor
$\boldsymbol{\epsilon}$, ϵ_{kl}	:	Second order strain tensor
L wave	:	longitudinal wave
$\boldsymbol{M}$	:	6×6 transformation matrix
qSV wave	:	quasi shear vertical wave
qL wave	:	quasi longitudinal wave
qT1 wave	:	quasi transverse wave 1
qT2 wave	:	quasi transverse wave 2
SDH	:	Sidedrilled hole
SH wave	:	shear horizontal wave
SV wave	:	shear vertical wave
$\boldsymbol{s}$	:	6×6 elastic compliance matrix (not a tensor)
s_{klij}	:	Fourth order elastic compliance tensor
$\boldsymbol{\sigma}$, σ_{ij}	:	Second order stress tensor
$\boldsymbol{u}$, u_i	:	displacement vector
$\boldsymbol{v}$, v_i	:	time derivative of displacement vector
v_l	:	longitudinal wave velocity
v_t	:	transverse wave velocity
$\boldsymbol{x}$, x_i	:	spatial coordinates

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