ANOVA tests for multiple d-primes

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Outline

One-way ANOVA

A one-way anova model has the form

$$y_{ij} = \alpha_i + \varepsilon_{ij}$$

Tests of interest:

- Any difference between α_i?
- Pairwise differences between α_i
- Comparing baseline α to the rest.

One-way ANOVA for d-primes

Conceptually:

$$y_{ij} = d'_i + \text{binomial-deviations}_{ij}$$

Formally:

$$y_i \sim \text{binom}(f_{m_i}(d'_i), n_i)$$

where

- m_i is the *method* (2-AFC, Triangle etc.)
- $f_{m_i}(d'_i)$ is the psychometric function for the ith method.
- y_i is the no. correct trials for the ith experiment.
- n_i is the total no. trials for the ith experiment.
- d'_i is d' for the ith experiment.

The common d-prime

$$d'_i = d'_c + e'_i$$

where

- d'_i is the d' from the ith experiment
- d'_c is the *common* d'
- e'_i are deviations from the *common* d'

Challenge:

- Binomial data: $(y_i,\ n_i) \rightarrow d'_i$
- What if n_i is large for some experiments, but small for others?
- What if different protocols are used?

Solution: estimate d'_c with Maximum Likelihood.

The any-differences hypothesis

$$H_0:\ d'_1 = d'_2 = \ldots = d'_n \qquad \text{versus} \qquad H_A:\ d'_i \neq d'_{i'}$$

for at least one pair of (i, i').

Examples in R

Estimation of common d'

Simple solution — a weighted average:

$$d'_{wa,e} = \sum_i w'_i d'_i$$

where $w'_i = w_i / \sum_i w_i$ are normalized weights.

Better solution: The ML estimator of d'_c:

$$\hat{d}'_c = \arg\max_{d'_c} \ell_0(d'_c; \boldsymbol{x}, \boldsymbol{n}, \boldsymbol{m}).$$

- $\boldsymbol{x}, \boldsymbol{n}, \boldsymbol{m}$: data from all experiments.

Testing the common d'

$$H_0: \; d'_c = d'_0 \qquad \text{versus} \qquad H_A: \; d'_c \neq d'_0$$

- where d'_0 is d' under the H_0.

For difference testing we want:

$$H_0: \; d'_c = 0 \qquad \text{versus} \qquad H_A: \; d'_c > 0$$

For similarity testing we want:

$$H_0: \; d'_c >= d'_0 \qquad \text{versus} \qquad H_A: \; d'_c < d'_0$$

Examples in R

All pairwise differences

Hypothesis test:

$$H_0: \; d'_i - d'_{i'} = 0$$
$$H_A: \; d'_i - d'_{i'} \neq 0$$

for some pair (i, i') where $i, i', = 1, \ldots, n$.

A *compact letter display* summarizes the pairwise tests:

1. A letter is assigned to all $i = 1, \ldots, n$ groups.
2. Two groups sharing a letter are *not* significantly different.
3. Two groups *not* sharing a letter are significantly different.

p-values are usually adjusted for multiplicity.

Difference from common d'

$$H_0: \; d'_i = d'_c \; \text{ for all } i$$
$$H_A: \; d'_i = d'_{c(i')}, \; \text{ for all } i \text{ except } i'$$

$d'_{e(i')}$ is the common d' considering all i except i'

Difference from specified value

$$H_0: \; d'_i = d'_0 \qquad \text{versus} \qquad H_A: \; d'_i \neq d'_0$$

where d'_0 is the value of d' under H_0.

Examples in R