Segment trees

Dynamic Range Minimum Queries

Segment trees

- Dynamic RMQ: Support following operations.
  - Add(i, k): Set A[i] = A[i] + k   (k can be negative).
  - RMQ(i,j)

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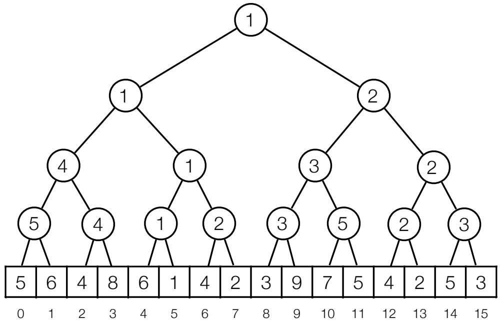
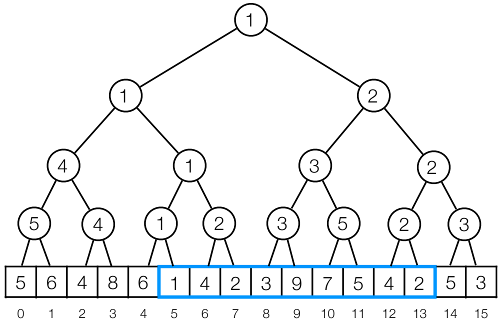


Segment trees

- Dynamic RMQ
  - RMQ(5,13) = ?



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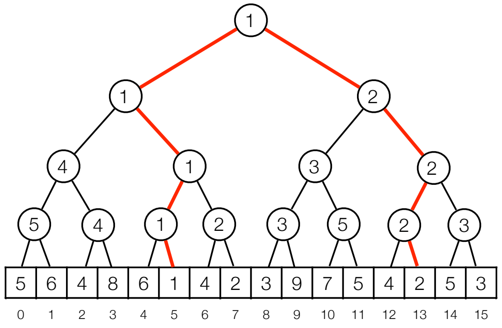
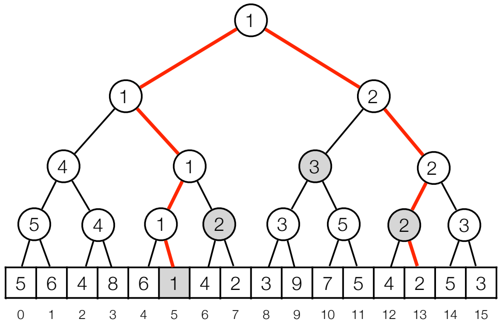


Segment trees

- Dynamic RMQ
  - RMQ(5,13) = Every interval can be composed of at most 2 log n intervals.



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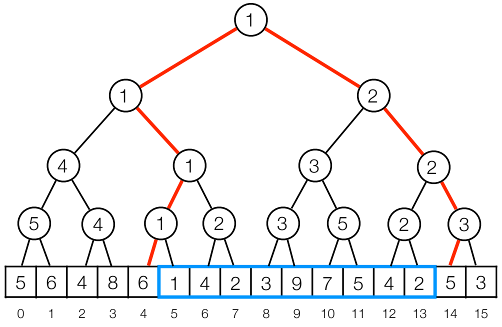

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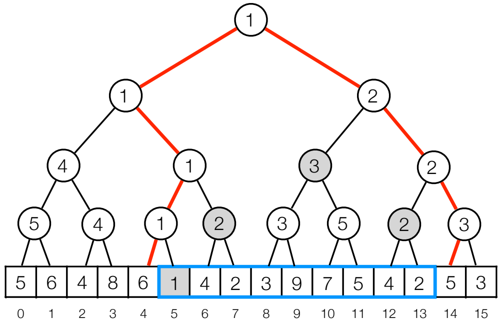
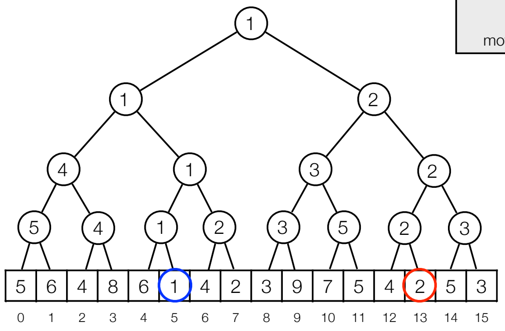


Segment trees

- Dynamic RMQ
  - RMQ(5,13) = INF

```
s = INF
while (a not right of b):
    if (a right child):
        s = min(s, tree[a])
        move a to the right
    if (b left child):
        s = min(s, tree[b])
        move b to the left
    move a and b to parents
```



0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
5	6	4	8	6	1	4	2	3	9	7	5	4	2	5	3

Segment trees

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  - RMQ(5,13) = 1

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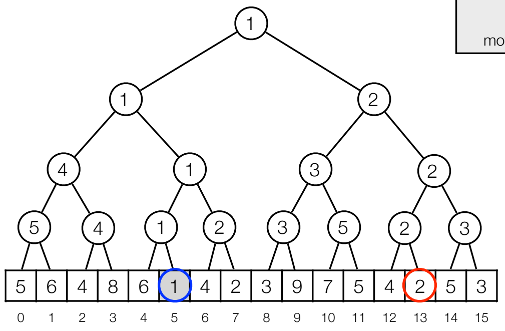

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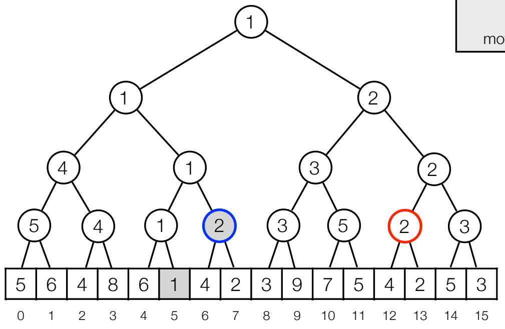

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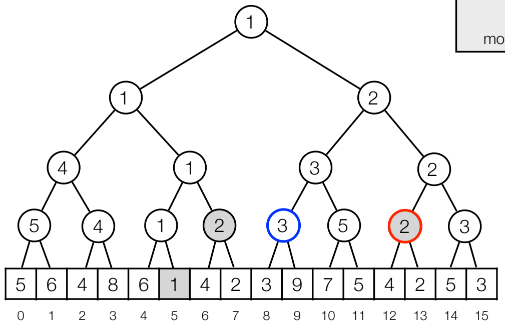

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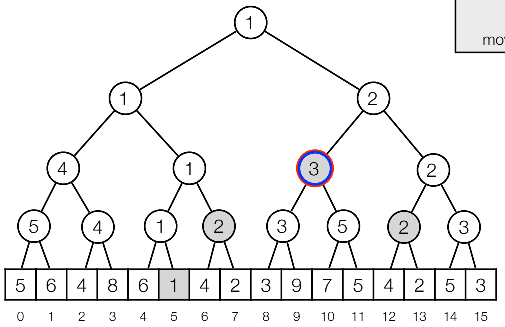

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
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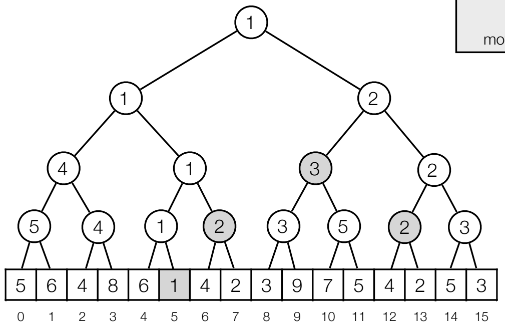

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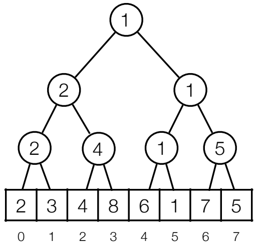
5	6	4	8	6	1	4	2	3	9	7	5	4	2	5	3

Implementation

- Implement tree using heap layout in array of length 2n:
  - Root at position 1.
  - Children of node i at position 2i and 2i+1.

```
m = INFINITY
a += n, b+= n
while (a ≤ b):
    if (a % 2 == 1):
        m = min(m, tree[a])
        a += 1
    if (b % 2 == 0):
        m = min(m, tree[b])
        b -= 1
    a = $\lfloor a / 2 \rfloor$
    b = $\lfloor b / 2 \rfloor$
return m
```



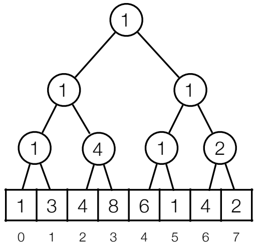
0	1	2	3	4	5	6	7
2	3	4	8	6	1	7	5

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
-	1	2	1	2	4	1	5	2	3	4	8	6	1	7	5

Space: O(n)
Time: O(log n)

Updates

- Add(5, 7)



0	1	2	3	4	5	6	7
1	3	4	8	6	1	4	2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
-	1	1	1	1	4	1	2	1	3	4	8	6	8	4	2

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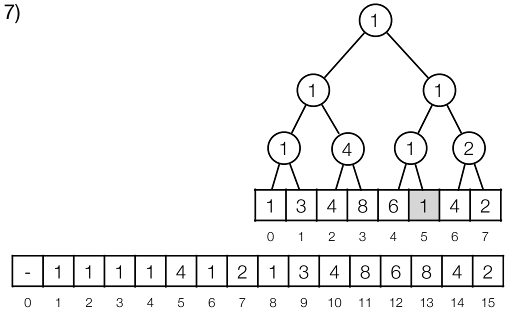

1	3	4	8	6	1	4	2
0	1	2	3	4	5	6	7

-	1	1	1	1	4	1	2	1	3	4	8	6	8	4	2
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

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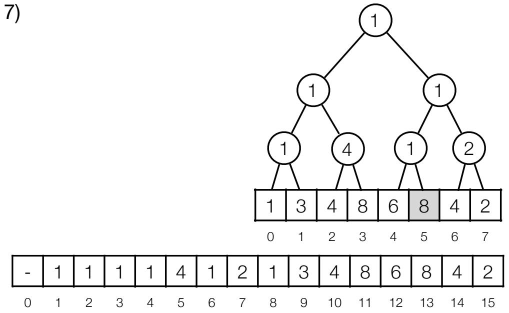

1	3	4	8	6	8	4	2
0	1	2	3	4	5	6	7

-	1	1	1	1	4	1	2	1	3	4	8	6	8	4	2
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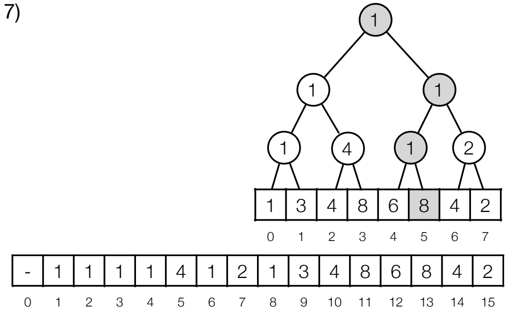

1	3	4	8	6	8	4	2
0	1	2	3	4	5	6	7

-	1	1	1	1	4	1	2	1	3	4	8	6	8	4	2
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

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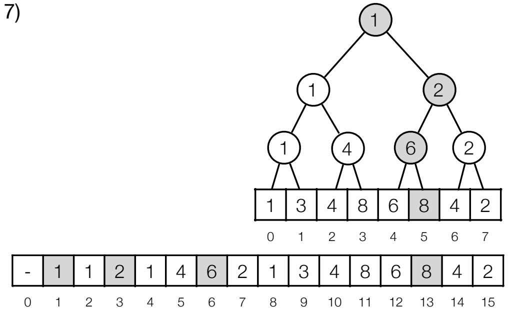

1	3	4	8	6	8	4	2
0	1	2	3	4	5	6	7

-	1	1	2	1	4	6	2	1	3	4	8	6	8	4	2
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

```
Add(i, k):
    i += n
    tree[i] += k
    i = ⌊i /2⌋
    while (i ≥ 1):
        tree[x] = min(tree[2*i], tree[2*i +1])
        i = ⌊i /2⌋
```

Time: O(log n)