Appendix A

Rotation matrices with Euler angles

The 6×6 matrix $\boldsymbol{M}$ entering the transformation law (2.22) has the following expression

$$
\boldsymbol{M} = \begin{pmatrix}
a_{xx}^2 & a_{xy}^2 & a_{xz}^2 & 2a_{xy}a_{xz} & 2a_{xz}a_{xx} & 2a_{xx}a_{xy} \\
a_{yx}^2 & a_{yy}^2 & a_{yz}^2 & 2a_{yy}a_{yz} & 2a_{yz}a_{yx} & 2a_{yx}a_{xx} \\
a_{zx}^2 & a_{zy}^2 & a_{zz}^2 & 2a_{zy}a_{zz} & 2a_{zz}a_{zx} & 2a_{zx}a_{zy} \\
a_{yx}a_{zx} & a_{yy}a_{zy} & a_{yz}a_{zz} & a_{yy}a_{zz} + a_{yz}a_{zy} & a_{yx}a_{zz} + a_{yz}a_{zx} & a_{yy}a_{zz} + a_{yx}a_{zy} \\
a_{zx}a_{xx} & a_{zy}a_{xy} & a_{zz}a_{xz} & a_{xy}a_{zz} + a_{xz}a_{zy} & a_{xz}a_{zx} + a_{xx}a_{zz} & a_{xx}a_{zy} + a_{xy}a_{zx} \\
a_{xx}a_{yx} & a_{xy}a_{yy} & a_{xz}a_{yz} & a_{xy}a_{yz} + a_{xz}a_{yy} & a_{xz}a_{yx} + a_{xx}a_{yz} & a_{xx}a_{yy} + a_{xy}a_{yx}
\end{pmatrix} \tag{A.1}
$$

where a_{ij} are the direction cosines as explained in section 2.4, page 19. As explained in that section, a general rotation of the crystal coordinate axes with respect to a fixed laboratory coordinate system may be described through the Euler angles (ϕ, θ, ψ). Thus, the matrix $\boldsymbol{M}$ may be considered of function of these three angles, $\boldsymbol{M} = \boldsymbol{M}(\phi, \theta, \psi)$. In the following, the matrix $\boldsymbol{M}$ is specified for a rotation around each of the tree laboraroty axes.

A rotation around the x-axis with the angle θ is described through the rotation matrix

$$
\boldsymbol{M}(\frac{\pi}{2}, \theta, -\frac{\pi}{2}) = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & \cos^2(\theta) & \sin^2(\theta) & \sin(2\theta) & 0 & 0 \\
0 & \sin^2(\theta) & \cos^2(\theta) & -\sin(2\theta) & 0 & 0 \\
0 & -\frac{\sin(2\theta)}{2} & \frac{\sin(2\theta)}{2} & \cos(2\theta) & 0 & 0 \\
0 & 0 & 0 & 0 & \cos(\theta) & -\sin(\theta) \\
0 & 0 & 0 & 0 & \sin(\theta) & \cos(\theta)
\end{pmatrix} \tag{A.2}
$$

A rotation around the y-axis with the angle θ is described through the rotation matrix

$$
\boldsymbol{M}(0, \theta, 0) = \begin{pmatrix}
\cos^2(\theta) & 0 & \sin^2(\theta) & 0 & \sin(2\theta) & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
\sin^2(\theta) & 0 & \cos^2(\theta) & 0 & -\sin(2\theta) & 0 \\
0 & 0 & 0 & \cos(\theta) & 0 & -\sin(\theta) \\
-\frac{\sin(2\theta)}{2} & 0 & \frac{\sin(2\theta)}{2} & 0 & \cos(2\theta) & 0 \\
0 & 0 & 0 & \sin(\theta) & 0 & \cos(\theta)
\end{pmatrix} \tag{A.3}
$$

Finally, a rotation around the z-axis with the angle ϕ is described through the rotation matrix

$$\boldsymbol{M}(0,0,\phi) = \begin{pmatrix} \cos^2(\phi) & \sin^2(\phi) & 0 & 0 & 0 & -\sin(2\phi) \\ \sin^2(\phi) & \cos^2(\phi) & 0 & 0 & 0 & \sin(2\phi) \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos(\phi) & \sin(\phi) & 0 \\ 0 & 0 & 0 & -\sin(\phi) & \cos(\phi) & 0 \\ \frac{\sin(2\phi)}{2} & -\frac{\sin(2\phi)}{2} & 0 & 0 & 0 & \cos(2\phi) \end{pmatrix} \tag{A.4}$$

Appendix B

SUNDT simulations

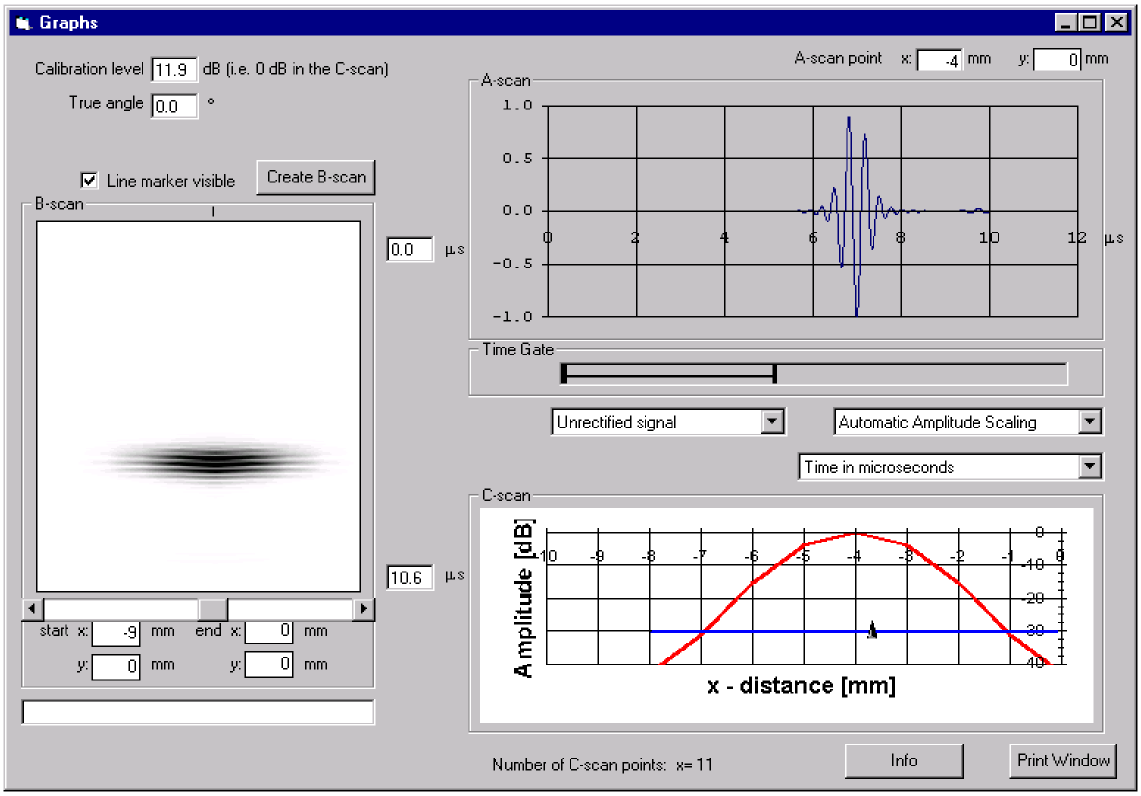

Figure B.1: SUNDT simulation of a horizontal rectangular crack. See subsection 5.3.2.

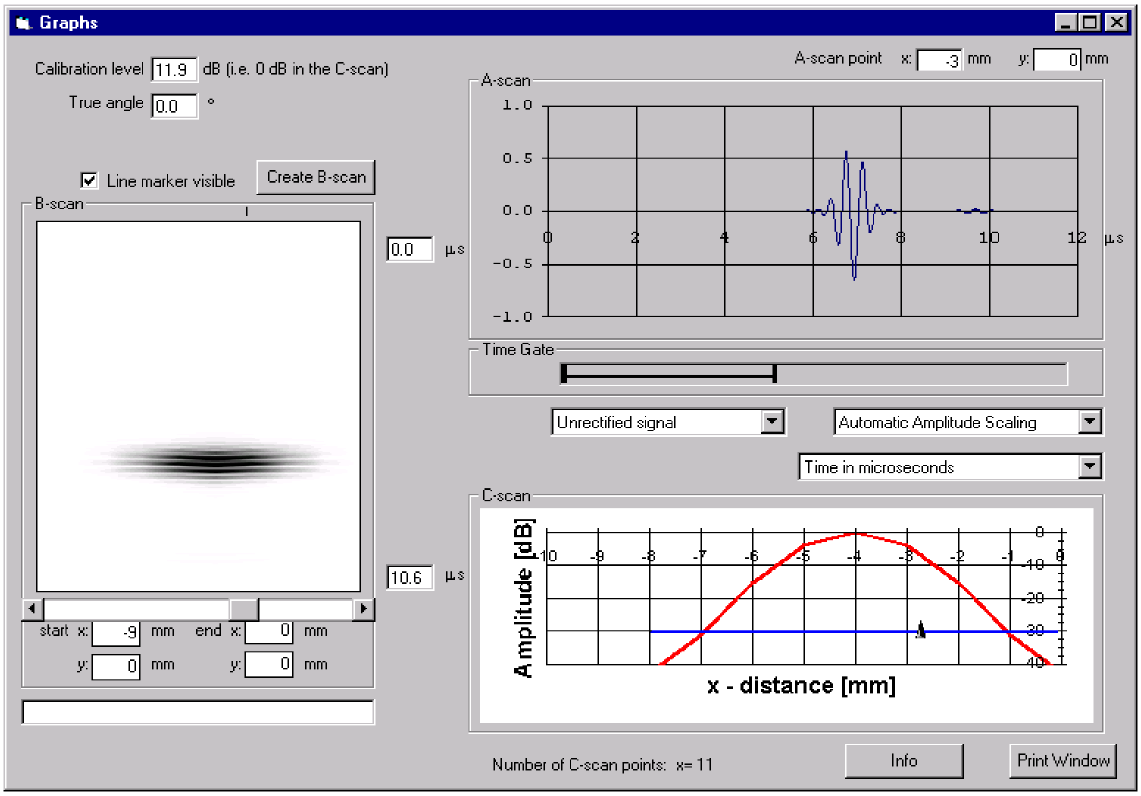

Figure B.2: SUNDT simulation of a horizontal rectangular crack. See subsection 5.3.2.

Appendix C

A-scans from case study

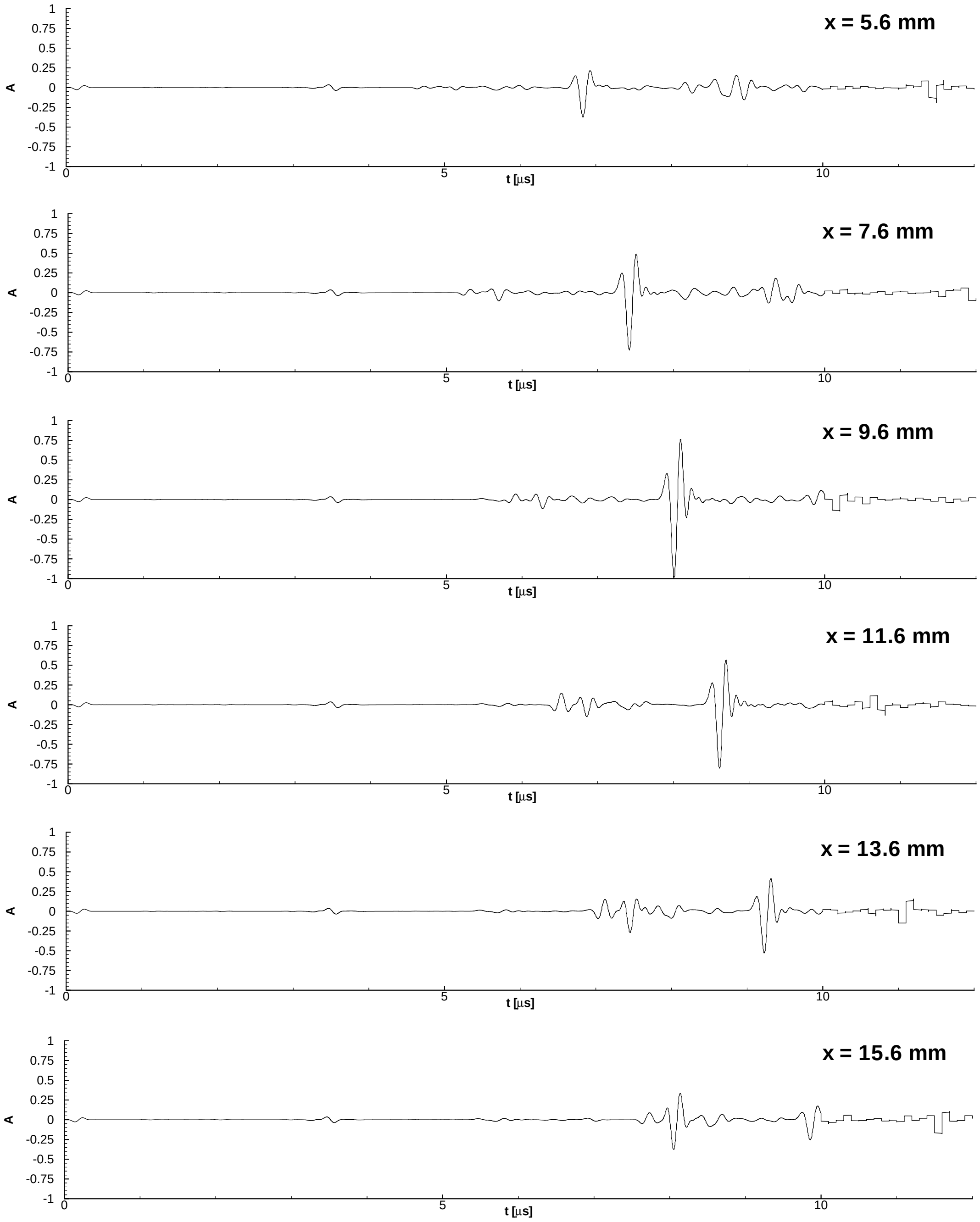

Figure C.1: Collection of A-scans recorded at different positions x on the component surface for the isotropic weld considered in section 7.2. See Fig. 7.6 for interpretation of echoes. Normalized with respect to the reference SDH.

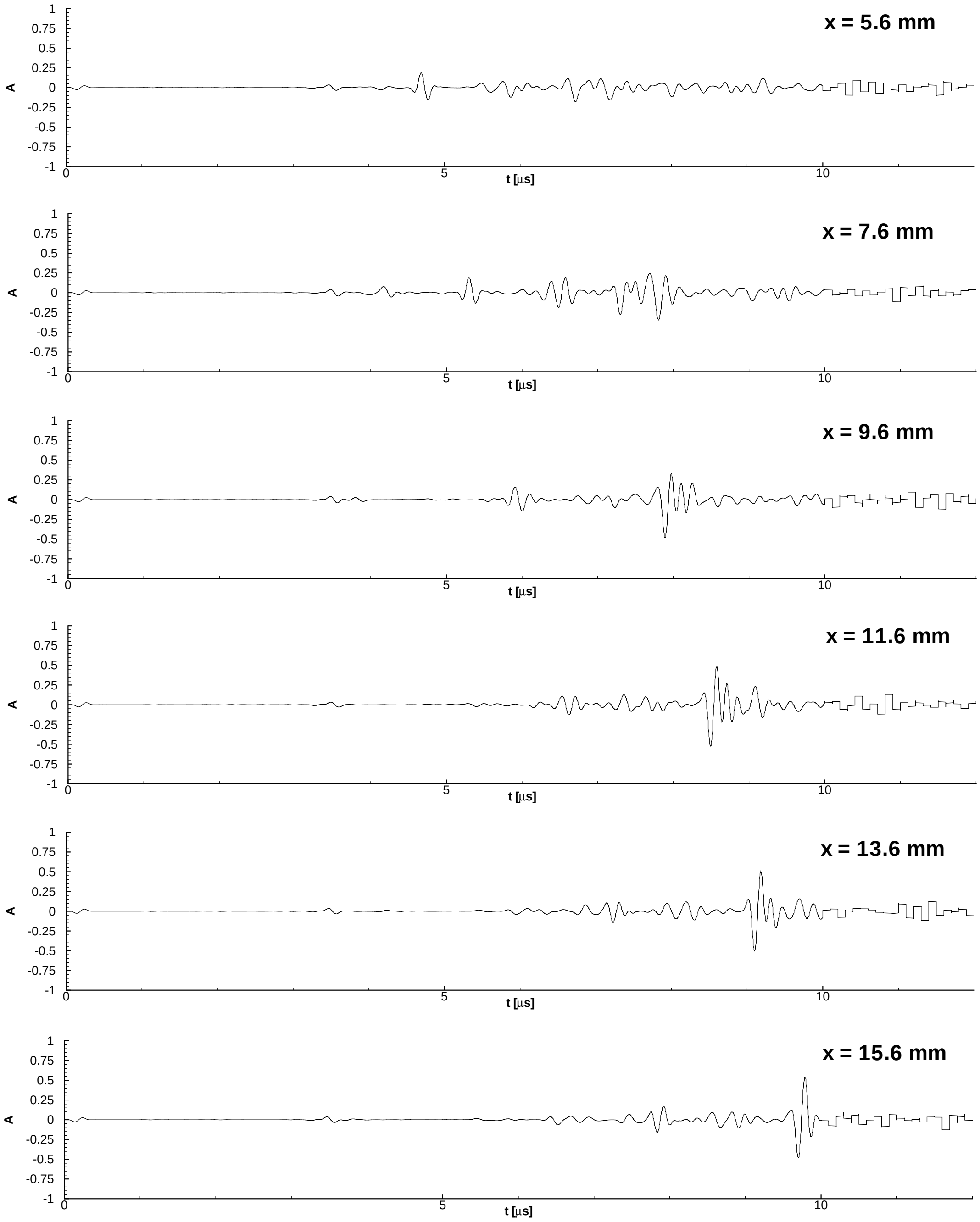

Figure C.2: Collection of A-scans recorded at different positions x on the component surface for the anisotropic weld considered in section 7.2. See Fig. 7.6 for interpretation of echoes. Normalized with respect to the reference SDH.