02158 CONCURRENT PROGRAMMING
FALL 2019

Auxiliary Exercises
Petri Nets

Exercise Petri.1

Five boats shuttle between two jetties, A and C, via a jetty B. Jetty A and C each has a capacity of two boats, whereas jetty B has a capacity of three boats. Make a Petri Net model of the boat traffic.

Exercise Petri.2

Four actions $A$, $B$, $C$, and $D$ are to be synchronized as follows:

After execution of $A$, either $B$ or $C$ is executed. Concurrently with this, $D$ is executed. All of this is repeated forever.

(a) Draw a Petri-net in which the actions $A$ to $D$ are represented by transitions and synchronized as described above.

(b) Which pairs of actions can be executed in parallel?

(c) Which interleavings (sequences of single transition firings) are possible for the first cycle of the execution?

Exercise Petri.3

Make a Petri Net for following process:

**Root Galettes**

- 1 leek
- 2 carrots
- 1 parsnip
- 2 large potatoes
- 1 egg yolk
- (1 tsp Maizena)
- salt, pepper
- olive oil

The leek is rinsed and finely chopped. The root vegetables are peeled and cut en julienne. Mix with egg yolk and optionally Maizena (cornflour). Season with salt and pepper and fry in hot oil like small pancakes, a couple of minutes on each side. Put on absorbing kitchen paper and serve.

Note: The paste is divided into 10 portions and there are two frying pans available.

*Problem originally due to the Danish cook, Claus Meyer.*
Exercise Petri.4

Draw the Petri Net $N = (P, T, F)$ where

\[
P = \{p_1, p_2, p_3\} \\
T = \{t_1, t_2, t_3\} \\
F = \{(p_1, t_1), (p_1, t_2), (p_3, t_2), (p_2, t_3), (t_1, p_2), (t_2, p_2), (t_2, p_3), (t_3, p_1)\}
\]

with the marking $M_0 = (2, 0, 1)$ (corresponding to $(p_1, p_2, p_3)$).

Write all simultaneous firings possible from $M_0$ using the notation $M_0 \overset{U}{\rightarrow} M'$.

Exercise Petri.5

Write down the mathematical model for Figure 1.1 in [Basic].
Transition Systems

Exercise Trans.1

Find all interleavings of the two processes:

\[ P_a: \ a_1, a_2, a_3 \]
\[ P_b: \ b_1, b_2 \]

Exercise Trans.2

Assume that two processes \( P_1 \) and \( P_2 \) consist of sequences of \( n_1 \) and \( n_2 \) actions respectively. Find an expression for the number of possible interleavings of \( P_1 \) and \( P_2 \).

Exercise Trans.3

Find all interleavings of the three processes:

\[ P_a: \ a_1 \]
\[ P_b: \ b_1, b_2 \]
\[ P_c: \ c_1 \]

Exercise Trans.4

Assume that \( P_i \) is a process consisting of \( n_i \) actions. Find an expression for the number of interleavings of \( P_1, P_2, \ldots, P_k \).

Exercise Trans.5

Draw transition diagrams for the two processes in the following program:

\[
\text{var } X, Y : \text{Integer}; \\
X := 1; \quad Y := 2; \\
\text{co } X := Y + 1 || Y := X - 1 \text{ oc}
\]

Now, draw (the reachable part of) the transition graph for the full program. The nodes should be states of the form \((X, Y, t_1, t_2, \pi_1, \pi_2)\), where \(t_i\) are local variables and \(\pi_i\) are the control variables of the two processes.

From the graph, determine the possible final values of \(X\) and \(Y\).

Exercise Trans.6

Make a transition diagram for each of the two processes in Figure 2.2 in [Andrews].
Shared Variables

Exercise Share.1 (Lock-step problem)

Write, using shared variables only, to pieces of program, \( SYNC_A \) and \( SYNC_B \), that synchronize to processes \( P_A \) and \( P_B \) such that they proceed in lock-steps. More precisely, if \( op_A \) and \( op_B \) are operations in \( P_A \) and \( P_B \) respectively, then the number of times these two operations have been executed must differ by at most one.

\[
\begin{align*}
\text{process } P_A; & \\
& \text{repeat} \\
& \quad \text{SYNC}_A; \\
& \quad \ldots \ \\
& \quad \text{op}_A; \\
& \quad \ldots \ \\
& \quad \text{forever}
\end{align*}
\]

\[
\begin{align*}
\text{process } P_B; & \\
& \text{repeat} \\
& \quad \text{SYNC}_B; \\
& \quad \ldots \ \\
& \quad \text{op}_B; \\
& \quad \ldots \ \\
& \quad \text{forever}
\end{align*}
\]

Exercise Share.2

In the following program, it is attempted to establish a critical region for two concurrent processes by using two shared boolean variables \( C_1 \) and \( C_2 \):

\[
\begin{align*}
\text{var } C_1, C_2 : \text{boolean}; \\
C_1 := \text{false}; & \quad C_2 := \text{false}; \\
\text{process } P_1; & \\
& \text{repeat} \\
& \quad \text{nc}_1: \text{non-critical section}_1; \\
& \quad \text{r}_1: \text{repeat} \\
& \quad \quad \text{a}_1: C_1 := \neg C_2; \\
& \quad \quad \text{until } \neg C_2; \\
& \quad \text{cs}_1: \text{critical section}_1; \\
& \quad \text{e}_1: C_1 := \text{false} \\
& \quad \text{forever;}
\end{align*}
\]

\[
\begin{align*}
\text{process } P_2; & \\
& \text{repeat} \\
& \quad \text{nc}_2: \text{non-critical section}_2; \\
& \quad \text{r}_2: \text{repeat} \\
& \quad \quad \text{a}_2: C_2 := \neg C_1; \\
& \quad \quad \text{until } \neg C_1; \\
& \quad \text{cs}_2: \text{critical section}_2; \\
& \quad \text{e}_2: C_2 := \text{false} \\
& \quad \text{forever;}
\end{align*}
\]

(a) Draw the transition diagrams for \( P_1 \) and \( P_2 \).

(b) Show that the program does not ensure mutual exclusion.

(c) Assume that \( a_1 \) and \( a_2 \) are executed as atomic statements instead, i.e. \( a_1: (C_1 := \neg C_2) \) and \( a_2: (C_2 := \neg C_1) \).

Determine whether the algorithm now ensures mutual exclusion.
Exercise Share.3

Consider the problem of establishing a critical region using a coordinator process addressed in Andrews Ex. 3.12. A proposal for the form of the processes is:

\[
\text{process } P[i : 1..n] = \\
\text{repeat} \\
\quad \text{non critical section}; \\
\quad \text{enter}[i] := true; \\
\quad \langle \text{await } in[i] \rangle; \\
\quad \text{critical section}; \\
\quad in[i] := false \\
\text{forever}
\]

(a) Write a proposal for the coordinator process.

(b) Express mutual exclusion among \( n \) process as an invariant.

(c) State and prove some auxiliary invariants of your program that may be combined to show mutual exclusion.

Hint: What can be said about \( in[i] \) in the critical section? What is known about the state of the coordinator process when \( in[i] \) is true?

(d) Is your algorithm fair?
Theory (Safety and Liveness)

Exercise Theory.1

Consider the concurrent program:

```plaintext
var x, y : integer := 0;
co
  repeat a1: (y < 2 → y := y + 1; x := y) forever ||
  repeat a2: (x = 0 → y := 0) forever ||
  repeat a3: x := 0 forever
oc
```

Question 1.1:

(a) Prove inductively that $I \triangleq 0 \leq x \leq y \leq 2$ is an invariant of the program.

(b) Draw the (reachable part of) the transition graph for the program. Since control remains at the $a$-actions, only the $(x, y)$ part of the state needs be shown.

(c) Determine whether $\neg(x = 1 \land y = 2)$ is an invariant of the program.

Question 1.2:

(a) Argue that $\square \diamond x = 1$ holds for the program under the assumption of weak fairness.

(b) Show that $\square \diamond x = 2$ does not hold, even under the assumption of strong fairness.

Question 1.3:

(a) Assume that the action $a_2$ cannot be considered atomic as a whole. Draw the transition diagram representing $a_2$ then and show that $I$ is no longer an invariant of the program.

(b) In the original program, assume that the action $a_1$ is replaced by the refinement:

```plaintext
b_1: await y < 2; c_1: t := y; d_1: y := t + 1; e_1: (x := y)
```

where $t$ is a local integer variable.

State a predicate $H$ that implies $I$, holds initially, and is inductive for the program (i.e. strong enough to be preserved by all atomic actions).
Semaphores

Exercise Sema.1

Three processes $P_1$, $P_2$, and $P_3$ execute three operations $A$, $B$, and $C$ respectively. The operations are to be synchronized using semaphores as follows:

```plaintext
var SA, SB, SC : semaphore;
SA := 0; SB := 0; SC := 0;

process PA;
repeat
  A;
  V(SC);
  P(SA);
forever

process PB;
repeat
  B;
  V(SC);
  P(SB);
forever

process PC;
repeat
  p(SC);
  V(SC);
  C;
  V(SA);
  V(SB);
forever
```

Draw a Petri net in which the operations $A$, $B$, and $C$ are synchronized the same way as in the above program. In the net, the operations must occur as transitions.

Exercise Sema.2

Recall the problem and solution to Exercise Petri.2.

Now, the four operations/actions are to be executed by four sequential processes $P_A$, $P_B$, $P_C$, and $P_D$ respectively. Write a program using semaphores to synchronize the four processes such that the operations $A$ to $D$ are synchronized as in the Petri-net. (The choice of which operation to execute, $B$ or $C$, need not be fair, and can be left to the semaphore mechanism.)

Exercise Sema.3

The meeting problem (barrier problem, lock-step problem) for two processes has been solved in Section 3.6 in [Basic] using general semaphores.

(a) Show that this solutions does **not** work with binary semaphores.

(b) Solve the meeting problem for two processes using binary semaphores only. Use the semaphore invariant to show that binary semaphores are sufficient.

Exercise Sema.4

Solve the meeting problem for three processes using semaphores.

Exercise Sema.5

Write a piece of code $SYNC_i$ that solves the meeting/barrier problem for an arbitrary number of processes $N$ using semaphores only.
Monitors

In all the below exercises you should assume Signal-and-Continue (SC) semantics of condition queues unless otherwise stated.

Exercise Mon.1

Write a monitor with two procedures $SYNC_A$ and $SYNC_B$ to be used by two processes $P_A$ and $P_B$ respectively. The monitor should synchronize the two processes, i.e. make them meet/wait for each other.

Exercise Mon.2

Now, the above problem is generalized to making $N$ processes meet before any of them can proceed (also known as the barrier problem). Write a monitor with a single procedure $SYNC$ to be used by all the processes for the synchronization.

Exercise Mon.3

The general meeting problem from the preceding exercise is now to be modified such that the $N$ processes not only meet but also “share the loot”. I.e. each process comes with a number (given as a parameter to $SYNC$) and get the mean value of all numbers back (as a return value). Write a monitor that solves this problem. Beware that due to the SC semantics, processes may call the monitor again before all have got their share of the loot.

Hint: Use an extra ”pre-queue” where processes may be delayed while the sharing takes place.

Exercise Mon.4

Write a monitor with two operations $sleep$ and $wakeup$ that implements the synchronization mechanism of Andrews Ex. 4.6.
Exercise Mon.5

Let $M$ be a positive constant. Consider the following specification of a *chunk semaphore*:

```
monitor ChunkSem;
var s : integer := 0;
procedure V() : (s = 0 → s := s + M);
procedure P() : (s > 0 → s := s - 1);
end;
```

(a) Implement the monitor.

(b) State and argue for a monitor invariant expressing the range of the variable $s$.

(c) State and argue for a monitor invariant expressing that calls of $P()$ do not wait unnecessarily.

(d) Suppose that $M$ is small compared to the number of processes that may call $P()$. Does your solution avoid unnecessary wakeups? If not, try to minimize the wakeups. Is the property from (c) still a monitor invariant? If not, try to remedy this.

(e) Determine if your monitor (and invariants) would be robust towards *spurious wakeups*.

(f) Describe how you would have to implement the monitor in Java.
CSP

From Concurrent Programming Exam, June 1994 (4-hours)

PROBLEM 3  (approx. 15 %)

Three CSP processes $P_1$, $P_2$, and $P_3$ perform three operations $A$, $B$, and $C$ respectively. The operations are to be synchronized which is accomplished by communication among the processes:

process $P_1 =$ repeat $P_2!();$ $A$ forever

process $P_2 =$ repeat if $P_1?()$ → skip $P_3?()$ → skip fi: $B$ forever

process $P_3 =$ repeat $P_2!();$ $C$ forever

Question 3.1:

Draw a Petri-net in which the three operations $A$, $B$, and $C$ are synchronized the same way as in the CSP program. In the net, the operations should be represented by transitions.

The operations are now to be executed by three sequential processes $P_A$, $P_B$, and $P_C$:

process $P_A =$ repeat $A$ forever

process $P_B =$ repeat $B$ forever

process $P_C =$ repeat $C$ forever

Question 3.2:

Show how semaphores can be used to synchronize the three processes such that $A$, $B$, and $C$ are synchronized in the same way as in the CSP program.
Rendezvous

Exercise Rendez.1

A monitor-implementation of a semaphore-like mechanism is given below. In the monitor, \textit{posinteger} is the type of all positive (\(> 0\)) integers.

\begin{verbatim}
monitor Event
    var S : integer := 0;
    Q : condition;

procedure Pass;
    if S = 0 then wait(Q)
procedure Clear(var r : integer);
    r := S;
    S := 0
procedure Release(v : posinteger);
    S := S + v;
    signal_all(Q)
end
\end{verbatim}

(a) Define a module with the same interface as \textit{Event} and implement it using rendezvous. Make sure that you get an effect similar to \textit{signal\_all} in \textit{Release}.

(b) The \textit{Event} monitor corresponds to the semaphore mechanism found in the (now bygone) operating system OS/2. Show how to use (an instance \textit{e} of) \textit{Event} to implement a classical semaphore \textit{s}. Hint: \textit{V}(s) can be implemented simply as \textit{e.Release}(1).
Deadlocks

From Concurrent Programming Exam, December 1998 (4-hours)

PROBLEM 4  (approx. 10 %)

In a system there is one instance of a resource type $A$, two instances of a type $B$, and three instances of a type $C$. The resources are used by four processes $P_1$, $P_2$, $P_3$, and $P_4$. The processes have declared their maximal resource demands as shown below. Furthermore, it is shown which resources have been allocated and which are requested at a certain moment.

<table>
<thead>
<tr>
<th></th>
<th>Max</th>
<th>Allocation</th>
<th>Request</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$A$</td>
<td>$B$</td>
<td>$C$</td>
</tr>
<tr>
<td>$P_1$</td>
<td>1</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>$P_2$</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$P_3$</td>
<td>1</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>$P_4$</td>
<td>0</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

Question 4.1:

Draw a resource allocation graph corresponding to this situation. In the graph, the expected, not yet requested, resource needs should be indicated by dashed arrows.

Question 4.2:

(a) Show that the situation is safe.

(b) Determine whether $P_4$ can be granted the requested $C$-instance according to the banker’s algorithm.
FURTHER SELECTED EXAM PROBLEMS

From Concurrent Systems Exam, December 2002 (4-hours)

PROBLEM 3  (approx. 15 %)

Below, a monitor implementation of a synchronization mechanism \textit{Gate} is shown. The gate may be opened or closed by an operation \textit{Set}. Processes call \textit{Pass()} to pass the gate and have to wait if the gate is closed. A special operation \textit{Go}(k) lets up to \textit{k} of the currently waiting processes pass through the gate.

\begin{verbatim}
monitor Gate
    var open : boolean := false;
    Queue : condition;

    procedure Pass() {
        if ¬open then wait(Queue);
    }

    procedure Set(b : boolean) {
        open := b;
        if open then signal\_all(Queue);
    }

    procedure Go(k : integer) {
        for j in 1..k do signal(Queue);
    }
end
\end{verbatim}

Question 3.1:

(a) Define a predicate \textit{I} expressing that calls of \textit{Pass()} do not wait unnecessarily.

(b) Argue that \textit{I} is an invariant of the monitor.

(c) Describe the effect of \textit{Go}(k) if there are less than \textit{k} calls of \textit{Pass()} currently waiting.

Question 3.2:

The functioning of the given monitor \textit{Gate} is now to be implemented by a module with the following specification:

\begin{verbatim}
module Gate
    op Pass();
    op Set(boolean);
    op Go(integer);
end
\end{verbatim}

Write a server process for the module \textit{Gate} that services the operations by rendezvous in such a way that it functions like the given monitor \textit{Gate} as seen from the calling processes.
From Concurrent Systems Exam, December 2003 (2-hours)

PROBLEM 2  (approx. 30 %)
The questions in this problem can be solved independently of each other.

Question 2.1:
A process $P$ uses three shared integer variables $x$, $y$, and $z$. The variable $x$ is both read and written by other processes, whereas $y$ and $z$ are only read by other processes. Determine which of the following statements in $P$ can be considered to be atomic.

\begin{align*}
  a: & \quad x := x + 1 \\
  b: & \quad x := y + 1 \\
  c: & \quad y := x + 1 \\
  d: & \quad y := y + 1 \\
  e: & \quad x := y + z \\
  f: & \quad z := y + z
\end{align*}

Question 2.2:
A concurrent program is given by:

\[
\begin{align*}
  \text{var} & \quad x, y : \text{integer} := 0; \\
  \text{co} & \quad x := y + 1 \parallel \{ y := x + 2 \}; x := 2 \text{ oc}
\end{align*}
\]

(a) Draw a transition diagram for each process.

(b) Determine all possible final states $(x, y)$ of the program.

Question 2.3:
Let $x$ and $y$ be integer variables. Determine which of the predicates $P$, $Q$, and $R$ are preserved by which of the actions $a_1$, $a_2$, and $a_3$, respectively:

\begin{align*}
  P & \triangleq x + y \geq 0 & a_1: & \quad y := 0 \\
  Q & \triangleq 0 \leq y \leq x & a_2: & \quad \{ y < 0 \rightarrow y := x + 1 \} \\
  R & \triangleq x \neq y & a_3: & \quad \{ y = 0 \rightarrow x := 0 \}
\end{align*}

Question 2.4:
Let $x$ and $y$ be integer variables and let the temporal logic formula $F$ be defined by:

\[
F \triangleq (\square y > x \geq 0) \land (\square \Diamond x = 0) \land (x = 0 \rightarrow x \neq 0)
\]

(a) Let states be given by pairs $(x, y)$. Give an example of an execution for which $F$ holds. The execution should be given as a short sequence of states which is repeated forever.

Now, consider each of the following actions within a program:

\begin{align*}
  a_1: & \quad \{ \text{await} \ x = 0 \} \\
  a_2: & \quad \{ \text{await} \ y > 1 \} \\
  a_3: & \quad \{ \text{await} \ x = 0 \lor y > 1 \} \\
  a_4: & \quad \{ \text{await} \ x = 0 \land y > 1 \}
\end{align*}

Assume that control has reached the particular action and that $F$ is valid for the program.

(b) Determine which of the actions will be eventually executed assuming weak fairness.

(c) Determine which of the actions will be eventually executed assuming strong fairness.
From Concurrent Systems Exam, December 2003 (2-hours)

**PROBLEM 3**  (approx. 20 %)

Let $N$ be a positive integer. The server-based module $Batch$ given below implements a synchronization mechanism that “collects” a batch of $N$ items provided by calls of $put()$ which may then be “removed” by a call of $unload()$.

```plaintext
module Batch
  op put();
  op unload();
body
  process Control;
    var count : integer := 0;
    repeat
      while count < N do
        in put() → count := count + 1 ni;
        in unload() → count := 0 ni;
      forever;
  end Batch;
```

**Question 3.1:**
Assume $N = 3$. Suppose that, concurrently, $unload()$ is called by two processes and $put()$ is called by five processes. Assuming no further calls, describe the overall effect of these seven calls.

**Question 3.2:**
Now, the module $Batch$ is to be replaced with a monitor which provides the same operations and behaves in the same way. Write such a monitor.
From Concurrent Systems Exam, December 2004 (2-hours)

PROBLEM 3  (approx. 25 %)

The server-based module $Latch$ given below implements a simple synchronization mechanism. The latch maintains a non-negative count which may be set to some value by $set(k)$ and decremented by $down()$. The operation $await()$ returns only when the count has reached zero.

```verbatim
module Latch
  op set(integer);
  op down();
  op await();
body
  process Control;
    var count : integer := 0;
    repeat
      in set(k : integer) → if k ≥ 0 then count := k
      [] down() → if count > 0 then count := count − 1
      [] await() and count = 0 → skip
    ni
    forever;
  end Latch;
```

Question 3.1:

The module $Latch$ is to be replaced with a monitor which provides the same operations and behaves in the same way. Write such a monitor.

Question 3.2:

In this question we consider $Latch$ to be a type of which distinct instances can be declared. A number of worker processes $P_1, P_2, \ldots, P_n$ need to establish a barrier to synchronize their rounds of work. The barrier is to be implemented by four $Latch$-instances and a coordinator process $Q$.

The code for the processes is shown below:

```verbatim
var latch1, latch2, latch3, latch4 : Latch;
latch1.set(n);  latch2.set(1);
process P[i : 1..n];
  repeat
    do work_i;
    SYNCHRONIZE:
      latch1.down();
      latch2.await();
      latch3.down();
      latch4.await();
  forever;
process Q;
  repeat
    ;
  forever;
```

Write the body of the coordinator process $Q$ such that the given SYNCHRONIZE code implements barrier synchronization for $P_1, P_2, \ldots, P_n$. 


From Concurrent Systems Exam, December 2006 (4-hours)

PROBLEM 2 (approx. 25 %)

In a system, a number of operations \( A_1, A_2, \ldots, A_n \) with corresponding successor operations \( B_1, B_2, \ldots, B_n \) \((n \geq 1)\) plus an operation \( C \) are to be executed the following way:

\((\ast)\) \( A_1, A_2, \ldots, A_n \) are executed concurrently. When \( A_i \) is has finished, the corresponding \( B_i \) is executed. As soon as all of \( A_1, A_2, \ldots, A_n \) have finished, \( C \) can be executed concurrently with the \( B \)-operations. When all the operations \( B_1, B_2, \ldots, B_n \), and \( C \) have finished, the execution starts all over again.

Question 2.1:
For a system with \( n = 2 \), draw a Petri Net in which the five operations \( A_1, A_2, B_1, B_2, \) and \( C \) are synchronized as described by \((\ast)\). In the net, the operations should be represented by transitions.

Question 2.2:
The operations are to be executed by \( n \) sequential processes \( P_1, P_2, \ldots, P_n \) plus a sequential process \( Q \). These processes have the form:

\[
\text{process } P[i : 1..n]; \\
\text{repeat} \\
A_i; \\
B_i; \\
\text{forever}
\]

\[
\text{process } Q; \\
\text{repeat} \\
C; \\
\text{forever}
\]

Show how to synchronize these processes using semaphores so that the operations \( A_1, \ldots, A_n, B_1, \ldots, B_n \) and \( C \) become synchronized as described by \((\ast)\).

Question 2.3:
The processes \( P_1, P_2, \ldots, P_n \) and \( Q \) are now to be synchronized using a monitor \textit{Sync}.

\[
\text{monitor Sync} \\
: \\
\text{end}
\]

\[
\text{process } P[i : 1..n]; \\
\text{repeat} \\
A_i; \\
B_i; \\
\text{forever}
\]

\[
\text{process } Q; \\
\text{repeat} \\
C; \\
\text{forever}
\]

Write a monitor \textit{Sync} providing appropriate synchronization procedures and show how these procedures are to be used by the processes \( P_i \) \((i = 1..n)\) and \( Q \) such that the operations \( A_1, \ldots, A_n, B_1, \ldots, B_n \) and \( C \) are executed as described by \((\ast)\).