Weekplan: Distributed Data Structures

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References and Reading

[1] Nearest Common Ancestors: A Survey and a New Algorithm for a Distributed Environment, S. Alstrup, C. Gavoille, H. Kaplan, T. Rauhe, TOCS 2004.

We recommend reading [1] in detail.

1 Labeling Schemes for Trees Let T be a rooted tree with n nodes. Consider the following queries:

- sibling(v,w): determine if v is a sibling of w.
- adjacency(v,w): determine if there is an edge (v, w).
- ancestor(v,w): determine if v is an ancestor of w.

Solve the following exercises.

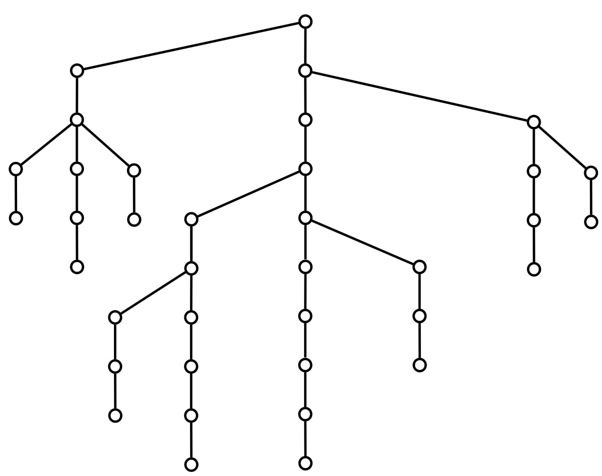
1.1 [w] Give an efficient labeling scheme for sibling queries.

1.2 [w] Give an efficient labeling scheme for adjacency queries.

1.3 Give an efficient labeling scheme for ancestor queries using labels of length $o(n)$ bits.

2 Label Length Encoding Consider the parent labeling scheme using $2\lceil \log n \rceil$ bit labels. The query algorithm assumes that we know the value $\lceil \log n \rceil$ to correctly extract the IDs. What if we do not know it? Show how to add $o(\log n)$ extra bits to the labels that will allow us to decode the label without knowning $\lceil \log n \rceil$.

3 Nearest Common Ancestor Labeling Schemes Consider the following tree T.



Solve the following exercises.

3.1 Show a heavy path decomposition of T.

3.2 Construct the labels of at least 4 non-trivial nodes in T using the $O(\log^2 n)$ bit labeling scheme.

3.3 Confirm that the scheme correctly computes nearest common ancestors for pairs of the constructed labels.

4 Query Algorithm Performance Consider efficient implementations of the query algorithms for various labeling schemes.

4.1 $[w]$ Give an efficient algorithm for the parent labeling scheme.

4.2 Give an efficient algorithm for the ID encoding nca labeling scheme.

4.3 Give an efficient algorithm for the heavy path decomposition nca labeling scheme.

5 Path Decompositions Let T be a tree with n nodes. Consider the following path decompositions.

- The *leaf-heavy decomposition* picks a *leaf heavy child* with maximum number of descendant leaves at each node and classify that as a *leaf heavy node*. The remaining nodes are *leaf light nodes*. The *leaf-lightdepth* of T is the maximum number of edges to leaf light nodes on a root-to-leaf path in T.
- The *long-path decomposition* picks a *long child* of maximum depth at each node and classify that as a *long node*. The remaining nodes are *short nodes*. The *shortdepth* of T is the maximum number of edges to short nodes on a root-to-leaf path in T.

Solve the following exercises.

5.1 Show that the leaf-lightdepth is at most $O(\log \ell)$, where ℓ is the total number of leaves in the tree.

5.2 $[*]$ What bounds can you give on the shortdepth of a tree?

6 Variable-Length Encodings Suppose a label stores the concatenation of a sequence $v_1, \ldots, v_k$ of variable length codes of total length ℓ. Show how to add $O(\ell)$ information to the label that will allow us decode the sequence.

7 Alphabetic Codes Let T be a tree with n nodes and let $h_1, \ldots, h_k$ be the heavy paths from the root of T to a node v. Consider the topmost nodes $v_1, \ldots, v_k$ on the heavy paths. Solve the following exercises.

7.1 $[w]$ Argue that $k = O(\log n)$ and $n = \text{size}(v_1) > \text{size}(v_2) > \cdots > \text{size}(v_k) > 0$.

7.2 Suppose we that for each node v_i, $1 < i \leq k$ store a code b_i of length at most

$$|b_i| \leq \log(\text{size}(v_{i-1}) - \log(\text{size}(v_i)) + O(1)$$

Show that $\sum_{i=2}^{k} |b_i| = O(\log n)$.

8 Lexicographic Comparison Let x and y be bitstrings stored in the rightmost (least significant) bits of two memory words. Given their lengths $|x|$ and $|y|$ show how to compare x and y lexicographically in constant time.

9 Range Minimum Queries Let A be an array A of n integers. Show how to preprocess A in $O(n)$ space to support the following *range minimum query* in constant time:

- $\text{RMQ}(i, j)$: Return the minimum element among $A[i], A[i+1], \ldots, A[j]$.

Hint: Find a connection to nearest common ancestors.