d-prime like interpretation of standard sensory and consumer linear mixed model results.

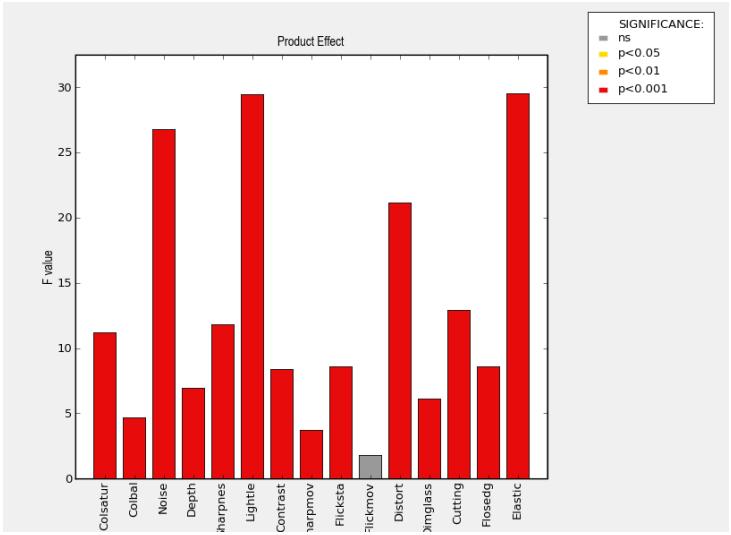
Brockhoff, P.B. Amorim, I ,Kuznetsova, A, Bech, S & de Lima, RR (2015).
delta-tilde interpretation of standard linear mixed model results. Submitted to FQP

Example for illustration: TV data (Bang and Olufsen)

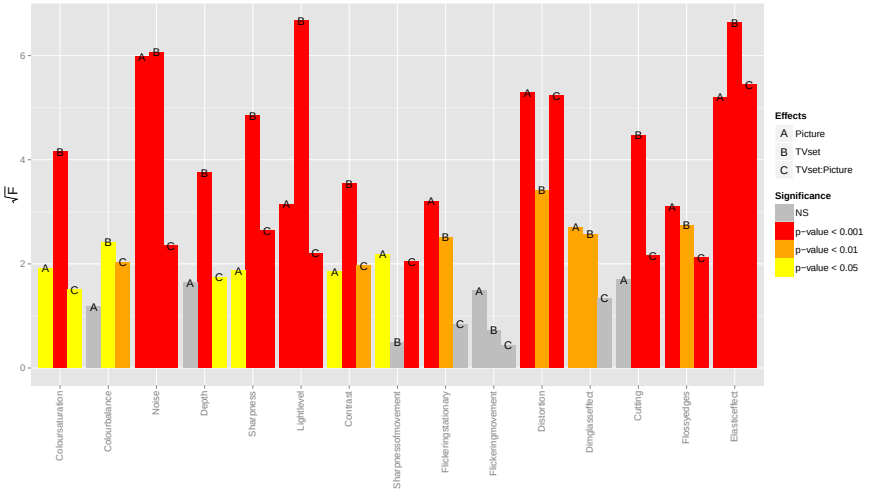
- 12 Products
  - 3 TVsets
  - 4 Pictures
- 8 Assessors
- 2 Replications
- 15 Attributes

From: Sensometrics Meeting, Norway
Kuznetsova, A., Brockhoff, P.B. & Christensen, R.H.B. (2013). lmerTest: Tests for random and fixed effects for linear mixed effect models (lmer objects of lme4 package).. R package version 2.0-0.
http://CRAN.R-project.org/package=lmerTest

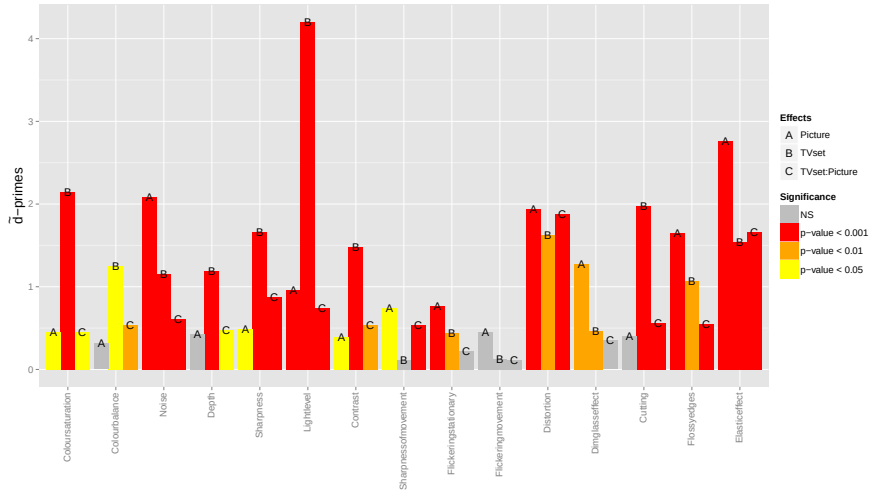
TV data, (PanelCheck) Product F plot



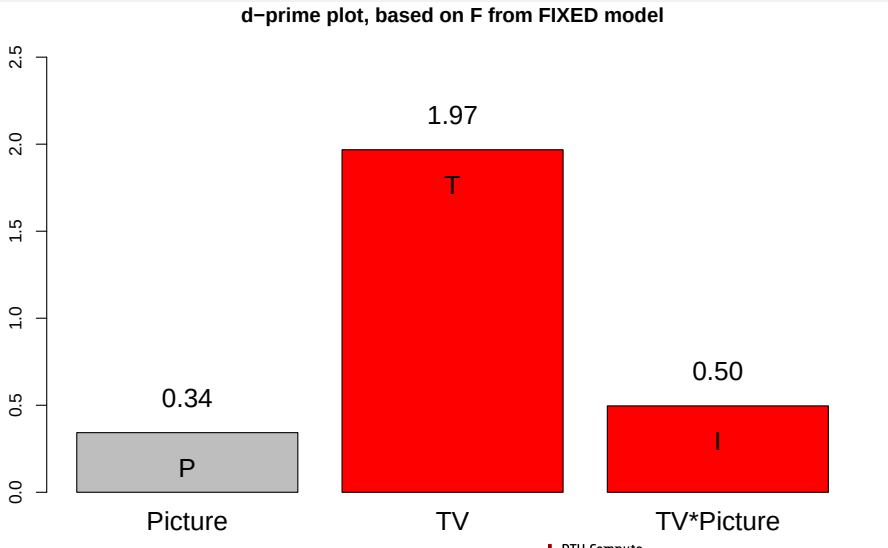
TV data, SensMixed Product root-F plot



TV data, dprime type plot



TV data, dprime type plot



TV data, Cutting, FIXED ANOVA table

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
TVset	2	365.89	182.95	124.95	0.0000
Picture	3	16.76	5.59	3.82	0.0124
TVset:Picture	6	40.52	6.75	4.61	0.0004
Assessor	7	788.01	112.57	76.88	0.0000
Picture:Assessor	21	40.84	1.94	1.33	0.1771
TVset:Assessor	14	128.01	9.14	6.24	0.0000
TVset:Picture:Assessor	42	60.33	1.44	0.98	0.5155
Residuals	96	140.57	1.46		

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Random effects

TV data, Cutting, FIXED ANOVA table

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Random effects

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Transforming Fs to d-primes:

Picture: $\tilde{d} = \sqrt{\frac{2}{48}} \cdot \sqrt{3.82 - 1} = 0.34$

TVset: $\tilde{d} = \sqrt{\frac{2}{64}} \cdot \sqrt{124.95 - 1} = 1.97$

TVset*Picture: $\tilde{d} = \sqrt{\frac{2}{16}} \cdot \sqrt{\frac{6}{11}} \cdot \sqrt{4.61 - 1} = 0.50$

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TVset*Picture: $\tilde{d} = \sqrt{\frac{2}{16}} \cdot \sqrt{\frac{6}{11}} \cdot \sqrt{4.61 - 1} = 0.50$

Main effect: $\hat{d} = \sqrt{\frac{2}{n}} \cdot \sqrt{F - 1}$

Interaction: $\hat{d} = \sqrt{\frac{2}{n}} \sqrt{\frac{DF}{K-1}} \cdot \sqrt{F - 1}$

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Transforming Fs to d-primes WITH mixed model bias correction

Picture: $\tilde{d} = \sqrt{\frac{2}{48}} \cdot \sqrt{3.82 - 1.33} = 0.32$

TVset: $\tilde{d} = \sqrt{\frac{2}{64}} \cdot \sqrt{124.95 - 6.24} = 1.93$

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Main effect: $\hat{d} = \sqrt{\frac{2}{n}} \cdot \sqrt{F_{Pic} - F_{Pic \times Assessor}}$

Interaction: $\hat{d} = \sqrt{\frac{2}{n}} \sqrt{\frac{DF}{K-1}} \cdot \sqrt{F_{Pi \times TV} - F_{Pi \times TV \times Ass}}$

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Transforming Fs to d-primes - how?

We use 3 basic relations:

ANOVA EMS

$$\sum_{i=1}^{k} (\mu_i - \bar{\mu})^2 = \sum_{i<j}^{k} (\mu_i - \mu_j)^2 / k$$

Cohen's d: $\frac{\mu_1 - \mu_2}{\sigma}$

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Transforming Fs to d-primes - how? (Main effects)

$$\begin{aligned} F &\approx \frac{n \sum_{i=1}^{k} (\mu_i - \bar{\mu})^2/(k-1) + \sigma^2}{\sigma^2} \\ &= n \sum_{i=1}^{k} \left(\frac{\mu_i - \bar{\mu}}{\sigma} \right)^2 / (k-1) + 1 \\ &= \frac{n}{2} \sum_{i<j}^{k} \left(\frac{\mu_i - \mu_j}{\sigma} \right)^2 / (k(k-1)/2) + 1 \\ &= \frac{n}{2} \text{(Average squared dprimes)} + 1 \end{aligned}$$

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d-primes, cutting, TV-effect

TV1	TV2	TV3
7.534	6.452	4.219

$$\hat{\sigma} = \sqrt{MSE} = \sqrt{1.464} = 1.210$$

$$\frac{\mu_1 - \mu_2}{1.210} = 0.89, \frac{\mu_1 - \mu_3}{1.210} = 2.74, \frac{\mu_2 - \mu_3}{1.210} = 1.85$$

$$\sqrt{\text{(Average squared dprimes)}} = \sqrt{\frac{1}{3}(0.89^2 + 2.74^2 + 1.85^2)} = 1.976$$

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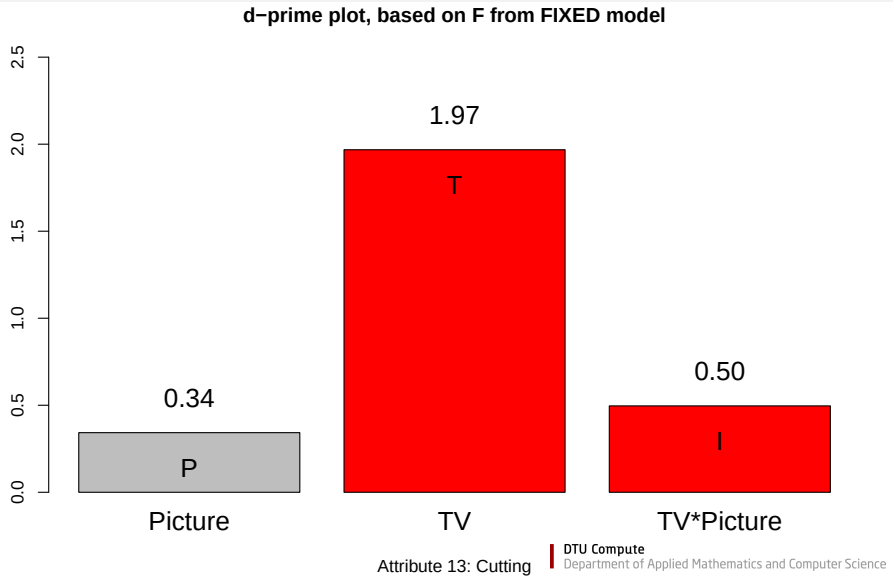
TV data, dprime plot version I



Compare with "REAL" d-prime from thurstonian model - SIMPLE example

Assessor	1	2	3	4	5	6	7	8	mean
TVset1	3.7	5.5	7.1	2.6	11.3	9.1	8.8	8.3	7.0500
TVset3	1.4	4.0	4.8	2.1	7.4	5.2	2.1	4.1	3.8875

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
TVset	1	40.01	40.01	6.38	0.0243
Residuals	14	87.85	6.27		

$$\hat{d} = \frac{7.05 - 3.8875}{\sqrt{6.27}} = 1.26$$

R-package `ordinal`

1. Categorize the data, e.g. 1-10:

Assessor	1	2	3	4	5	6	7	8
TVset1	3	5	6	2	10	8	8	8
TVset3	1	3	4	2	7	5	2	4

2. Use the ordinal package to do the thurstonian model:

Christensen,R.H.B.(2013). ordinal - Regression Models for Ordinal Data. R-package version 2013.9-30
`http://www.cran.r-project.org/web/packages/ordinal/`

REAL d-primes from thurstonian model:

```
> clm1 <- clm(Cutting_ord ~ TVset,link="probit",data = TVbo_reduced)
> round(coef(clm1),2)
    1|2     2|3     3|4     4|5     5|6     6|7     7|8     8|10 TVsetTV3
  -2.41   -1.42   -1.04   -0.68   -0.27   -0.07    0.17    1.23    -1.25
```